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Electric Circuits and Machines

by

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AND

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SECOND EDITION

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PREFACE TO SECOND EDITION

To the Instructor

Prior to 1932, electrical and mechanical engineering students in the University of California at Berkeley were together during the junior year in classes on electrical subject matter. Since that time they have had separate courses, the mechanicals having been given their work substantially as in this book with the requirement that they receive the same depth of training as though they were electricals. These students take a full year of electrical engineering—three hours each week in class and an associated weekly laboratory of three hours. The aim of the course has been to give these students a strong minor in electrical engineering; hence the book has been written for the electrical as well as for the student in another engineering field.

For the first few years, the general subject material in the book appeared in a 200-page mimeographed syllabus. From 1941 to 1949 it was published in an expanded photolithed book, and since that time it has been in printed form. The revision has involved a survey of the entire book. Changes in the discussion of certain subject matter aim at a clearer and better presentation. A few sections have been added in appropriate places so as to bring the book technically up to date. An example is the inclusion of transistors and transducers. Problems have been added throughout the book with some of the older ones replaced or revised in view of experience.

The revision adds particularly an equivalent chapter (Appendix I) on the magnetic circuit, a subject requested by other instructors. The presentation is appreciably advanced over that of elementary physics and includes a modern discussion of the magnetic circuit as well as the application. It also provides a basic introduction to field theory. Attention has been given in the revision to suggestions of other instructors outside the University of California who have used the book and the authors greatly appreciate their helpful comments.

The general scope of the book includes the major subjects of circuits, machines, electronics, and instruments. The book has been organized so that a variety of presentations can be devised to meet the objectives of particular groups of students. Experience has shown more than sufficient subject material for a full-year course and the authors have never attempted to cover all sections. Depending upon the goal of the instructor and the time available, some sections may be given merely as reading assignments and others omitted. Considerable stress is placed on single-phase circuit theory because it is fundamental to studies on polyphase circuits, electronics, machines, instrumentation, and also to problems on dynamics and vibration in other fields of engineering.

The authors normally conclude polyphase circuit theory by the end of the first semester. In the second semester it has been found optional whether the work start with machines or with electronics. The choice may depend upon the stress and amount of time the instructor wishes to give these components. Laboratory experiments have been easily regrouped to accompany the classroom work.

In general both reading and problem assignments are given each class period for discussion in the subsequent meeting. Much of the classroom instruction is done through problems. In handling problems, strong emphasis is given the approach—the principle or principles to be applied, the character of the answer desired, where to start, and the selection of an appropriate pathway to the solution.

To the Student

The aim of this book is to present you with those few basic concepts that underlie electrical circuits, machines, and electronics, and to point the way to your use of them in practical problems. To this end the problems in the book have been taken from practice so far as possible and therefore are of the nature you may meet in the field after you leave college. The emphasis of the book is placed on the alternating-current circuit, particularly single-phase, because voltages and currents are variable far more than they are steady. The direct-current system and, in general, electronic circuits are but special cases of the single-phase circuit, the polyphase system is only an extension, and the electrical machine is merely an application.

The student who masters the techniques of electric circuit analysis has acquired a more powerful tool than, at first, he might realize. These techniques are applicable directly to the solution of problems involving mechanical or acoustical systems. The methods used today in the treatment of mechanical vibration problems are exactly those that have been developed for alternating-current circuits.

Very little, if any, subject matter in undergraduate engineering is complex, and electrical engineering is no exception. But a handful of physical principles underlie all fields, and you should approach each division with the thought that your study is a further means of viewing them all, and of seeing them at a different angle. The stronger your grasp of the few general fundamental concepts of engineering, the less is your need for details and the less also is there any urge for you to memorize. In engineering practice no two problems ever are exactly the same, not too often do they pertain to but one field, and seldom are they so simple as are required for illustrative examples in the first exposition of a subject. For these reasons, it is all the more important that you train yourself to see engineering broadly, to observe ideas and methods, and to realize that the things you learn in any of the various divisions have application to them all.

You have learned by now that to obtain any lasting knowledge on any subject you have to teach yourself to a great extent even though you have instructors and textbooks to guide you. Considerable responsibility for your college studies has been placed on your shoulders and you alone will have to carry it. In order to assist you, several suggestions are given here. In the first place, you must *study* any textbook, thinking as you go along. Merely reading a book is not sufficient because there is a distinct difference between reading and studying. While speed-reading of fiction and popular stories may be quite acceptable, use your own judgment on its application in studying technical literature. For a new assignment by your instructor it is well to read the subject matter through somewhat rapidly for the over-all viewpoint, the manner of approach, new terminology, and general methods of calculation. But then start back and *study* the material as you read it over again.

Take advantage of the opportunity to teach yourself by completing the details of an analysis when intermediate steps have been omitted. Unless these small gaps are completely apparent, bridge them yourself, either by an indicated procedure accompanying the development or by your own methods.

Extensions of various developments have been left purposely for you to fulfill. This gives you opportunity to enlarge upon the treatment, to participate along with the authors, and to obtain a better understanding of the concepts involved. The greater your scope of the work the simpler will be the assignments of your instructor. In the same way that corollaries to a geometric theorem serve to strengthen and to fix the theorem itself, so you will discover in expanding upon a subject that the many related ideas help to establish and make clearer the basic procedure.

Whenever an example is given, make the actual calculations yourself in order to tie down the theory and the method of computation that is being illustrated. In addition, perhaps, carry through a parallel example with somewhat different numbers or slight change of circuit. You then become your own instructor as well as student. In time you will gain assurance on the practicality of a circuit you may devise, or representative magnitudes of its elements. In the meantime, your instructor will discuss with you questions on these items.

Many of the phases of discussion on the main stems of subject matter in the book have been placed purposely in the problems so as to relieve the context of detail and to allow you the opportunity of extending the theoretical developments. Again, some problems are given for your instructor to discuss with you and to expand upon in view of his experience. He will recognize these and take them into consideration in his assignments.

In solving any problem, view it first from the principles on which it rests not on the numbers that attend it. It cannot be overly stressed that understanding an idea and recognizing its application are of major concern in engineering. Numerical accuracy always is presumed, of course. Divorce the problem from all so-called formulas and think it through in terms of its physica aspects. You should not start any problem solution for quantitative result until you have done this. A few minutes on the qualitative analysis of a problem before you write down any equations with numbers usually will save you far more than that amount of time after you begin the numerical work.

As questions arise on a statement in the textbook or on a problem, write each question out in words. You will find pleasantly surprising the extent to which the clarification required of you in stating the question properly forces your attention on the major points involved and, in a large number of instances, leads you to the answer. Further, writing out your question in order to fix the kernel of it in your mind, or to bring it to the instructor, or to raise it in class discussion, is training in orderly thinking.

In general, there is more than one way to solve a problem, and consequently you usually have means of checking your results. A second approach should start from the beginning and not at some point part way through the solution after a false premise or a bad calculation has been introduced. Ability to recognize other forms of solution comes through training that you can give yourself. Observe the manner in which your instructor discusses the subject and the problems from different angles.

At the end of any solution ask yourself whether the results appear reasonable in view of the initial data and the general character of the problem. This is a necessary part of good engineering, and since you cannot escape it in the field you should begin now to develop that ability to size up your final answer. Do not be content simply with some ultimate numbers. Also, after you have completed a problem summarize it in your mind to fix its principles and procedures. A few minutes of retrospection on one problem may be of greate value than the time spent in carrying through another.

Because of time, parts of the book may be omitted by your instructor, thes sections then to be viewed as reference material should you ever need them Again, it is not intended that you attempt every problem in the book. Man have duplicate features, and a judicious selection of one or two in a group of lik problems is sufficient. However, it is well to read the others to see the natur of differences that may appear or to observe the character of the application Understand that practice problems are the means by which you test yourse in applying knowledge of the subject matter.

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University of California
January, 1957

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CHAPTER I

ELECTRICAL POWER

1.1. Use of Electrical Power. Although electrical power does not exist in nature as a primary source, a large percentage of the total power used in the United States appears at some stage of its use in the electrical form. Ease of generation, ability to send it over long distances, high efficiency in transmission and in the operation of electrical devices, adaptability in its use, flexibility, and low cost are chiefly responsible for its wide application. Electric motors in industries in the United States, for example, represent approximately 45 per cent of the total installed horsepower. Further, the cost of the electrical energy required to drive these motors amounts to less than 25 per cent of the total cost of fuel and power consumed in the industries.

Applications of electrical power lie at every hand. It is met in the home by way of lighting, heating, cooking, refrigeration, air conditioning, telephones, radios, small-appliance motors, etc. On the farm it has added services in water pumping, soil heating, dairy cooling, and the operation of many types of machines. It is fundamental to communication systems and is essential in methods of transportation, as exhibited in the automobile and gas-electric busses, and in city, interurban, and main electric railways.

Industrially, there are literally hundreds of applications, ranging from signal and control circuits taking but a fraction of a watt to installations of single machines handling many thousand kilowatts. Treating but a few of the many industrial classifications, electricity is found in the preparation and manufacture of foods; in plants involving metalworking and the manufacture of all types of articles and machines from metals; in the process, textile, and wood industries; and in quarrying, mining, and metal refining. A great number of miscellaneous fields include mercantile establishments, the motion picture industry, printing, publishing, ice and refrigeration plants, etc. The use of electricity is universal.

1.2. Central Power Systems. The greatest amount of electrical power is generated and distributed by the large power systems, whether private or public enterprises. Primary reasons favoring the large system lie in the economics of plant design, equipment, and system operation, and in the reliability of service. Each power utility company normally supplies broad areas within one distribution system. Its generating plants are basically steam or hydroelectric installations, the fuel for the steam stations being mainly coal, oil, or gas, with some of the more recent installations using nuclear reactors.

The initial cost of the generating equipment per unit of capacity and the cost of operation per unit of energy decrease appreciably as the size of the plant increases. Part of the latter is due to higher efficiencies obtainable from the larger units of apparatus. The ratio of the average load to the maximum load demanded of the station also favors the larger plant. This ratio is called the load factor and, in general, increases as the number of customers increases.

Engineering developments have been able to keep pace with the demand for larger units to the extent that 200,000-kva (160,000-kw) single-shaft steam-turbine-driven generators have been in operation for many years. 250,000-kw, 3,600-rpm units have been built for the Tennessee Valley Authority. Turbo-generator sets in tandem, with the turbines taking steam at high, intermediate, and low pressures, can produce 300,000 kw total. The significance of this amount of power is better understood when it is realized that a city of 500,000 population with an average industrial load might be expected to require about 250,000 kw.

Waterwheel generators likewise have increased in size. At Hoover Dam, for example, the vertical-shaft generators have a rating of 82,500 kva.

A group of much larger generators, each of 108,000-kva rating and driven by a 150,000-hp hydraulic turbine, are installed at Grand Coulee. In addition, the associated pumping plant has 65,000-hp synchronous motors which drive the pumps that lift water from 270 to 365 ft above Roosevelt Lake behind Grand Coulee Dam. This plant is but one of several in the Columbia River Project for power generation and for irrigation. For mechanical reasons, the speeds of steam turbines are relatively high, whereas those of water wheels and large pumps are greatly reduced.

Small arc-light and Edison companies were the original central stations, competition being mainly for direct-current residence lighting. Starting about 1890 the Edison groups won out in the central station field, the arc-light companies being absorbed by electrical manufacturing organizations which began their growth near this period. The name Edison remained associated with the central station industry as witnessed, for instance, by the Consolidated (New York), Detroit, Commonwealth (Chicago), Southern California, etc., Edison companies. In addition, of course, is the greater number of similar private power concerns bearing names indicative of their localities. Of more recent origin are the large government projects such as Tennessee Valley, Bonneville, and Grand Coulee.

In the future we can expect a decided increase in the percentage of energy coming from nuclear power plants. The nuclear reactors in these plants, regardless of type, will ultimately deliver their energy from electric generating equipment. Electrical power surpasses all other forms in its flexibility and ease of control.

For economic reasons, private power companies are monopolies closely regulated by the State in which they exist. Each company serves a given geographical district free of competition except perhaps for publicly owned concerns also within the area. The regulation of the private companies is a function of the State, hence public utility commissions, set up by the legislatures, have jurisdiction over the general financial structure, rates, kinds and character of service, and questions of safety as these problems affect the public. Such commissions regulate all forms of utilities including gas, light, power, and water; express companies, railroads, street railways, and all common carriers; telephone and telegraph systems; and many smaller groups of like nature but of lesser concern.

A public utility commission has general authority over disputes between the utilities and its customers and the public, and between utilities. It also has authority over financial and operational procedures of the utilities. It decides what shall be a fair rate of return on the investment of the company. The acts of the commission are usually final, subject only to review by the State Supreme Court on questions of authority. No commission, of course, of this kind has jurisdiction over publicly owned enterprises, nor can it in any way interfere with Federal regulations or State constitutional rights of organizations or individuals.

1.3. Energy Flow. In nearly all of its applications electrical energy is an intermediate stage between the primary source of energy and the final desired form. The primary source may be coal, oil, gas, or water, while the final energy may be mechanical, heat, light, or sound. Between the two end-points the energy is changed in form several times.

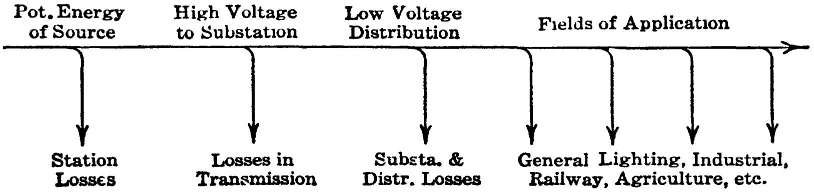


FIG. 1.1. Power flow diagram of an electrical system.

Figure 1.1 is a single-line, power flow diagram to show the elements of an electrical power system. The initial energy is potential in the fuel used for the prime mover or in water stored in reservoirs. This energy is converted to kinetic energy in the driving machines and is transformed from mechanical in the connecting shafts to electrical at the terminals of the generators. Transmitted and distributed in the electrical form, the energy again is converted at the load, a great part of it directly to heat in heating and lighting loads. Another great part is changed first to mechanical energy in electrical motors that drive mechanical machines and thence passes to heat.

Not all of the initial potential energy existing in the primary source is put to practical use. Some of it may not be available, and part of it may be converted to potential energy of heat and lost, for example, the heat in stack gases. Another portion may be changed first to the kinetic form and left unclaimed, as is shown by the velocity of discharge of water wheels. Still another part is lost as heat in mechanical friction and in analogous electrical friction in conducting and magnetic circuits that give rise to copper and iron losses. Hence, the quantity of energy in the flow decreases with distance from the source, as it does also with the number of conversions between the supply and the load. In large power systems perhaps 75 to 90 per cent of the energy available at the switchboards in the generating stations ultimately may be sold, 8 to 10 per cent being lost in the high-voltage transmission systems alone. Electrical energy is a commodity that cannot be stored but must be produced as required. As the load varies, the input to the prime movers must be increased or decreased in proportion to the demand. The flow of energy at all points of a system thus rises and falls in time and in direct ratio with load conditions.

1.4. Costs and Efficiencies. Since the first commercial installations of power plants, the cost of converting the primary form of energy to electrical energy has decreased steadily. Mainly because of better methods in heat transfer developed by the mechanical engineer, boiler efficiencies have risen greatly. In the last 20 years the cost of electrical energy in units of fuel per kwhr at the switchboard has been more than halved, and at present less than 1.5 lb of coal, or its equivalent in gas or oil, are required per kwhr. This figure applies, of course, to steam plants. Efficiencies have been improved, as well, in hydroelectric power generation but to a much less degree and are now practically at their peak.

For a particular system, a diagram like that of Figure 1.1 could show at each step the kilowatt or percentage losses, or it could indicate the increasing cost of energy as one moves from the source to the load. In the latter case, the cost would include the fuel and all investments with their attendant charges and taxes for each component part of the system.

Electrical energy can be produced at the switchboards of large modern steam or hydroelectric generating stations for a cost as low as 2 to 2.5 mils per kwhr. Depending upon many factors affecting the technical and economic aspects in the transmission, distribution, and desired reliability of electrical energy, and upon the locality, the initial cost increases to a value roughly between 3 and 10 cents per kwhr for the energy alone delivered to the home. Plants that take large blocks of energy usually may obtain it at considerably reduced rates, perhaps from a little less than one cent per kwhr to a very few cents. The rates of private utilities are subject normally to approval of a regulatory body—a public utilities commission—acting for the state. Engineers also are found handling many of the general problems of such groups.

High over-all efficiencies of systems have come about, to a great extent, by better utilization of materials in mechanical and electrical machines, and through engineering design which has raised the efficiency of each component of the system. For example, results achieved in striving for more efficient electrical apparatus show large transformers to have maximum efficiencies only a few tenths below 100 per cent—almost complete energy transformation. Rotating machines in the larger sizes have losses of only 1 or 2 per cent of their ratings. Although increased efficiencies of apparatus have helped in lowering over-all costs of a power system, better methods of manufacture and a smaller price per unit of apparatus because of quantity production also have played an important part. In addition, of course, principles of economics applied to system layout and installation have aided greatly in cost reduction.

1.5. Units and Dimensions. We shall use mainly the meter-kilogram-second (mks) system of units. This system is the standard generally accepted for electrical engineering work.[1] Actually, the majority of the units of the mks system are practically identical to the familiar practical units long used by electrical engineers and are more commonly used than the cgs electrostatic or cgs electromagnetic units. (The mks units are based on the absolute ohm and ampere, whereas the older practical units are based on the international ohm and ampere; the corresponding quantities, however, have nearly equal values.) Also, the sizes of the mks units are better suited to the majority of engineering work than are those of the cgs system. Except where expressly otherwise indicated, all derivations and basic relations will be in the rationalized mks system, which will be introduced through its use in the text. Rationalization means adopting the ampere-turn as the unit of magnetomotive force, and associating one line of electric flux with each unit of electric charge. This rationalization removes the factor 4π which would otherwise appear in many of the basic equations and places it in the values assigned to the permeability and dielectric constants of free space.

In computations on electrical machines and in a large part of the literature, other units (such as the horsepower, the "line" of magnetic flux, and the gauss) are so well established that the student should be familiar with them and be able to make such conversions as are necessary. In recognition of this, many problems throughout the text are stated in units other than the mks. Conversion factors for the units more commonly used are given in Appendix I.

Often, long and tedious developments can be checked dimensionally for correctness. In addition, dimensional forms may be used at times as an aid in deriving physical laws. To establish a complete system of dimensions associated with electrical units, it is convenient to choose one electrical dimension as a fourth fundamental unit. Appendix I gives the dimensions of the majority of electrical quantities in terms of mass, length, time, and charge.

1.6. Energy, Work, and Power. The energy of a body is the ability of that body to do work. If the energy is due to position or condition, for example, the energy in an elevated tank of water or in a mountain reservoir, or the heat energy of a body at high temperature, the energy is said to be potential in character. If the energy is due to motion, such as in a moving flywheel, it is said to be kinetic energy. Each form of energy has units by which it is expressed in quantity, for example, heat units such as calories and British thermal units; mechanical units such as joules, ergs, foot-pounds, and horsepower-seconds; and electrical units such as watt-seconds and kilowatt-hours. Since the several forms of energy are mutually convertible, and there are conversion factors relating the units one to another, the units may not necessarily be peculiar to a particular form of energy.

---

[1] A. E. Kennelly, "I.E.C. Adopts MKS System of Units," *Elec. Eng.*, **54** (1935), 1373–1384.

Work is the result of motion against a resisting force, and generally it is given by the product of distance and the component of force along the path traversed. For a body in rotation, work is given by the product of torque and angle through which that torque is applied. Work and energy have the same units and, of course, the same conversion factors among the various units.

Power is the time rate of doing work and is usually the work done in one second. It is expressed in terms both of units of work and of time. Power is stated in many ways, with conversion factors existing among the several units of power. If the unit of time is the same for two units of power which are to be compared, the conversion factor is the same for power as for the corresponding units of work or energy.

In engineering, mechanical power is most frequently given by foot-pounds per second or per minute (ft-lb per sec or min), or in terms of *horsepower* (hp), a larger and more convenient unit. One horsepower is equal to 550 ft-lb of work per sec or 33,000 ft-lb per min, that is,

$$\text{horsepower} = \frac{\text{foot-pounds per second}}{550} = \frac{\text{foot-pounds per minute}}{33{,}000} \tag{1.1}$$

While equation 1.1 is general, for rotating apparatus it usually is written

$$\text{horsepower} = \frac{\text{torque} \times \text{angular velocity}}{33{,}000} = \frac{2\pi TN}{33{,}000} \tag{1.2}$$

where T is the torque in pound-feet and N is the speed in revolutions per minute (rpm). Equation 1.2 is the more useful form of the *horsepower equation.*

In the mks system of units, mechanical power can be expressed either as newton-meters per second, or watts, the two being identical without any conversion factor, that is,

$$1 \text{ watt} = 1 \text{ newton-meter per second}$$

A newton is that force which, acting on a one-kilogram mass, will produce an acceleration of one meter per second per second. One newton is equal to 0.2248 lb.

1.7. Voltage, Potential Difference, and Electromotive Force. Because of flux cutting in a generator or of chemical action in an electric cell or battery, there is produced between the terminals of these sources of electrical energy a potential difference. This potential difference is expressed in volts, and *voltage* is defined as *work per unit charge,* that is, *watt-second per coulomb.* An equivalent statement defines the volt as the difference of potential between two points of a circuit carrying a constant current of one ampere when the power dissipated between these points is equal to one watt. Passing through the energy source from the negative to the positive terminal, which is the conventional direction of positive current flow, there is a rise in potential. If the external circuit is closed so that current can flow, there exists a potential difference between any two points along the path. From the positive terminal of the source back to the negative terminal, the potential drop is equal to the net potential rise through the supply. The voltage, or potential difference, between any two points in a circuit carrying current is the work required to move one coulomb from one of those points to the other.

Within the windings of a generator there is a force acting on any free electrons to move them toward the negative terminal. Within the electrical cell a force exists to move the positive and negative ions in opposite directions through the solution. Again, if a source of electrical energy is placed across the electrodes of an electronic tube, the electric field between those electrodes produces a force on the electrons liberated at the cathode. The driving force in these instances is a true force and can be expressed in mechanical units if desired.

In early studies on electricity, the electrical circuit was compared with the hydraulic system, and the assumption was made that there existed an electrical pressure analogous to the pressure created by a pump. From these associated physical ideas came the term electromotive force (emf) as that mechanism causing current to flow, just as a pressure difference causes water to flow. In this sense, the force action discussed in the preceding paragraph could properly be called an electromotive force.

The unit of electromotive force, however, also is called the volt. It should be recognized that when electromotive force is thought of as an electrical pressure, it is not synonymous with potential difference, which is a measure of the work done on a unit charge in its motion. When electromotive force is treated interchangeably with potential difference, as is quite usual in practice, it then may be said to be the total work required to move one coulomb through the entire circuit. The ideas underlying potential difference and electromotive force are based on different physical concepts.

Voltage gradient is the ratio of work per coulomb to distance. In an electrostatic field, for example, in a vacuum tube or in the space surrounding a transmission-line conductor, the gradient is expressed in terms of volts per inch or per meter. The gradient gives directly the intensity of the field and is a measure of the force on a unit charge at the point in question, that is,

$$F = q\mathcal{E} \text{ newtons} \tag{1.3}$$

where $\mathcal{E}$ is the intensity of the electric field in volts per meter and q is the value of the charge in coulombs.

1.8. Current, Charge, and Quantity. It will be recalled from physics that electrical current can be defined in terms of the force action between current elements. This is the basis for defining the mks unit of current, the *ampere*, which is defined as the current which, flowing in two parallel conductors of infinite length and separated by a distance of one meter, produces a force between these conductors of 2×10^{-7} newtons per unit of length. The value of the ampere can be determined accurately by the Rayleigh current balance.[2] This balance consists of a movable coil placed between two fixed coils, all of them carrying the same current. The electromagnetic force (which can be calculated with a high degree of accuracy) on the movable coil is balanced by the gravitational force on a known mass placed on the pan of the balance to which the moving coil is attached. The mks ampere is sometimes referred to as the *absolute ampere* to distinguish it from the long-used practical unit, the *international ampere*, which is defined by means of the silver voltameter. The difference between the values of the two amperes is too small to be of importance in engineering work, that is,[3]

$$\text{1 mean international ampere} = 0.999853 \text{ absolute ampere}$$

The word "mean" in the expression refers to the weighted average of several national laboratories. It is not necessary to use the prefix "absolute" when it is understood that the mks units are being used.

Current exists in a circuit only when that circuit is closed and is acted upon by a voltage, either directly applied or induced by mutual action. As a result, current is conceived to be the flow of electricity, that is, *charges*, the flow being possible because of the closed conducting path. The analogy is obtained from the flow of water through pipes. Similar to the definition for quantity of water as that amount which passes a given point in a given time, electrical *quantity* is the time integral of the current in a circuit. The practical unit of electrical quantity, the *coulomb*, is that quantity which is represented by a flow of one ampere for one second. *One coulomb is one ampere-second.*

Symbolically,

$$q = \int i\,dt \text{ coulombs, or ampere-seconds} \tag{1.4}$$

from which

$$i = \frac{dq}{dt} \text{ amperes} \tag{1.5}$$

where q is the quantity of electricity, or charge, that gives rise to the current i because of its flow. Although the definition for q is fundamental, the expression for i does not indicate the initial manner of defining current, even though it often is given in terms of the time rate of flow of quantity. Since these relations are equivalent, the definition of only one unit is possible from them.

In work on electronics, it is often more convenient to express current in terms of linear charge density and the velocity of the charges. Thus, equation 1.5 can be transformed as follows:

---

[2] H. L. Curtis, "Review of Recent Absolute Determinations of the Ohm and Ampere," *Jour. of Research of the Nat. Bur. of Stds.* **33,** No. 4, Oct. 1944, 235–254. This paper gives the methods used by the various national laboratories in determining the values of the electrical units.

$$
\begin{aligned}
i &= \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\rho\, \Delta x}{\Delta t} \\
&= \rho v \text{ amperes} \qquad (1.6)
\end{aligned}
$$

where ρ is the linear charge density in coulombs per meter and v is the velocity of the charges in meters per second.

1.9. Electrical Power and Energy. All forms of energy are mutually convertible, and electrical energy can be transformed to mechanical energy, as in the electric motor, or to heat energy, as in the resistor elements of an electric stove. Although by far the major part of electrical energy supplied a lamp appears as heat, a small portion is converted to light energy. As observed in nature, all electrical energy is transformed ultimately into heat.

Electrical power is the time rate at which electrical energy is transferred through a circuit, finally to be transformed to mechanical energy or to heat. From the preceding definitions of voltage and current, electrical power is expressed as the product of these two quantities. Remembering that one joule of mechanical energy is equal to one watt-second of electrical energy,

$$\text{voltage} \times \text{current} = \frac{\text{watt-second}}{\text{coulomb}} \times \frac{\text{coulombs}}{\text{seconds}} = \text{watts electrical power}$$

In usual symbols,

$$P \text{ (power)} = e \text{ (volts)} \times i \text{ (amperes)} = \text{watts} \qquad (1.7)$$

Although the elementary unit of electrical power is the *watt*, a larger and more convenient unit used in practice is the *kilowatt* (kw), which is 1,000 watts. Corresponding to the units for power, electrical energy is expressed in terms of *watt-seconds* or *kilowatt-hours* (kwhr) as the usual larger unit.

The magnitudes of mechanical and electrical units are such that

$$1 \text{ horsepower} = 746 \text{ watts } (\tfrac{3}{4} \text{ kilowatt, approx.})$$

and

$$\text{horsepower} \times 0.746 = \text{kilowatts} \qquad (1.8)$$

This relationship between these units of power is of concern when making calculations on motors and generators.

It might properly be remarked at this point that the product of voltage and current for a circuit always is *volt-amperes.* At the same time, this product is also watts, if the instantaneous voltage and current are being considered, giving in this case instantaneous power. If the voltage and current do not vary with time over a certain time interval, as is generally true in direct-current systems, the average power over this time interval is equal to the instantaneous power. In the alternating-current circuit, as we shall see later, because of phase relations between voltage and current (the current does not necessarily reach its maximum at the same time as does the voltage), and because of the particular definitions given the magnitudes of voltage and current, the number of watts of average power may not be equal to the number of volt-amperes. The ratio of watts active power, which is the average rate at which electrical energy is transferred through a circuit or is being transformed to other forms, to the volt-amperes of the circuit is called the power factor (pf). Power factor varies between the limits of zero and unity; that is, the number of watts in a circuit is never greater than the number of volt-amperes.

1.10. Nameplate Information on Electrical Apparatus. Electrical power of commercial concern is produced through chemical action, for example, the dry cell and the storage battery, or by the electrical generator. Chemical action accounts for only a small fractional part of the electrical power generated. Most electrical power is obtained from generators driven by water wheels, steam turbines, or oil or gas engines. A generator may be designed mechanically to give a direct-current (d-c) voltage at its terminals, or an alternating-current (a-c) voltage. About 95 per cent of the electrical power is produced by a-c generators, or alternators, and this is almost wholly of 3-phase character.

In general, electrical power is transformed at the generator terminals to a higher voltage and lower current, transmitted over long lines, and converted back to a lower voltage and higher current at load or distribution centers. This applies, of course, almost solely to a-c systems. A-c voltages vary over a wide range, from 115–230 volts for home circuits to 250,000 volts or more on long transmission lines. D-c voltages seldom reach beyond 3,000 volts, the greater part of d-c power being produced and consumed at voltages less than 550 volts.

Each piece of electrical apparatus carries nameplate information specifying its operating voltage and power capacity, information of primary concern in the application of the device. Nameplates on rotating machines may be much more detailed. A d-c machine will have designated its rated voltage, rated load current in amperes, speed in revolutions per minute (rpm) at rated load, rated load in *horsepower output for motors* or rated *kilowatt output for generators*, temperature rise at normal load, the manner in which the machine is wound, and whether or not the duty is continuous, intermittent, etc. A-c machines carry similar information and, in addition, the frequency and number of phases. One also may find the power factor and the excitation voltage and current, depending upon the type of machine. A-c generators—and some a-c motors—may be rated in terms of kilovolt-amperes (kva) rather than kilowatts (kw) because of the power factor for which the machine is designed. A kilovolt-ampere rating on a motor will refer to electrical input, the output always being given in terms of horsepower. For illustrations of nameplate data, see problems 7 and 15.

Nameplate data give direct information on the conditions under which a machine or piece of equipment is to be used and the ranges of instruments necessary for making measurements. Specifically, nameplate data are for rated, or normal, or 100 per cent values of voltage, current, etc. These all mean the same thing and state the conditions under which the apparatus can be kept in continuous service without overheating or damage and with the performance characteristics for which it was designed. In using electrical apparatus, it is essential that reference be made to the nameplate, and that there be an understanding of the information it provides.

PROBLEMS

1. The force on an electron due to the electric field is given by the dimensional equation: force = $k \times$ volts $\times$ coulombs. What must be the units of k? Interpret the terms in the equation.

2. The force on an electron due to a magnetic field is given by the dimensional equation: force = $k \times$ webers per square meter $\times$ coulombs. Determine the units of k and interpret the various terms.

3. Is the equation derived from the following statement dimensionally correct? The capacitance of a condenser formed by two concentric cylinders is equal to the product of the length of the cylinders and the dielectric constant of the separating medium divided by the difference of the two radii.

4. What is the efficiency of an electric motor which does 335,000 newton-meters of work per min with an input of 6.5 kw?

5. An electric motor has an output of 3,350,000 newton-meters of work in 3 min. What horsepower is it delivering? What kilowatt input is required if the motor efficiency is 88%?

6. The total load of an electrically-driven elevator is 4,000 lb which is raised at a speed of 250 ft per min. If the efficiency of the hoisting mechanism is 75%, and that of the motor is 86%, what horsepower is required of the motor? What is its kilowatt input? What is the over-all efficiency of the drive?

7. Each of the Hoover Dam generators is rated 82,500 kva; 16,500 volts; 180 rpm; 60 cycles; 3 phase; 1.0 power factor. They are driven by 115,000-hp water turbines having a full-load efficiency of 92%. Calculate the rated delivered torque of one of these turbines, the generator efficiency, and the over-all efficiency of turbine and generator.

8. For an operating head of 500 ft, calculate the required flow in cubic feet of water per second for one of the Hoover Dam turbines referred to in problem 7.

9. The storage pond for an electrical generating station holds 50,000 acre-feet of water. If the average height of the water above the water wheels is 750 ft, what potential energy in kilowatt-hours is stored in the reservoir? If the over-all efficiency of one of the generating units is 86.7%, how long could this single set run while delivering 25,000 kw?

10. A water wheel in the Pit No. 1 plant of the Pacific Gas and Electric Co. works under a head of 420 ft. If the wheel efficiency is 90%, and the generator efficiency is 96%, what is the kilowatt output of the generator for a flow of 900 cu ft of water per sec?

11. The town of Grand Canyon, Arizona, on the Colorado River installed motor-driven pumps to raise water from springs 3,400 ft below the canyon rim. Two pumps operate in series with a lift of 1,700 ft each. Calculate the horsepower requirement of each motor for a flow of 85 gal of water per min, and a pump efficiency of 60%.

12. In the Grand Coulee irrigation project, 12 pumps, each operated by a 65,000-hp motor, will lift 16,000 cu ft of water each second from the main dam 280 vertical ft to the irrigation reservoir. Assuming the motors will operate at rated output, calculate the efficiency of the pumps.

13. A coal car weighing 12,500 lb runs up an incline onto an 18-ft dumping platform which weighs 2 tons. If the car is stopped at the middle of the track on the platform, and one end of the platform is to be raised 8 ft in 15 sec, what horsepower is required of the hoisting motor?

14. A d-c generator for marine service is rated 400 kw, 240 volts, 1,200 rpm. What is rated current for the machine? What torque is required to drive the generator if its efficiency is 93%?

15. A d-c motor has a nameplate rating of 55 hp, 220 volts, 207 amp, 1,140 rpm, continuous duty, 40°C temperature rise. What is the manufacturer's guaranteed efficiency at full load? What is the delivered output torque of the motor at rated load?

16. A 220-volt d-c motor draws 23.4 amp line current while running at 1,740 rpm and supplying a net force of 60 newtons at the end of a 0.4-meter brake arm. What is the motor efficiency for this load?

17. The following data are obtained at one load point on a Prony-brake test of a d-c motor: 115 volts, 50.5 amp, 21 lb 7 oz scale weight, 1,155 rpm. Tare weight is 1 lb 13 oz, and the lever arm is 18 in. What is the motor efficiency at this load?

18. A 230-volt d-c motor drives a 120-volt d-c generator at 850 rpm. The generator load is composed of fifteen 100-watt lamps, forty 60-watt lamps, and 4 kw of heating and appliance units. If the respective efficiencies are 89 and 91%, what are the line currents of each machine and what is the shaft torque?

19. A 230-volt d-c motor drives a hoist through a gear box giving a 35 to 1 speed reduction from motor to a 20-inch diameter hoisting drum. The motor efficiency is 88%, and that of the gear box and hoisting mechanism is 70%. If the hoisting load is 3,000 lb and is raised at a speed of 2 ft per sec, what is the motor speed, the shaft torque between motor and gear box, and the motor line current?

20. A 5-gallon hot water heater uses a 120-volt, 1,200-watt heater element. Inlet water temperature is 55°F, and outlet temperature is set at 140°F. Calculate the time required for the 85°F temperature rise. For a cost of 3.2 cents per kwhr, calculate the energy cost per gallon of hot water.

21. A type 891 water-cooled tube has a plate dissipation of 10 kw and a filament heating power of 1,320 watts, both of which must be dissipated by the cooling water. For a water flow of 4 gal per min between the anode and jacket, and an inlet temperature of 25°C, calculate the outgoing temperature.

22. A type 880 water-cooled tube is used as a radio-frequency amplifier with an output of 45 kw. The plate supply voltage and current are 10,000 volts and 6 amp. The filament voltage and current are 12.6 volts and 320 amp. The cooling water must dissipate the plate loss and the filament heating power. Calculate the rate of water flow for a difference of 10°C between outlet and inlet water temperatures.

23. A type 9C21 water-cooled tube is delivering 100 kw of radio-frequency power at a plate-circuit efficiency of 74.5%. The filament power is 8.1 kw. Calculate the temperature rise of the cooling water, due to the plate loss and filament power, for a flow of 15 gal per min.

24. A 35,000-kw turbine-generator set was sold with a guaranteed heat consumption of 10,640 Btu per kwhr. The acceptance test showed an actual heat consumption of 10,720 Btu per kwhr. If the set is to operate at full load 4,200 hours per year, and energy costs 38 cents per million Btu, what is the yearly cost of the differential in heat intake?

25. A 25-hp motor, to run 4,500 hours per year, is given the following quotation by two manufacturers: (a) $475, 87% efficiency; (b) $550, 89% efficiency. If electrical energy is sold for 1.5 cents per kwhr to the customer, what would be the yearly difference in energy cost for the two motors?

26. A $7\frac{1}{2}$-hp motor has efficiencies of 65, 81.5, 86, 86, and 84% at quarter-load points from $\frac{1}{4}$ to $1\frac{1}{4}$ rated horsepower output. If it is to operate at full load for 2 hours each day, at $\frac{3}{4}$ load for 4 hours, and at $\frac{1}{2}$ load for 2 hours, what will be the monthly energy charge for a cost of 1.2 cents per kwhr?

27. A 220-volt, 1,140-rpm, 10-hp d-c motor has efficiencies of 70, 81, 85, 85, 83.5, and 82% at the quarter-load points from $\frac{1}{4}$ to $1\frac{1}{2}$ times rated horsepower output. The motor has a linear speed drop of 8% of rated speed from no load to full load. On the same graph, plot the curves of kilowatt input, delivered torque, efficiency, and speed, all as a function of horsepower output. (This set of curves shows the performance characteristics of the motor.)

28. Electrical energy is generated by a large power company at an average rate of 250,000 kw for the year. What is the yearly output in kilowatt-hours? Assuming an annual charge of 12% on an investment of $110,000,000 for the plant, what is the cost of energy in mils per kilowatt-hour?

29. If a barrel of oil contains 6,250,000 Btu, and the over-all efficiency of energy conversion is 24.6%, how many kilowatt-hours are realized at the switchboard per barrel of oil? If certain stand-by losses are represented by $1\frac{1}{4}$ barrels of oil per kw-yr, what is the plant economy in Btu per kwhr?

30. What does it cost to operate five 25-watt lamps, four 60-watt lamps, and a 550-watt electric heating unit for 2 hours if electrical energy costs 4 cents per kwhr?

31. The power bill for a home is $7.47 for a given month. The separate costs of the bill are 70 cents service charge, 4.4 cents per kwhr for the first 40 kwhr, 3.1 cents per kwhr for the next 60 kwhr, 2.5 cents per kwhr for the next 100 kwhr, and 1.2 cents per kwhr for all kwhrs in excess of 200. How many kwhrs of energy were consumed in this given month?

32. The meter readings at the end of successive months for a home are 8,978 and 9,324 kwhr. Calculate the power bill for the month using the rate schedule given in problem 31.

33. A home burned 14,600 cu ft of natural gas in one month for heating and cooking. The monthly charge was based on $1.12 for the first 200 cu ft, 5.85 cents per 100 cu ft for the next 2,300 cu ft, 5.55 cents per 100 cu ft for the next 17,500 cu ft, and 5.25 cents per 100 cu ft for the next 80,000 cu ft. What was the bill for gas for that month? If the gas contains 1,175 Btu per cu ft, how many pounds of coal containing 12,500 Btu per lb would have the same heat content? What would the bill have been if the total energy had been supplied by electricity at the rate of 1.2 cents per kwhr?

CHAPTER II

GENERATION OF ELECTRICAL POWER

2.1. Historical. With every electrical circuit carrying current, no matter what the configuration of that circuit may be, there is an associated magnetic field. Conversely, with every magnetic field there will be found a related current (or currents) which gives rise to it. Even in permanent magnets, physical theory supposes circulating currents in the poles or electron motion in their respective orbits together with electron spin, these being productive of the magnetism.

Although Oersted discovered in 1819 that a current in a wire deflected a magnetic needle near it, Arago (1820) was the first to announce that a magnetic field surrounded a wire when it carried current. Arago, also, independently of Gay-Lussac and of Davy, discovered that a bar of steel could be magnetized if placed in a coil carrying current. In the same year, Ampère and Biot-Savart stated quantitatively the laws underlying the force action of one current-carrying conductor upon another. The fundamentals of electricity and magnetism were being uncovered rapidly at this time.

On Christmas, 1821, Faraday showed his wife the motion of a magnetic needle when near an electric current. In following up his argument that as a current-carrying circuit affects a magnet so a magnet should react upon a current-carrying circuit, he was led to the discovery of induced electric currents which he published in 1831, the year before Henry published the results of his investigations on the same phenomena. It should be remembered that electric currents in those times were produced by voltaic cells, voltage-producing machines as yet not being in existence.

2.2. Induction by Magnets. If a magnet is thrust into a coil of wire the ends of which are connected to a galvanometer, there will be a momentary deflection shown by the galvanometer needle. The phenomenon is an experimental fact which shows that with the magnet in motion with respect to the coil there is an induced electromotive force in the wire giving rise to a current when the circuit is closed. Removal of the magnet from the coil causes the galvanometer needle to deflect in the opposite direction, thus indicating a direction of induced emf (electromotive force) corresponding to the direction of motion of the magnet. Further experimentation will show that the rate of motion of the magnet into or out of the coil will influence the magnitude of the induced emf—the more quickly the magnet is moved, the greater the galvanometer deflection. The same effects are found if the magnet is held and the coil thrust over it. The magnitude of the deflection also is proportional to the pole strength.

2.3. Induction by Currents. The magnet may be replaced by a coil of wire carrying a steady current, the thrust of this second coil near or into the first coil producing effects on the galvanometer the same as those mentioned above. The relative motion, in speed and direction, between the two coils gives rise to results of the same character as those exhibited by a magnet and a coil.

If the coils are placed in close proximity to each other, and the current in either changed from one value to another, the needle of the galvanometer connected to the second coil will deflect momentarily. The direction of the change in magnitude of the controlled current in the first coil determines the direction of motion of the needle, and the rate at which the current is altered determines the magnitude of the needle deflection. A current varying periodically in the first coil, with both coils fixed in position, will result in a to-and-fro motion of the galvanometer needle with the same periodicity as the varying current which is controlled. The first coil may be said to be the primary, and the coil in which the phenomena are observed then may be called the secondary. Together, with their mutual reactions, they constitute the essentials of the transformer. The voltage in the secondary is produced by mutual induction.

As already stated, it is a physical fact that a magnetic field accompanies every current, the strength of the field increasing and decreasing as the current increases and decreases. If saturation does not exist, the relation between current and the flux it produces is linear. As the current in a coil or circuit varies, the flux created by the current, in building up about that circuit and in collapsing upon it, induces a voltage in those conductors. This voltage is one of self-induction, usually treated as an effect distinct from the voltage of mutual induction discussed in the preceding paragraph. Self-induction, however, is but the mutual effects of the several turns of the same circuit upon each other and, hence, is but a special case of general mutual induction.

A variation of the experiments described is to pass a conductor, the ends of which are connected to a galvanometer, through a magnetic field. The field may exist because of permanent magnetism or it may be produced electromagnetically, that is, by a current flowing through a coil of wire. The direction of motion of the conductor and its speed determine respectively the direction of deflection of the galvanometer needle and the magnitude of that deflection. This experiment illustrates the principle of generator action which takes place in all electric generators and motors. It is, of course, immaterial in the generation of voltage whether the conductor or the flux, or both, move with respect to each other.

2.4. Opposing Force Developed in a Generator. An electric generator converts mechanical energy into electrical energy by the electromagnetic induction resulting from the relative motion of conductors and a magnetic field. When electrical power is taken from such a machine, an electrical current flows in the conductors. The presence of this current develops a mechanical force on the conductors in a direction that opposes their motion. This reaction of the machine makes it necessary to supply a mechanical force, and hence mechanical energy, in order to maintain the conductors in motion. From the Law of the Conservation of Energy, the mechanical energy input must equal the electrical energy output in the ideal machine which has no losses. In all electrical generators we are concerned with these two effects:

1. The voltage generated by electromagnetic induction.
2. The reaction of the machine, when the conductors are carrying current, resulting in a force opposing the motion.

It follows that there must be two fundamental equations expressing the principles of the generator, that is, the voltage equation and the force equation.

These same equations are also fundamental in expressing the principles of motors since the generator and the motor are essentially the same machine. Under load, the conductors of both machines are carrying current and are cutting lines of flux. By virtue of flux cutting, both machines have voltages induced within them. Further, because the current-carrying conductors are in a flux field, force action, hence torque, is produced in the air gaps of both machines.

The voltage generated in a single conductor is given by the experimentally determined relation

$$e = \beta l v \text{ volts} \tag{2.1}$$

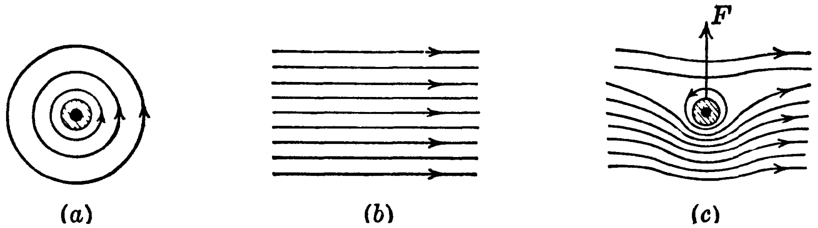
where β is the flux density in webers per square meter (the weber is defined in Section 6), l is the conductor length in meters, and v is the conductor velocity in meters per second. Equation 2.1 is restricted to the condition that β, l, and v are all mutually perpendicular. Under this condition, the right-hand side of equation 2.1 expresses the rate at which the conductor is cutting flux, that is,

$$\begin{aligned} e &= \beta l v \text{ volts} \\ &= \text{limit of } \frac{\beta l \, \Delta x}{\Delta t} \\ &= \text{limit of } \frac{\Delta \Phi}{\Delta t} \\ &= \frac{d\Phi}{dt} \text{ volts} \end{aligned} \tag{2.2}$$

where Φ is the flux in webers and t is in seconds. This equation expresses the voltage induced in a conductor as the time rate at which that conductor is cutting flux. Equation 2.2 is not limited by the restrictions of equation 2.1 and in this respect might be considered as a more general statement of the latter equation. The direction of the induced voltage is best shown by considering the reaction when the conductor is carrying current, as is explained in this and the following section.

As already stated, a conductor carrying current is surrounded by a magnetic field. The direction of this field may be determined easily by the *right-hand rule* which may be stated thus: *If a conductor is grasped by the right hand so that the fingers encircle the conductor in the direction of the lines of flux, the thumb will point in the direction of the current flow.* Hence, known direction of current flow also will tell the direction of the flux encircling the conductor. In applying the right-hand rule to a coil, the thumb will point in the direction of the flux passing through the coil if the coil is grasped so that the fingers point in the direction of the current flow. It should be remarked that the right-hand rule is simply an arbitrary convention in general use which associates direction of current flow and direction of flux thereby created. Obviously, it rests upon our adopted conventions on polarity of magnetic fields and currents. This single rule will enable one to determine the direction of induced voltage and torque in either a generator or a motor. This approach is preferable to depending upon memory for a different rule for each case.

Figure 2.1a shows a conductor carrying current. The direction of current flow is out of the paper, as indicated by the dot within the conductor which represents an arrowhead. Figure 2.1b shows a uniform field. When the current-carrying conductor is placed in the flux field, the resultant field produced by the combination of the main field and that due to the current in the conductor is as shown in Figure 2.1c. The fields add below the conductor and subtract above the conductor with the result that there is a stronger field below and a weaker field above. The force acting on the conductor will tend to move it from the stronger toward the weaker field. This direction of motion would tend to straighten the lines of flux. It is evident that a reversal of either the direction of flux *or* the direction of current would reverse the direction of the force.



(a) (b) (c)

FIG. 2.1. Force acting on a conductor.

The force F tending to move the conductor has been found experimentally to be given by the equation

$$F = \beta l i \sin \alpha \text{ newtons} \tag{2.3}$$

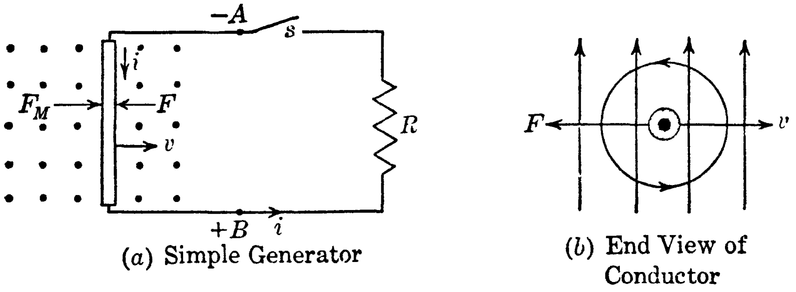
where β is the flux density in webers per square meter, l is the length of the conductor in meters, i is the current in amperes, and α is the angle between l and β.

It is, perhaps, well to digress at this point and show how the force exerted on free charges moving in a magnetic field, such as in a cathode-ray oscillograph, can be obtained from equation 2.3 and equation 1.6 of Chapter I. Substituting the definition of current as given by equation 1.6 in equation 2.3, the force exerted on an elemental length, Δx, of moving charges of density ρ is

$$\begin{aligned} F &= \beta\ \Delta x \rho v \sin \alpha \text{ newtons} \\ &= \beta q v \sin \alpha \text{ newtons} \qquad (2.4) \end{aligned}$$

where q is the charge in coulombs, v is the velocity of the charge in meters per second, and α is the angle between the magnetic flux and the velocity of the charge.

2.5. Fundamentals of the Elementary Generator. The simplest type of generator consists of a straight conductor cutting a magnetic field, as shown in Figure 2.2. The lines of flux are out from the paper, as indicated by the arrowheads. The conductor has a voltage generated in it, as given by equation 2.1.



(a) Simple Generator

(b) End View of Conductor

Fig. 2.2. Voltage induced in a single conductor and the reaction of the field on the conductor.

When the switch s is open, no current flows and the reaction of the field on the conductor is zero. Since there is no force opposing the motion, no external mechanical force is required to maintain this motion. Stated in another way, this merely means that with the switch open no mechanical work is required to generate the voltage, but, at the same time, the generator is delivering no electrical output.

When the switch is closed, a current, i, flows in the conductor and the generator delivers electrical energy to the load resistance R. The field reacts with the current in the conductor to develop a force, F, opposing the motion. An external mechanical force, F_M, equal and opposite to F, must be applied in order to maintain the motion. The mechanical work done on the conductor is given by the relation

$$\text{mechanical work} = F_M\ \Delta x \text{ newton-meters, or watt-seconds}$$

where F_M is the applied force in newtons, and Δx is the distance in meters through which the force is exerted.

The power (rate of doing work) can be expressed as

$$P = F_M \frac{\Delta x}{\Delta t} = F_M v \text{ watts}$$

From the Law of Conservation of Energy, the mechanical power input must equal the electrical power output. Expressing this in equation form,

$$P = (F_M)(v)$$

$$= (\beta li)\left(\frac{e}{\beta l}\right) = ei \text{ watts} \tag{2.5}$$

Equation 2.5 is a very important and general equation in the solution of electrical circuit problems. The relation may be stated as follows: The instantaneous power in watts is equal to the product of the instantaneous voltage in volts and the instantaneous current in amperes. If the magnitudes of neither current nor voltage vary with time, the average power is equal to the instantaneous power. The direction of the induced voltage is taken as the direction it drives the current through the load. The direction of current flow must be such that a force is developed between the conductor and the main field that opposes the applied force F_M. This necessitates a current flow, as indicated by the arrows. The end view of the conductor, Figure 2.1c, is helpful in determining this direction of current flow. Since, by convention, the positive (plus) terminal of a voltage source is the terminal by which the current leaves the source, the B terminal is positive for the generator shown in Figure 2.2.

2.6. Expressions for Induced Voltage. Although the expression for the voltage induced in a circuit, as given by equation 2.2, is quite general, other mathematical statements and physical interpretations of the law of induction are often more convenient to use. Reference to Figure 2.2a shows that $\Delta\Phi = \beta l \, \Delta x$, as given in equation 2.2, is the change of flux linking the circuit because of the conductor moving a distance Δx. Equation 2.2, therefore, can be interpreted as expressing the voltage induced in a *single turn* or loop of a circuit as the time rate of change of the flux passing through the turn. In Figure 2.2a the loop is formed by the conductor and its connections to the load resistance R. Figure 2.3a, which is equivalent, shows the conductor bent into a single-loop circuit. The induced voltage is the time rate of change of flux enclosed by the turn. If the flux threads N loops of a circuit (a coil of N turns), Figure 2.3b, the symbolic statement of the voltage induced is

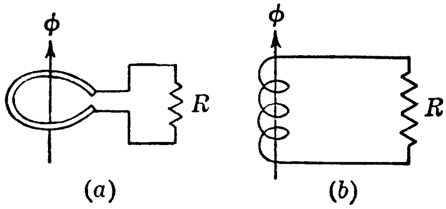


FIG. 2.3. Voltage induced in a coil.

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{d(N\Phi)}{dt} \text{ volts per coil} \tag{2.6}$$

The product $N\Phi$ is called the *flux linkages of the circuit*. Equation 2.6 serves as the definition of unit magnetic flux. The weber is the magnetic flux which, traversing a circuit of a single turn, will induce an electromagnetic force of one volt when reduced to zero at a uniform rate in one second.

It should be remembered from physics that the induced voltage in a circuit is in such a direction that the current it causes to flow will produce effects *opposing* the action which induced the voltage. This physical observation is known as Lenz's law. Thus, in Figure 2.2 the motion of the conductor decreases the main body of flux linking the conductor. The current which flows because of the induced voltage has such a direction that its flux adds to that within the loop, thereby tending to maintain the initial flux linking the circuit. Expressions for induced voltages often are preceded by a negative sign in order to indicate this opposing action. However, with the understanding that the associated phenomena are to be recognized, the algebraic sign has been omitted from equations for induced voltages in this and subsequent discussions.

In the sense that a body in motion tends to maintain its momentum, which is one of Newton's Laws of Motion, so a closed electrical circuit tends to maintain constant the number of lines of flux interlinking that circuit. These are analogous physical observations, the second leading to the principle of Constant Flux Linkages. The electromagnetic field exhibits the property of electromagnetic inertia, and energy is required to overcome this inertia and change the initial magnetic relations of the circuit. The energy may be measured by the mechanical work required to move the conductor or by the electrical energy which appears in the coil. Essentially, the process is the transformation of mechanical energy into electrical energy.

The flux which links the turns of a coil may be produced either by current carried by that coil or by current in a neighboring circuit. Not all the flux, however, will link all the turns, as is implied by equation 2.6. For many circuits, for example, a coil compactly wound, this may be practically true, but for others, a long solenoid for instance, calculations based on such an assumption may be far from correct. Some of the flux does link all the turns, but some flux links fewer turns, and still fewer turns are linked by less flux. These partial linkages, if they may be called that, contribute their share to the total induced voltage just as do the others; consequently, a broader definition for induced voltage is to state it as the time rate of change of the total flux interlinkages of the circuit. Practically, the total interlinkages may be given as the product of the actual number of turns and an equivalent flux which is assumed to link them all, that is,

$$\text{total interlinkages} = \Omega = \Sigma n\Phi = N\Phi_e$$

That part of the total flux which does not link all the turns is a leakage flux with respect to that part which is enclosed by all the turns. In the same sense, the total flux produced by one of two neighboring circuits may be thought to have two components. The first component links with the turns of the second circuit and is called a mutual flux. The other part does not link the second circuit in any way and is called a leakage flux of the first circuit with respect to the second. In a generator, the mutual flux of each field coil with respect to the armature winding is the air-gap, or useful, flux since it leads to the generated voltage. The leakage flux of the field coils usually is expressed as a percentage of the useful flux.

Change of interlinkages may be due to change in position of the circuit with respect to the flux, change of current in the circuit, change of current in a near-by circuit, change in configuration of the magnetic path, or change in permeability of the magnetic circuit. The induced voltage is variable when the time rate of change of the flux linkages is variable. The preceding equations give the instantaneous induced voltage if instantaneous values of β, v, and $d\Phi/dt$ are used in the expressions.

The average voltage induced in a circuit may be written from equation 2.6 in the form

$$\begin{aligned} e_{\text{avg}} &= \frac{N}{t}\int_0^t \frac{d\Phi}{dt}\,dt = \frac{N}{t}\int_{\Phi_1}^{\Phi_2} d\Phi \\ &= \frac{N[\Phi_2 - \Phi_1]}{t} = \frac{N\Phi_2 - N\Phi_1}{t} \qquad (2.7) \\ &= \frac{\text{net change in flux linkages}}{\text{time for change of linkages}} \text{ volts per coil} \end{aligned}$$

For a single *conductor* moving across a field of flux the expression for the average induced voltage becomes

$$e_{\text{avg}} = \frac{\Phi}{t} \text{ volts per conductor} \qquad (2.8)$$

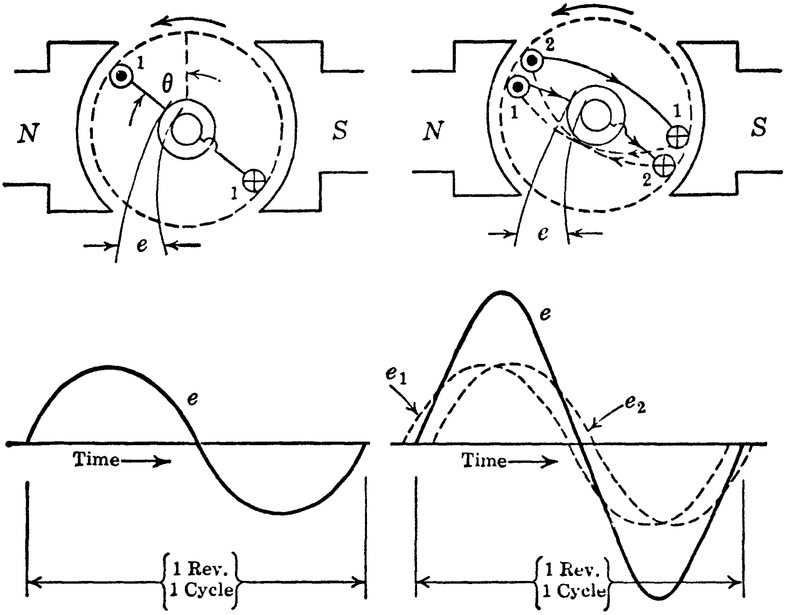
where Φ is the flux in webers cut by that conductor in t seconds. For a *coil* of N turns the average induced voltage is

$$e_{\text{avg}} = N\frac{\Phi}{t} \text{ volts per coil} \qquad (2.9)$$

where Φ is the equivalent flux change through the coil in t seconds. In general, either of these two equations, with proper interpretation, can be used for a specific problem.

2.7. The Elementary Generator. A simple type of generator, and one which comprises the essential elements of the modern electrical machine, is the single loop of wire rotated through a magnetic field produced by two poles, as shown in Figure 2.4a. The loop is placed in slots on a laminated iron rotor and moves with uniform velocity about an axis at right angles to the magnetic field and in the plane of the loop. As the sides of the coil sweep past the pole faces, they cut the flux crossing the air gap from the pole faces to the rotor, and a voltage is produced between the collector rings to which the coil ends are fastened. Sliding contacts, called brushes, allow this voltage to be carried to an external circuit where it is utilized. Mechanical energy required to drive the coil is transformed into electrical energy.

It will be observed from Figure 2.4a that the coil sides are always cutting flux in opposite directions and that their small voltages are additive around the loop. Further, by following the coil during one revolution, it will be seen that the polarity of the leads changes. This type of generator produces a voltage wave which is alternating in character at the machine terminals, and it means that current will flow through the coil one way half of the time and in the reverse direction the other half of the time. Because the voltage and current are alternating at the machine terminals, the a-c generator also is called an alternator.



(*a*) Single Coil (*b*) Two Coils in Series

FIG. 2.4. Elements of the 2-pole a-c generator (alternator).

Figure 2.4b shows the connections for a 2-coil circuit, the instantaneous voltages for each coil, shown dotted, and the resultant voltage of the combination. Because of mechanical displacement of corresponding coil sides in adjacent slots, there is consequently an angular difference in time between the individual coil voltages. It should be understood that each coil may have many turns, and that a practical machine has all the armature slots filled with coils arranged in the general manner shown in Figure 2.4b.

2.8. Equation for the Alternating Voltage. A-c wave shapes are sinusoidal at generator terminals so far as is practicable. This result can be accomplished by proper shaping of the faces of the field poles. In commercial alternators, however, the sinusoidal wave form results not only from the flux distribution, but also from the use of a large number of coils properly distributed among the slots of the armature. Sinusoidal voltages are produced through the mechanical design of the machine, and quantities relating to them yield readily to mathematical analyses and to computations. In addition, sine waves incur less losses in power transmission than do other wave forms. The harmonic content of power system voltages is very low. Sine-wave generators for testing purposes have been built with only the third harmonic detectable, and of magnitude one-third of one per cent of the fundamental.

It is of note that electrically oscillatory systems containing inductance and capacitance, and mechanically oscillatory systems containing inertia and restoring force, give rise to sinusoidal vibrations when left to themselves. In fact, the majority of physical phenomena of oscillatory character follow sine-wave variations. A discussion on sinusoidal motions and wave shapes is in no way peculiar to the electrical machine.

If the flux density in the air gap is sinusoidal over the pole pitch of 180 degrees, it can be expressed as

$$\beta = \beta_m \sin \theta$$

where β_m is the flux density per unit area at the pole center. The instantaneous voltage generated in a conductor of length l moving at a uniform velocity v is

$$e = \beta_m l v \sin \theta \text{ volts per conductor} \tag{2.10}$$

The coil voltage is the sum of the voltages of each conductor in the coil. If the coil has but one turn, and the coil sides are spaced 180 electrical degrees apart, the instantaneous voltage is

$$\begin{aligned} e &= 2\beta_m l v \sin \theta \\ &= E_m \sin \omega t \text{ volts per turn} \end{aligned} \tag{2.11}$$

where

$$E_m = 2\beta_m l v \tag{2.12}$$

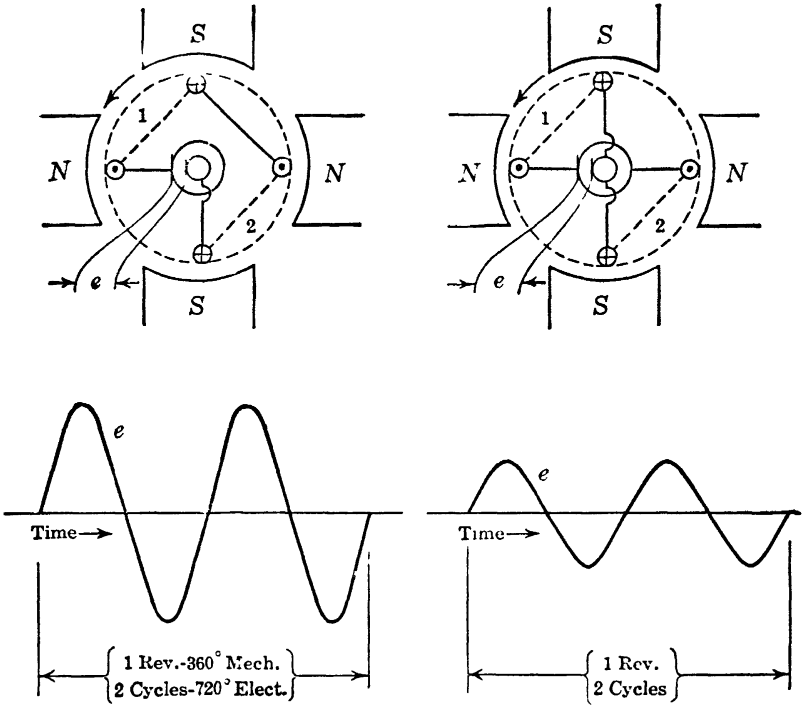
The angle θ varies with time t as the coil rotates and may be written in the form

$$\theta = 2\pi \frac{t}{T} = 2\pi f t = \omega t \text{ electrical radians} \tag{2.13}$$

By definition, T is the time in seconds to complete one *cycle* (two alternations) of the voltage wave (one revolution for a 2-pole machine) and is called the *period* of the wave; $f = 1/T$ is the number of cycles completed in one second and is called the *frequency* of the generated voltage; and $\omega = 2\pi f$ is the *angular velocity* of the wave in electrical radians per second.

Although the development is based on the simple construction shown, wherein a coil is rotated through a field of flux, the same result also is achieved in modern a-c generators in which the field of flux is moved past conductors placed in slots on a stationary armature, called the stator (see Figure 9.1b). It makes no difference electrically whether, by mechanical construction, the coil or the flux field is in motion, the other being fixed in position.

2.9. Multipolar Machines. The coil of the generator of Figure 2.4a has passed through one cycle, or 2 alternations, when it has made one complete revolution and is in position to repeat the form of the generated voltage wave.



(*a*) Coils 1 and 2 in Series (*b*) Coils 1 and 2 in Parallel

FIG. 2.5. Simplified circuits for the 4-pole machine.

The coil has swept out 360 mechanical degrees and each coil side has passed 2 poles of opposite polarity. The great majority of machines have 4, 6, or more poles, and many large motors and generators have been built having more than 100 poles. The armature coils are so placed in the slots that both sides of every turn cut flux, and the span of the coil is never more than the distance between centers of adjacent poles, that is, one pole pitch. The fundamentals of the 4-pole machine are illustrated in Figure 2.5. Although in an actual machine the armature, which may be the rotor or the stator, would have many slots to accommodate the many armature coils (see Figure 9.1), only 4 slots and 2 coils (4 coil sides) are shown in order to simplify the fundamental ideas. The 2 coils can be connected in series (single circuit), in which case the terminal voltage of the machine would be twice the coil voltage. With this connection, the current rating of the machine would be equal to the current rating of a coil, since each coil must carry the load current. If the 2 coils are connected in parallel (2 circuits), the terminal voltage would be that generated in one coil. However, for this latter connection, the current rating of the machine would be twice that of one coil, inasmuch as the load current would divide equally through the two parallel paths. It should be noted that for any manner of connecting the coils properly the volt-ampere rating of the machine will be the same. The coil arrangement for other multipolar machines follows that for the 4-pole generator of Figure 2.5.

In a machine having 4 poles, consecutive poles being of opposite polarity (this is true for all machines), each coil side, in making one revolution of 360 mechanical degrees, generates 2 cycles of the induced voltage wave because the second pair of poles acts exactly as does the first pair. By definition, *one cycle of emf is 360 electrical degrees;* hence, any conductor on a 4-pole machine sweeps out 720 electrical degrees, or 2 cycles, in one complete revolution. A 6-pole machine gives rise to 3 cycles per revolution, etc. The number of electrical degrees, therefore, is equal to the product of the number of mechanical degrees passed over and the number of pairs of poles:

$$\text{number of electrical degrees} = \text{number of mechanical degrees} \times \frac{P}{2} \tag{2.14}$$

Since the number of cycles per revolution is equal to the number of pairs of poles, it follows that the frequency increases directly with the speed. Hence, frequency in cycles per second is

$$\begin{aligned} \text{frequency} &= \text{number of pairs of poles} \times \text{revolutions per second} \\ &= \frac{P}{2} \times \frac{\text{rpm}}{60} \text{ cycles per second} \end{aligned} \tag{2.15}$$

Commercial frequencies in this country are mainly 60, 25, and 50 cycles per second, with a few odd frequencies found occasionally. For an a-c machine, the speed in equation 2.15 is called the *synchronous* speed.

2.10. Average Values. On any machine, the average flux density in the air gap over a pole pitch of 180 electrical degrees is

$$\beta_{\text{avg}} = \frac{1}{\pi} \int_0^\pi \beta \, d\theta$$

where θ is measured in electrical radians, and $\theta = 0$ midway between poles. If the flux distribution is sinusoidal, then

$$\beta_{\text{avg}} = \frac{1}{\pi} \int_0^\pi \beta_m \sin\theta \, d\theta = \frac{2}{\pi} \beta_m \tag{2.16}$$

The total useful flux per pole is the product of the average density and the area along the air gap over one pole pitch. In terms of β_m,

$$\Phi = \frac{2}{\pi}\beta_m \frac{2\pi r}{P} l \text{ webers per pole} \tag{2.17}$$

where r is the radius of the air gap, and l is the axial length of the pole body.

Selecting one pole pitch as the unit of distance, the linear velocity of a conductor on the rotor of a P-pole machine, or the corresponding relative velocity between a stationary armature conductor and a moving field pole, is

$$v = \frac{\text{distance}}{\text{time}} = \frac{\text{one pole pitch}}{\text{time for } \frac{1}{2} \text{ cycle}} = \frac{2\pi r/P}{1/(2f)}$$

Substituting B_m from equation 2.17, and the above expression for velocity, in the first of equations 2.11, the instantaneous generated voltage can be expressed also as

$$\begin{aligned} e &= \omega\Phi \sin \omega t \\ &= E_m \sin \omega t \text{ volts per turn} \end{aligned} \tag{2.18}$$

For a sine wave, the average value is fixed with respect to the maximum value; that is,

$$e_{\text{avg}} = \frac{2}{\pi} E_m \tag{2.19}$$

Over a complete cycle, the average value is, of course, zero; hence, this ratio is restricted in its application to but one alternation (half cycle). From equation 2.18, for a single-turn coil of 2 conductors,

$$e_{\text{avg}} = \frac{2}{\pi}\omega\Phi = 4f\Phi \text{ volts per turn} \tag{2.20}$$

The average voltage generated in one conductor is

$$e_{\text{avg}} = 2f\Phi = \frac{\Phi}{1/(2f)} \text{ volts per conductor} \tag{2.21}$$

Equation 2.21 is but a special case of equation 2.8. Φ is now the total air gap, or useful, flux per pole of a machine, and $1/(2f)$ is the time in seconds for a conductor to pass one pole. For a 2-conductor coil, the time required for the coil to move from that position in which it encloses maximum pole flux to that position at which it encloses zero flux is $T/4 = 1/(4f)$ seconds, or 90 electrical degrees. This statement leads directly to equation 2.20. Equations 2.20 and 2.21 apply to all machines, a-c or d-c, regardless of the number of poles. They are the most useful expressions for the calculation of induced voltages. Equation 2.20 applies also to a transformer in which Φ is the maximum flux in the iron core during the cycle.

2.11. The Direct-current Machine. To show the elements of the direct-current machine, let the collector rings in Figure 2.4 be replaced by insulated segments, as shown in Figure 2.6. The stationary brushes now are so placed that they exchange contact with the segments just as the coil ends fastened to them are midway between poles and are cutting no flux. The voltage wave, in following through one revolution of the coil, now will be as illustrated in Figure 2.6.

The segments are called commutator bars, and the assemblage of bars, insulation, clamping rings, spider, etc., forms the complete commutator. The commutator is really a mechanical switch which automatically, regardless of speed, always connects the external circuit to coil ends having a given polarity. Modern machines, of course, generally are multipolar and have many coils on the armature (rotor) connected in various arrangements so as to produce required voltages. Many coils also lead to a smoother voltage wave. Further, the pole faces usually are concentric with the armature, thereby leading to flat-topped waves and a consequent decrease in voltage ripple at the machine terminals.

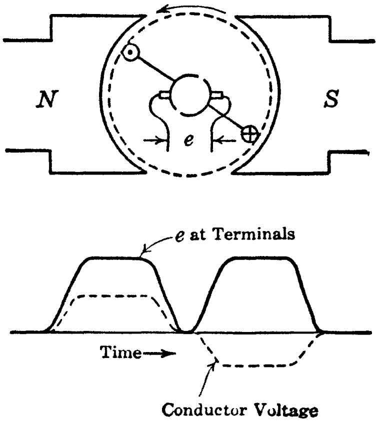


FIG. 2.6. Elements of the 2-pole d-c generator.

All coils in a d-c machine generate an alternating voltage, and at a frequency dependent upon the number of poles and the speed, as is the case with the a-c machine. The action of the commutator is to reverse every negative alternation of the voltage wave, as indicated in Figure 2.6. It is only at the terminals of the generator, and by virtue of the commutator, that a true direct or continuous voltage is obtained. The conductor, turn, or terminal voltages of the d-c machine are calculated exactly as are the average voltages for the a-c machine.

2.12. Effective Values. The practical magnitude of a sine wave is based upon its effective value rather than upon its average or maximum value. The *effective ampere*, for example, is defined as that current which, flowing through a pure resistance, will produce heat at the same average rate as a direct-current ampere. The a-c and d-c amperes, therefore, are equal in their abilities to convert electrical energy into heat energy. Since electrical energy is transformed into heat at the instantaneous rate of i^2r, the effective value I of current $i = I_m \sin \omega t$ must be defined by the relation

$$\begin{aligned} I^2R &= \frac{1}{2\pi}\int_0^{2\pi} i^2R\, d(\omega t) \\ &= \frac{1}{2\pi}\int_0^{2\pi} I_m{}^2R \sin^2 \omega t\, d(\omega t) \\ &= \frac{I_m{}^2R}{2} \end{aligned}$$

The effective value of a pure sine wave of current, therefore, is related to the maximum value as given by the expression

$$I = \frac{I_m}{\sqrt{2}} \tag{2.22a}$$

Similarly, for a sine wave of voltage $e = E_m \sin \omega t$, the effective value E of the voltage is

$$E = \frac{E_m}{\sqrt{2}} \tag{2.22b}$$

Alternating-current voltmeters and ammeters are calibrated to indicate the effective values of voltage and current and generally will do so for non-sinusoidal waves as well as for those which are sinusoidal.

Observe that in the above steps the ordinates of the initial current wave over a complete cycle were squared, and the area under the squared curve obtained by integration. Dividing this area by the interval 2π for the cycle, there results the average, or mean, height of the squared curve. The square root of this average ordinate therefore becomes the effective current. In other words,

effective value = square root of the average square

= root-mean-square (rms)

and is true for any type of wave shape.

2.13. Application of Rms Values. Although the development has been for sine waves only, the effective value for any complex current or voltage wave shape can be found by similar steps. If the wave is a repeating function, the analysis is carried out over one cycle. The abscissae of the wave may be expressed in radians, degrees, or seconds. The following steps show the procedure in calculating the rms (effective) value:

1. Select a convenient cycle for the wave.
2. Square the ordinates of the wave over the cycle.

3. Obtain the area under the squared curve.
4. Obtain the average height of the squared curve.
5. Extract the square root of this average ordinate—it is the effective value of the initial wave.

For waves which do not repeat, the procedure is the same and is carried out over the time interval desired. Depending upon the nature of the problem, the work may be performed in some cases by integral calculus, in others by graphics or graphical approximations.

Effective, or rms, values apply not only to currents and voltages, but to other quantities as well, for example, to duty cycles for electrical and mechanical machines. The ordinates may be torque, horsepower, etc., the abscissae usually time in seconds. The horsepower rating of an electric motor required to drive a mechanical machine is obtained from the rms of the horsepower duty cycle for that mechanical system. Such applications are very frequent and form a good part of application work in practice. Calculations of rms values for duty cycles are perhaps more frequently met by the engineer in industry than are computations on effective values of current and voltage. The general steps in any of these calculations are those given above.

In calculating the rms horsepower required of a motor for a duty cycle, it must be remembered that the rating is to be based on the heat produced in the motor by way of the I^2R in its windings, and the ability of the machine to dissipate that heat during the cycle. In a motor having constant flux, for example, the developed torque and the current are directly proportional to each other, and the rms of either the torque or current cycles is then representative of the other. The direct product of the torque and speed curves for the cycle gives the developed horsepower for the motor. The rms rating, however, must be based on the equivalent horsepower cycle which has the same shape as the torque and current curves and which assumes constant normal running speed over the complete cycle. The difference between the developed and delivered horsepower represents the change in stored energy of the motor and its connected load during the periods of acceleration and retardation. Very high torque peaks in a cycle may be of far greater concern in determining the motor rating than calculations on rms values.

Since a motor cannot dissipate heat so well during acceleration or retardation or at rest as it can when running at normal speed, the rms horsepower calculated from the equivalent horsepower cycle must be increased. This may be done by raising the rating a given percentage, 20 per cent for example, or as indicated by experience and judgment. Again, the final rating may be calculated from the equivalent horsepower cycle with the time intervals modified. During acceleration and retardation, a motor may be able to dissipate heat only about one-half as fast as at full speed and, while at rest, only about one-third as fast. These factors will vary with the type of enclosure for the motor and are obtained from experience. The corresponding time intervals for such periods of the cycle then are decreased by these factors so as to give a reduced over-all time for the cycle. It is an equivalent time during which the motor could dissipate the heat produced were it to run at full speed throughout. For example, consider the duty cycle shown in Figure 2.7. The average rate of heating during this cycle is proportional to

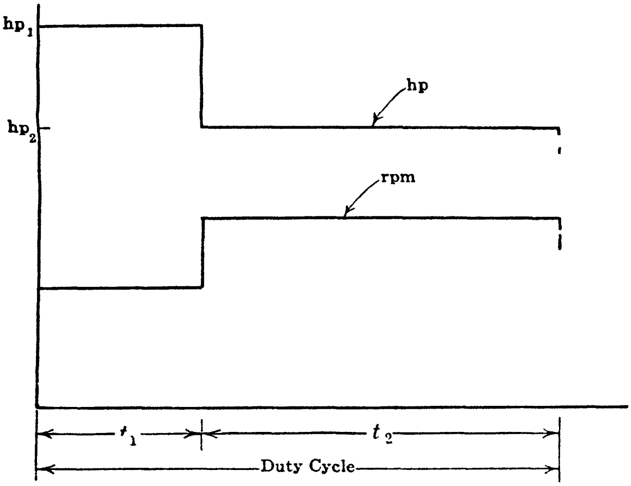


FIG. 2.7. Elementary duty cycle.

$$\frac{(\mathrm{hp}_1)^2 t_1 + (\mathrm{hp}_2)^2 t_2}{t_1 + t_2}$$

To accommodate safely this heating, a motor must be chosen which has been designed to have an average rate of cooling equal to this average rate of heating. These statements expressed in equation form become

$$\frac{(\text{hp rating})^2(kt_1 + t_2)}{(t_1 + t_2)} = \frac{(\mathrm{hp}_1)^2 t_1 + (\mathrm{hp}_2)^2 t_2}{(t_1 + t_2)}$$

or

$$\text{hp rating} = \sqrt{\frac{(\mathrm{hp}_1)^2 t_1 + (\mathrm{hp}_2)^2 t_2}{kt_1 + t_2}} \tag{2.23}$$

where k is a factor, less than unity, which takes into account the decrease in motor ventilation at the lower speed.

2.14. Form Factors. The ratio of the effective value of any cycle to the average value is called the *form factor* of the wave and simply is a number by which the average is multiplied to give the effective value. For sine waves, the form factor is 1.11, a number which will be used later in studies on machines.

For waves which are nonsinusoidal, the form factor is an index to its general shape. Form factors greater than 1.11 imply peaked waves, and factors less than 1.11 indicate flat-topped waves.

Referring to equation 2.20, the effective value of voltage for an N-turn coil now can be given as

$$E = 4.44\, f\Phi N \text{ volts effective} \tag{2.24}$$

Equation 2.24 applies to coils on a transformer as well as to those on machines. Φ is the maximum flux which threads the coil.

2.15. Example. The effective voltage for a 5-turn coil on a 6-pole generator is 10.6 volts. The flux per pole is 0.00795 weber. Calculate the frequency and the speed of the machine, and write the equation for the voltage per coil assuming it to be sinusoidal.

Solution:

The average voltage per conductor is

$$E_{\text{avg}} = \frac{10.6}{5 \times 2} \times \frac{1}{1.11} = \frac{0.00795}{t} \text{ volts per conductor}$$

$$t = 0.00833 \text{ second per half cycle}$$

$$f = \frac{1}{2t} = 60 \text{ cycles per second}$$

$$= \frac{6}{2}\,\frac{\text{rpm}}{60}$$

Therefore,

$$\text{speed} = 1{,}200 \text{ rpm}$$

and

$$e = 10.6\sqrt{2} \sin 2\pi 60t$$

$$= 15 \sin 377t \text{ volts}$$

If the average voltage per turn is used, the time taken to change the flux through the loop from maximum to zero is 0.00417 sec, corresponding to $\frac{1}{4}$ cycle. Whereas one conductor moves one pole pitch of 180 electrical degrees in cutting the flux of one pole, the coil (of two conductors) need move only 90 electrical degrees to cut the same flux.

An alternate method of solution is to use the expression for the maximum voltage generated. The maximum voltage generated in a coil of N turns is

$$E_{\max} = N\omega\Phi$$

$$\sqrt{2}(10.6) = 5 \times 2\pi f(0.00795)$$

$$f = \frac{15}{0.25} = 60 \text{ cycles per second}$$

PROBLEMS

1. What is the action of a compass (a) placed on a conductor carrying direct current (b) placed within a coil carrying current, the axis of the coil being in the plane of the compass needle?

2. A coil carrying direct current is wound around one leg of a horseshoe magnet. What is the action on the needle of a galvanometer in the electrical circuit if the armature of the magnet is pulled away suddenly?

3. Two coils are placed closely parallel to each other, one of them carrying direct current and the other closed through a galvanometer. What is the direction of swing of the galvanometer needle when the coils are separated suddenly compared with the direction of needle swing when the current in the first coil is suddenly established?

4. An armature conductor cuts 0.01 weber of flux in $\frac{1}{100}$ of a second. What is the average induced voltage in the conductor?

5. A circular coil of one turn and 3 ft in circumference is rotated about a diameter in a magnetic field of uniform strength and of density 1.5 webers per square meter. If the axis of rotation is perpendicular to the magnetic field, derive an expression for the generated voltage in terms of the velocity of rotation (rpm). Determine the average voltage for 1,200 rpm.

6. A wire 2 ft in length and bent in a semicircle is moved normal to the plane of its arc through a uniform magnetic field of density 56,000 lines per sq in. at a velocity of 15 ft per sec. The plane of the semicircle is parallel to the magnetic field, and a line joining the ends of the curved section of wire is normal to the field. What is the voltage induced in the 2-ft length of wire?

7. The flux per pole of a 6-pole, 600-rpm generator is 0.02 weber. What voltage is produced at the terminals of the machine if the individual voltages of 516 conductors on the armature are directly additive?

8. The average flux density of a 6-pole, 1,200-rpm generator is 0.7 weber per square meter over a pole pitch of 180°. The length of an armature conductor is 4.2 in. and the conductor is 10 in. from the center of the shaft. What average voltage is induced in the conductor?

9. A test coil of 15 turns is wrapped around the stator of an alternator (only one side of the coil passes through the air gap, parallel with the shaft). If the rotor has 4 poles, each carrying 600,000 lines of flux, and rotates at 1,800 rpm, what voltage is induced in the test coil?

10. The generated voltage per conductor of a 4-pole d-c generator is 2.5 volts. If the average flux density over the pole pitch is 0.65 weber per square meter, and the armature revolves at 1,750 rpm, what is the lateral area of the armature?

11. The maximum flux through a 200-turn transformer winding is 0.005 weber. If the flux variation is at 60 cycles per second, what average voltage is induced in the winding? What is the maximum voltage? What is the ratio of the average voltage to the maximum voltage? (Leakage flux is considered negligible.)

12. A coil of 80 turns produces a flux of 110,000 lines. Part of this flux links a second coil of 35 turns. When the current in the first coil is reduced to zero in 0.1 sec, the average voltage induced in the second coil is 0.3 volt. Calculate the percentage of the flux that is mutual.

13. A sinusoidally varying flux of 35,000 lines maximum completely links 2 coils placed side by side. If the coils are connected in series, a maximum voltage of 30 volts is generated within them, whereas with one coil reversed in connection, the maximum induced voltage is 15 volts. How many turns are in each coil if the frequency of flux variation is 60 cycles per second?

14. The voltage generated in a 4-turn armature coil on a 6-pole machine is given by the expression: $e = 8.4 \sin 251t$ volts. What is the flux per pole? Write the equation for the induced voltage of the coil if the speed is increased 50%.

15. A single-phase generator built for spark-gap radio transmitters has 24 poles and runs at a normal speed of 2,500 rpm. What is the flux per pole if an armature conductor has a maximum induced voltage of 2.83 volts at 1,000 rpm? What is the maximum rate of change of voltage at a speed of 2,500 rpm? Assume a sine wave of voltage.

16. The machine of problem 8 has 100 conductors. Calculate the torque developed when each conductor is carrying 10 amp. Calculate both the mechanical power input and the electrical power output and compare these values. (Neglect losses.)

17. A 26,700-kw alternator runs at 300 rpm. Calculate the input torque. If the armature diameter is 124 in. and the effective conductor length is 96 in., what is the force per square inch of armature surface? Assume an efficiency of 92%.

18. The voice coil of a loudspeaker has 20 turns and a diameter of 2 in. The flux density is one weber per square meter. Write the expression for the force for a current of $0.6 \sin 2{,}510t$ amps. If the unit is used as a microphone and develops a voltage $e = 0.001 \sin 2{,}510t$ volts, write the expression for the velocity of the voice coil.

19. For frequencies of 25 and 60 cycles per second, tabulate the corresponding speeds of machines for numbers of poles from 2 to 36.

20. A 24-pole alternator generates a 500-cycle voltage. At what speed does it run? What is the angular velocity of the voltage wave in electrical radians per second? How many mechanical degrees does one pole span?

21. An early 10-pole generator had stamped on its nameplate, "7,200 Alternations per Minute." What was its speed? How many electrical degrees were passed over in one revolution of the rotor?

22. A d-c motor having 6 poles runs at 750 rpm. What is the frequency of the induced voltage in the armature conductors? What is the frequency of induced emf in a 2-pole small-appliance motor running at 9,000 rpm?

23. How many electrical degrees are contained in one revolution of a 12-pole machine? What is the time for one alternation if the speed is 500 rpm?

24. What is the speed of a 10-pole machine that generates a voltage wave $e = 625 \sin 314t$?

25. An alternator produces a terminal voltage of 14,000 volts effective at 25 cycles. Write the equation for the instantaneous voltage. Determine the maximum time rate of change of voltage. Measured from the beginning of a cycle, what time elapses until the voltage is half maximum?

26. What will be the effective value of an alternating current which contains a prominent 3rd harmonic and is given by the equation $i = 25 \sin \omega t + 10 \sin 3\omega t$?

27. A current is given by $i = 15 - 10 \sin \omega t$. What is its effective value?

28. Determine the effective values of the following a-c wave forms, where in each case the maximum value is 10 amp: (a) rectangular; (b) semicircular; (c) triangular (any form of this); and (d) trapezoidal in which the base is twice the parallel side.

29. What is the maximum value of the equivalent sine wave which will give the same effective voltage as a triangular voltage wave of 100 volts maximum?

30. A vacuum-tube rectifier produces a half-sine wave of 100 ma peak value. Compute the average and effective values of this current.

31. The starting current of a motor is 80 amp which decreases exponentially to 5 amp in 10 sec. Calculate the rms value during this interval.

32. The equivalent horsepower duty cycle for a constant-speed motor is as follows: 90 hp for 6 sec, linear decrease from 60 hp at 6 sec to 20 hp at 24 sec, 10 hp from 24 sec to 30 sec, no load from 30 sec to 40 sec. Compute the required horsepower rating for the motor.

33. A mill motor has the following duty cycle: 120 hp equivalent over the first 15-sec interval while accelerating; 90 hp for the next 30 sec while running at constant speed; zero hp for the next 8-sec interval while being brought to rest; and a rest period of 30 sec. If the motor can dissipate heat 60% as well when accelerating and retarding as it can when running at normal speed, and 35% as well when at rest, calculate the required horsepower rating of the motor for this application.

34. A hoist motor for mine service is to have the following duty cycle: the torque is constant at 240 lb-ft over the first 10 sec during acceleration; from 10 sec to 60 sec, under normal speed, the torque decreases linearly from 160 lb-ft at the rate of 4 lb-ft per sec; from 60 sec to 68 sec the torque is constant at -80 lb-ft per sec while retarding; and from 68 sec to 80 sec the power is shut off while the motor is at rest. The ratio of torque to current is 5 lb-ft per amp. Plot the torque and current cycles for the motor and calculate the rms current. Assuming a normal running speed of 850 rpm, plot the curves of developed and equivalent horsepower for the motor. Calculate the required rating of the machine on the assumption that the motor can dissipate its heat 70% as well when accelerating and retarding as it can at full speed, and 50% as well when at standstill.

35. Show that the form factor of a sine wave is 1.11.

36. Determine the form factor of (a) a rectangular wave, (b) a triangular wave.

37. Sketch the waveform of $i = \sin \omega t + 0.3 \sin 3\omega t$. Determine the form factor.

38. Repeat problem 37 for a current of $i = \sin \omega t - 0.3 \sin 3\omega t$.

39. Compare the effective values of the currents of problems 37 and 38.

CHAPTER III

CIRCUIT CONSTANTS

3.1. The Circuit Constants. From the elementary study of mechanics in physics, it will be remembered that the motion of a body always is opposed in part by *frictional* forces. Furthermore, physical bodies possess the property of *mass* or *inertia* which tends to maintain the initial conditions of motion, the position of rest being but a limiting case. At the same time, all bodies possess *elasticity*, or spring action, such that, when deformed, forces exist tending to restore them to their unstrained positions. These phenomena all are directly taken into account in physical laws in mechanics. The first two are included in the first two laws of motion stated by Newton, and the third is fundamental to Hooke's law. These three circuit parameters for the mechanical system relate completely the component reactive forces, manner of motion, and power and energy, to given applied forces.

Electrical circuits possess properties analogous to those of the mechanical system. Electrical *resistance* takes the place of mechanical friction, *inductance* which portrays electrical inertia plays the counterpart of mechanical inertia, and *capacitance* indicates the measure of electrical displacement with the applied electromotive force. Resistance, R, is a property of the conducting circuit, and its effects are observable only when currents are flowing. Electric charges in motion constitute an electric current, and associated with the current there is a magnetic field to which is ascribed the phenomena taking place in the medium surrounding the current, and the reactive effects upon this current or upon other currents. Inductance, M (mutual) and L (self-), is a coefficient expressed in terms of these effects and which serves to characterize the magnetic field. Reactions of the magnetic field are observed only while it is changing, that is, while the current producing it is varying, and, consequently, inductance is of prime concern in all a-c circuits. Surrounding electric charges, either at rest or in motion, is also an associated electric field, differing from the magnetic field. The greater the voltage between two separated conducting plates, for example, the greater is the accompanying charge on the plates and the intensity of the electric field of force they produce about them. Capacitance, C, is a measure of the charge stored to the corresponding voltage, and serves to characterize the electric field and its effects. These parameters of the electric circuit determine completely component reactive voltages, the manner of current variation, and power and energy, for the voltage impressed on the circuit.

It should be remarked at the outset that more than being merely representative of circuit properties which simply may influence the flow of current or the magnitude of voltage, they characterize methods of energy transference or transformation. Resistance always is associated with the dissipation of electrical energy in heat (Joule's law). Inductance, both self- and mutual, connotes the storage of electrical energy as kinetic energy in magnetic fields. Capacitance gives a measure of electrical energy stored as potential energy in electrostatic fields. These last points of view are the broadest and most fundamental which may be given the circuit constants.

Although referred to as constants, these quantities are not strictly constant in the sense that their magnitudes may be invariable. All will vary somewhat in any circuit, but the variations may be so small that in the great majority of problems the differences may be quite properly ignored. This is the general reasoning behind the terminology *constant*. Strictly speaking, circuit constants are only constants so-called.

The constants may be lumped or distributed. Actually, all constants are distributed throughout the circuit, but they may be given lumped values when analysis allows their total effects in a branch to be considered at a fixed point. For example, the total resistance of a short loop of wire in general may be treated by assigning it a single value which may be thought to exist at any point along the loop, but the resistance of a long transmission line must, in general, be treated per unit length of line. Again, high frequencies may require analysis by way of the distributed effects of resistance. It should be apparent that the method of employing lumped constants, whenever legitimate, leads to simpler mathematical analyses.

3.2. Resistance and Resistors. Resistance is a property of a conductor or conducting path which opposes the flow of current; also it is that property by virtue of which electrical energy is transformed into heat. This second characteristic leads to a better and more general understanding of resistance, because inductance and capacitance also have effects on the magnitude and manner of current flow but do not possess the property of converting electrical energy into heat. All materials have resistance, although in widely varying magnitudes. Some, such as metals and many chemical solutions, conduct electricity readily and are said to have low values of resistance. Others, such as glass, polystyrene, bakelite, mica, etc., are practically nonconducting and are called insulators. There is no material, however, which is either a perfect conductor having zero resistance or a perfect insulator having infinite resistance.

Circuit elements in which the residual inductance and capacitance are negligible at low frequencies (every resistor has a certain amount of inductance and capacitance associated with it), are called resistors, and they may have fixed or variable magnitudes of resistance. Fixed resistors may be built to have but a few millionths of an ohm, others to have a resistance of many million ohms. Variable resistors, called rheostats, may be single resistors or a group of resistors so arranged that the over-all resistance can be varied, either by moving an arm or by switches. The variations usually are in steps. Water rheostats give a uniformly smooth gradation in current. Wire-wound resistors usually are alloys of copper, nickel, manganese, chromium, iron, etc., having high specific resistances. Molded resistors, widely used in electronic circuits, are of the character of a baked mixture of powdered carbon and a ceramic material. These resistors also are usually much smaller in size and much higher in resistance than those used in power circuits. The magnitudes of their resistances are given by a color code, with perhaps a second designation indicating tolerance.

There is no material which does not possess the property of resistance, and, although resistance may be undesirable in some circuits, it is purposely used to a very great extent as a means of control. Its applications lie in limiting the magnitude of current and in adjusting current to a given value in a circuit or in an associated branch by permitting control of that current over a chosen range or in a desired manner. Also, resistance has a specific application in heating, for example, electric furnaces, baking ovens, dryers, heating pads, and a great variety of uses by way of small low-power units. For load tests on some generators the resistance between electrodes immersed in a tank of water (water rheostat) is used to vary the current and to dissipate the output of the machine.

3.3. The Ohm. Experiment shows that for a given d-c circuit the potential difference required to force current through the circuit is directly proportional to the magnitude of the current. The constant of proportionality is the circuit resistance, and in practical units is given by the relation

$$R \text{ (ohms)} = \frac{E \text{ (volts)}}{I \text{ (amperes)}} \tag{3.1}$$

This expression, in any arrangement of the terms, is Ohm's law, derived initially for direct current, although applicable with modification to a-c circuits as well.

Experiment also shows that for all electrical circuits

$$\begin{aligned} H \text{ (heat energy)} &= \int_{T_1}^{T_2} e_R i \, dt \\ &= \int_{T_1}^{T_2} \frac{e_R^2}{R} \, dt = \int_{T_1}^{T_2} i^2 R \, dt \text{ watt-seconds or joules} \end{aligned} \tag{3.2}$$

where $e_R = iR$ and is that component of the impressed voltage consumed in driving the instantaneous current i through the circuit resistance R. The above expression, showing the equivalence of electrical and heat energies, is known as Joule's law.

The ohm may be defined in any one of several ways, all of which are essentially equivalent. One ohm is the ratio of one volt to one ampere. It also is the proportionality constant in Joule's law and is that resistance in which one watt-second of heat energy is produced by one ampere flowing for one second. This latter statement defines the mks ohm. In high-frequency circuits it is customary to consider the effective resistance to be defined and calculated as the ratio of power transferred to the square of the current, a definition very closely allied to that obtained from Joule's law.

The international ohm, as established in 1893, was defined by the resistance of a specified column of mercury. Each national standards laboratory maintains wire resistance standards which have extreme permanency and the values of which were originally determined by comparison with the mercury standard. With the adoption of the mks system of units, it became necessary to evaluate these resistance standards in terms of the absolute ohm. In the method employed by the Bureau of Standards,[1] the inductance of a self-inductor in international henries is compared with its value in absolute henries. The ratio of these values of inductance is the same as the ratio of a resistance in international ohms to its value in absolute ohms. The value in international henries was determined by bridge measurements which involve the mean solar second and the international ohm. The inductance in absolute henries was calculated from physical dimensions with a probable error of less than 4 parts per million. With the high accuracy that can be obtained in determining the value of the absolute ohm, there is no longer any need of making measurements on the mercury ohm. Commercial resistance standards are available that can be certified by the Bureau of Standards to within a few parts in a million.

3.4. The Circular-mil-foot. The resistance of any conductor is found by experiment to vary directly with length and inversely with cross-sectional area. Further, for a given length and area of conductor, resistance depends upon the kind of material used and upon the temperature. These experimental results are expressed by the relation

$$R = K\frac{l}{A} \text{ ohms} \tag{3.3}$$

where K is a characteristic constant of the material at a given temperature and is called the specific resistance or resistivity. It is the resistance of a piece of material of unit length and unit area at a specified temperature. Specific resistance is given in ohms per meter cube, in which case the calculation of resistance from the above equation requires that l and A be in meter units. The specific resistance of the internationally recognized annealed-copper standard (corresponding to 100 per cent conductivity) at 20°C is 1.7250×10^{-8} ohm-meter. Copper of other quality and other conductors are conveniently expressed in terms of their conductivity (the reciprocal of resistivity) relative to this standard. Commercial copper conductors have a conductivity varying from approximately 100 per cent for annealed copper to 97 per cent for hard-drawn copper. Aluminum has a conductivity of approximately 65 per cent.

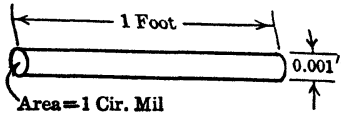


FIG. 3.1. The circular-mil-foot.

In practice, it is found more convenient to express the specific resistance of conductors in ohms per circular-mil-foot at a given temperature. The foot is the adopted unit of length, and the circular mil the unit of area; hence, resistivity is resistance of one foot of material having a

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[1] H. L. Curtis, *loc. cit.*

cross-sectional area of one circular mil and at a stated temperature, Figure 3.1. For example, at 0°C the resistance of a copper conductor (100 per cent conductivity) one foot long and one circular mil in area is 9.56 ohms; at 20°C it is 10.38 ohms; and at 60°C it is 12 ohms.

The circular mil, Figure 3.2, is a unit of area and is used in practice to state and to compare conveniently the areas of conductors, particularly wires and cables. The circular mil is an *area* equal to the area of a circle one mil, or one thousandth of an inch, in diameter. The number of circular mils in a circular conductor therefore is equal to the square of the diameter in mils. The square mil is an area equal to the area of a square one mil on each side. The square mil thus is the larger unit, and $4/\pi$ circular mils are contained in one square mil. The number of circular mils in a conductor having any cross-sectional shape therefore is equal to 1.273×10^6 times the area of that conductor in square inches.

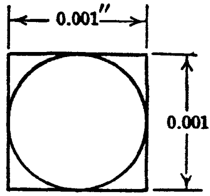

FIG. 3.2. Relation of the circular mil to the square mil.

3.5. Effect of Temperature on Resistance. Resistance also is a function of temperature, and for most materials it increases with the temperature. The practical empirical relation expressing the temperature effect is

$$R_T = R_0 (1 + \alpha_0 T) \tag{3.4}$$

R_T is the desired resistance at temperature T in degrees centigrade, R_0 is the resistance at 0°C, and α_0 is the temperature coefficient of resistance based on 0°C. More general yet,

$$R_2 = R_1[1 + \alpha_1(T_2 - T_1)] \tag{3.5}$$

In this last expression, R_2 is the resistance at temperature T_2°C, R_1 is the known resistance at T_1°C, and α_1 is the temperature coefficient based on temperature T_1. The coefficient α_1 varies with the initial temperature T_1. For copper, it has values of 0.00427, 0.00393, and 0.00364 based on initial temperatures of 0°C, 20°C, and 40°C, respectively. For example, the resistance at 60°C of a conductor having a resistance of 10 ohms at 20°C is

$$\begin{aligned} R_{60} &= 10[1 + 0.00393(60 - 20)] \\ &= 11.57 \text{ ohms} \end{aligned}$$

Since the specific resistance of copper is 12 ohms at 60°C, which is the normal operating temperature for much electrical apparatus, very conveniently,

$$\begin{aligned} R_{60} &= 12 \frac{\text{length in feet}}{\text{area in circ. mils}} \\ &= \frac{\text{length in inches}}{\text{area in circ. mils}} \text{ ohms} \end{aligned} \tag{3.6}$$

WIRE TABLES *

STANDARD ANNEALED CIRCULAR COPPER WIRE

AMERICAN WIRE GAGE (B. & S.)

Gage Number	Mils Diam.	Circ. Mil. Area	Abs. Ohms per 1,000 ft †		Pounds per 1,000 ft
			20°C	75°C	
0000	460	212,000	0.0490	0.0596	641
000	410	168,000	0.0618	0.0752	508
00	365	133,000	0.0780	0.0948	403
0	325	106,000	0.0983	0.120	320
1	289	83,700	0.124	0.151	253
2	258	66,400	0.156	0.190	201
3	229	52,600	0.197	0.240	159
4	204	41,700	0.249	0.302	126
5	182	33,100	0.314	0.381	100
6	162	26,300	0.395	0.481	79.5
7	144	20,800	0.498	0.606	63.0
8	129	16,500	0.629	0.764	50.0
9	114	13,100	0.793	0.964	39.6
10	102	10,400	0.999	1.22	31.4
11	90.7	8,230	1.26	1.53	24.9
12	80.8	6,530	1.59	1.93	19.8
13	72.0	5,180	2.00	2.44	15.7
14	64.1	4,110	2.53	3.07	12.4
15	57.1	3,260	3.19	3.88	9.86
16	50.8	2,580	4.02	4.89	7.82
17	45.3	2,050	5.07	6.16	6.20
18	40.3	1,620	6.39	7.77	4.92
19	35.9	1,290	8.06	9.80	3.90
20	32.0	1,020	10.2	12.4	3.09
21	28.5	810	12.8	15.6	2.45
22	25.4	642	16.2	19.6	1.95
23	22.6	510	20.4	24.8	1.54
24	20.1	404	25.7	31.2	1.22
25	17.9	320	32.4	39.4	0.970
26	15.9	254	40.8	49.7	0.769
27	14.2	202	51.5	62.6	0.610
28	12.6	160	64.9	79.0	0.484
29	11.3	127	81.9	99.6	0.384
30	10.0	101	103	126	0.304
31	8.93	79.7	130	158	0.241
32	7.95	63.2	164	200	0.191
33	7.08	50.1	207	252	0.152
34	6.31	39.8	261	318	0.120
35	5.62	31.5	329	400	0.0954
36	5.00	25.0	415	505	0.0757
37	4.45	19.8	523	637	0.0600
38	3.97	15.7	660	803	0.0476
39	3.53	12.5	832	1,010	0.0377
40	3.15	9.89	,050	1,280	0.0299

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* Based on a table published by U. S. Bureau of Standards.

† Resistance at the stated temperature of a wire whose length is 1,000 ft at 20°C.

3.6. Wire Tables. Most of the information needed on conductors for use in calculations on resistance is contained in wire tables. Such tables show wire diameters and areas, resistances per 1,000 feet or per mile at given temperatures, and weights per 1,000 feet or per mile. Many tables, particularly those published by wire and cable manufacturers, also show wire diameters inclusive of various insulating coverings as well as diameters for the bare wire. It should not be inferred that cylindrical wires only are drawn. A very great many are not of that shape, and practically any desired form of cross section can be obtained.

The most common type of electrical conductor, though, is standard annealed copper wire of circular cross section. The size of the wire is usually designated by a gage number. The American Wire Gage, formerly called Brown and Sharpe, is the most widely used and is based on a constant ratio between cross sections of successive gage numbers. Likewise, the resistances of successive gage numbers are in constant ratio. This ratio of resistances is 1.261. Since the cube of this value is 2.005, it follows that the resistance approximately doubles every 3 gage sizes. Also, for every 10 gage numbers the resistance increases approximately by the factor 10. The approximate constants for any gage number can readily be obtained without reference to the wire table by using these relations and No. 10 gage wire as a reference. No. 10 gage wire has the following approximate relations that are relatively easy to remember: diameter of 0.1 in., resistance of 1 ohm per 1,000 ft, and a weight of 10π lb per 1,000 ft.

3.7. A-c, or Effective, Resistance. All currents produce magnetic fields which vary in strength with the magnitudes of those currents. In a conducting material, the changing magnetic field created by a varying current gives rise to circulating, or eddy, currents in that conductor and in neighboring circuits. In the case of armature conductors in machines, eddy currents, caused by the conductor currents, are developed in the adjacent iron as well. Armature cores are laminated primarily to reduce the magnitudes of eddy currents, howsoever they may be produced.

Eddy currents exist because of transformer action, and their presence means a conversion of electrical energy to heat. This additional energy is taken into account through an a-c, or effective, resistance such that the square of the measured line current times this effective resistance will equal the total power loss, the rate at which electrical energy for the entire system is being transformed into heat. The effective resistance of a machine, for example, may be from 5 to 15 per cent greater than the d-c resistance. It is difficult to calculate this equivalent resistance for even very simple cases, but it often can be measured. In such instances, it is obtained as the ratio of power to the square of the current.

At high frequencies, the current is not uniformly distributed over the cross section of the conductor but tends to be concentrated near the surface, thereby leading to an increased current density at the surface, an equivalent decrease in conductor area, and a corresponding increase in resistance. This phenomenon, called skin effect, is due to the emf induced by the magnetic flux within the conductor, that is, the internal inductance of the conductor. If the conductor be considered as a group of parallel current filaments, the current filaments near the center have the greatest amount of magnetic flux surrounding them and hence the greatest inductive reactance. The current division through the filaments will obey the usual laws of parallel circuits and, as a result, the filaments having the highest reactance will carry the least current. This effect increases with the cross-sectional dimension of the conductor and with the frequency and may give rise to an a-c resistance many times the d-c resistance of the conductor. The table gives the ratio of the a-c resistance to the d-c resistance for several copper conductors. The figures of the table indicate that, at power frequencies, the effect is only of importance for the larger conductors, whereas, at radio frequencies, it is a major factor and special stranded conductors are commonly employed. Hollow tubing is also used since the central part of a solid conductor is not effective in carrying current under high frequency. The effective resistance of a coil at radio frequencies may be many times the d-c resistance because of skin effect in the conductor of each turn and also because of the proximity effect of adjacent turns which produces an additional non-uniformity in the current distribution over the cross section of the conductor. Losses in dielectrics and metal objects in the vicinity of the coil also increase its effective a-c resistance. Effective resistance in capacity circuits may take into account dielectric losses as well as those due to the main conducting currents.

RATIO OF A-C TO D-C RESISTANCE

Frequency	$\frac{R_{ac}}{R_{dc}}$ for #0000 AWG	$\frac{R_{ac}}{R_{dc}}$ for #10	$\frac{R_{ac}}{R_{dc}}$ for #30
60	1.004	1.000	1.000
1,000	1.64	1.003	1.000
10,000	4.68	1.245	1.000
100,000	14.20	3 34	1.003
1,000,000	45.0	10.0	1.248

3.8. Measurement of Resistance. Resistance is measured in several ways in practice. Common methods employ any one of several forms of bridge circuit, for example, the Wheatstone bridge, or the Kelvin bridge, or variations of these circuits. Other methods employ the voltmeter and ammeter on a d-c test, the wattmeter and ammeter on an a-c test, comparison with a known resistance, the ohmmeter, the megger, and—for radio-frequency resistance—special shielded bridges, a substitution test, or the calorimeter. These methods vary widely and may be greatly restricted in use. The particular one chosen will depend upon the magnitude of the resistance to be measured, the conditions under which it is to be measured, and the accuracy desired. The bridge circuit, when applicable, is the most accurate of all.

3.9. Examples. The field coils of a 6-pole a-c motor have 330 turns each of No. 7 wire. The mean length of turn (MLT) is 46.5 in. What is the 60-degree resistance of the field coils, and what is the weight of the copper they contain?

$$\text{resistance of coils} \quad \frac{6 \times 330 \times 46.5}{20{,}800} = 4.43 \text{ ohms total}$$

$$\frac{6 \times 330 \times 46.5}{12 \times 1{,}000} \times 0.577 = 4.43 \text{ ohms}$$

(0.577 ohm = resistance per 1,000 ft of No. 7 wire at 60°C)

$$\text{weight of copper} = \frac{6 \times 330 \times 46.5}{12 \times 1{,}000} 63 = 483 \text{ pounds}$$

An armature winding of an a-c generator contains 108 6-turn coils arranged in 3 groups of 36 coils each, and each group consists of 2 parallel paths of 18 coils each. All coils have a mean length of turn of 62 in., and a cross-sectional copper area of 0.200 × 0.080 in. Calculate the 60-degree resistance of one group of coils and the weight of copper in the entire winding. (Copper weighs 0.32 lb per cu in.)

$$\text{resistance per coil} = 6 \frac{62}{0.200 \times 0.080 \times 1.273 \times 10^6} = 0.01824 \text{ ohm}$$

18 coils are in series in each path, and the over-all resistance of 2 parallel paths is half that of either path, hence

$$\text{resistance per group} = 0.01824 \times 18 \times \tfrac{1}{2} = 0.164 \text{ ohm}$$

$$\text{total weight of copper} = 108 \times 6 \times 62 \times 0.200 \times 0.080 \times 0.32 = 206 \text{ pounds}$$

3.10. Mutual and Self-induction. If two circuits are in close proximity to each other, the current in either one will produce flux which links both circuits. These flux linkages are mutual linkages and the circuits are said to be coupled by way of the flux, Figure 3.3. An alternating current in one circuit will induce a voltage in the second circuit because of the accompanying change in flux linkages, the voltage being the *voltage of mutual induction*. Such a phenomenon is voltage generation by transformer action, already discussed. Many circuits may be coupled to each other, each one inducing mutual voltages in all its neighboring circuits when it undergoes any change in current. Each induced voltage has its own direction which must be recognized.

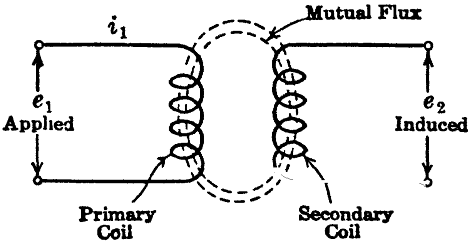


FIG. 3.3. Mutual flux in a coupled circuit. The voltage of mutual induction is e_2.

All flux which any circuit creates must, of necessity, link that circuit regardless of the fact that some of the flux also may link with other circuits. As the current varies, the changing magnetic field in being built up about the circuit which produces it cuts those conductors and induces a voltage in them by self-action, Figure 3.4. This induced voltage is the *voltage of self-induction.*

The voltages of mutual and self-induction exist only when current and its associated flux are changing, and these voltages inherently are in opposition to that changing current and flux. With a rising current and consequent increase in flux linkages the induced voltages are in such directions that the currents they would cause to flow in their respective circuits would tend to prevent the establishment of those flux linkages. Decreasing linkages induced by falling currents produce effects tending to maintain the existing flux linkages. Electrical circuits hence exhibit the characteristics of electrical inertia because changes of current induce voltages which oppose those changes of current.

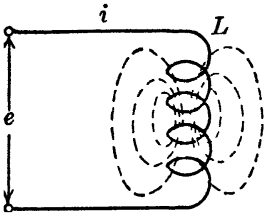


FIG. 3.4. Self-induced flux in a single coil. In magnitude, e is both the applied voltage and the voltage of self-induction.

It should be apparent that mutual induction is the general physical observation, and that self-induction is but the special case wherein only a single circuit is considered. Although these two types of induction often are treated distinctly from each other, because of the manner of their use in the analysis of electrical circuits and apparatus, the fundamental principle is that of *induction.* The discussion of inductive effects in one circuit, or in two circuits, etc., then becomes but a specific application of the general principle. Obviously, self-induction always is present with changing flux linkages whether or not mutual induction takes place between distinct circuits. Also, the total self-inductive effects of a coil really take into account, although not stated as such, the mutual reactions among its many turns.

3.11. Coefficients of Induction. All induced voltages are directly proportional to the time rate of change of flux linkages. Consequently, where flux is a linear function of the current which produces it, the voltage induced in a circuit because of a current changing in a neighboring circuit is directly proportional to the time rate of change of that current. This is an experimental fact. The voltage of mutual induction is expressed symbolically by the relations

$$\begin{aligned} e_2 &= M_{12}\frac{di_1}{dt} \text{ volts} \\ e_1 &= M_{21}\frac{di_2}{dt} \text{ volts} \end{aligned} \tag{3.7}$$

where $M_{12} = M_{21} = M$ and only two circuits are considered.

The factor of proportionality M is the coefficient of mutual induction, or simply the *mutual inductance* of the one circuit with respect to the other. In the order of the subscripts, M_{12} denotes the effect of the current in circuit 1 in inducing a voltage in circuit 2. Since action and reaction are equal between two circuits, the coefficients have the same magnitude. The advantage of retaining the subscripts is apparent only when more than two circuits are taken into account. Because of the reciprocal actions taking place between any two circuits, the same time rate of change of current in either will induce a voltage of the same magnitude in the other.

Mutual inductance is defined as the ratio of the voltage produced in the second circuit to the time rate of change of current in the first. The henry is the practical unit of inductance, and mutual inductance has a magnitude of one henry when the current in a circuit changing at the rate of one ampere per second induces one volt in the second circuit.

For a single circuit in which the effects of self-induction are considered, the voltage of self-induction is

$$e = L\frac{di}{dt} \text{ volts} \tag{3.8}$$

where the factor of proportionality L is the coefficient of self-induction, or simply the *self-inductance* of the circuit. Self-inductance is defined as the ratio of the self-induced voltage to the time rate of change of current which produces that voltage. Self-inductance has a value of one henry when current changing at the rate of one ampere per second induces one volt.

As with resistance, inductance is a proportionality factor. It is a derived unit relating the more fundamental units of voltage, current, and time. Since the coefficients of induction characterize the behavior of circuits under the influence of varying currents, their use is indispensable in the analysis of alternating-current systems.

3.12. Relation of Self-inductance to Magnetic Circuit. Because the magnitudes of induced voltages depend upon the total flux produced by a given current, as well as upon the time rate of change of that flux, it follows that the configurations of the electrical circuits and the magnetic paths influence the self- and mutual inductances. From the results of Ampère's experiments and the extension of the basic ideas by Maxwell, the relation between the magnetic field and the electric circuit can be expressed by

$$\oint H_l\, dl = i \text{ amperes} \tag{3.9}$$

and

$$\beta = \mu H \text{ webers per square meter} \tag{3.10}$$

The student should realize that these relations and other basic equations are but mathematical formulations of physical phenomena that express the results of experimental investigations. The line integral in equation 3.9 is to be taken around a closed path. H_l is the component of the magnetic field intensity measured in the direction of dl and is expressed in ampere turns per meter (or simply amperes per meter), dl is in meter units, and i is the total current (conduction and displacement) which is linked by the path of integration. Equation 3.9 may be stated as follows: The work required to carry a unit magnetic pole around any closed path is equal to the total current linked by that path.

The permeability μ in equation 3.10 is defined by the relation

$$\mu = \mu_r \mu_0 \text{ henry per meter} \tag{3.11}$$

where μ_r is the permeability relative to free space (a numeric), and $\mu_0 = 4\pi \times 10^{-7}$ henry per meter is the permeability of free space.

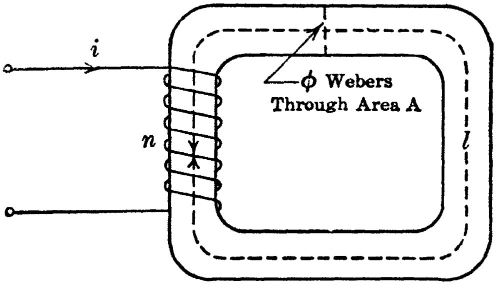


FIG. 3.5. An iron-core inductance.

For a coil of n turns carrying i amperes and surrounding a magnetic circuit of length l and cross section A, Figure 3.5, the magnetic flux can be obtained from equations 3.9 and 3.10 as follows:

$$H = \frac{ni}{l} \text{ ampere turns per meter} \tag{3.12}$$

$$\beta = \mu \frac{ni}{l} \text{ webers per square meter} \tag{3.13}$$

$$\Phi = A\mu \frac{ni}{l} = \frac{ni}{l/\mu A} \text{ webers}$$

$$= \frac{ni}{\mathcal{R}} \text{ webers} \tag{3.14}$$

$$= \frac{\text{magnetomotive force}}{\text{reluctance}} \text{ webers}$$

Equation 3.14 is sometimes called the expression for Ohm's law of the magnetic circuit because of its similarity to his law for the electrical circuit.[2]

---

[2] A more complete treatment of the magnetic circuit is presented in Appendix I.

The voltage of self-induction for a coil is

$$e = L\frac{di}{dt} = n\frac{d\Phi}{dt} = n\frac{d\Phi}{di}\frac{di}{dt} = \frac{n^2}{\mathcal{R}}\frac{di}{dt} \tag{3.15}$$

whence

$$L = n\frac{d\Phi}{di} = \frac{n^2}{\mathcal{R}} \text{ henries} \tag{3.16}$$

From this last relation, it is observed that the self-inductance of a coil varies directly as the square of the number of turns, and inversely as the reluctance of the magnetic circuit. The coefficient of self-induction therefore is a function of the geometry of the magnetic circuit and physically is representative of the configurations of the electric and magnetic systems. This is true also for mutual inductance. It is by equations developed from elementary expressions of the above type that inductance is calculated. Inductance formulas are available in the various engineering handbooks for the accurate calculation of both mutual and self-inductance of various configurations. The value of an inductance can be readily determined in the laboratory either from a-c bridge measurements or by measuring the rate of rise of current when the circuit is connected to a d-c voltage source, as is described in Section 3.18.

3.13. Example on Inductance Calculation from Circuit Dimensions. A transmission line consists of 2 parallel cylinders, each having a diameter of 0.2 cm and separated 40 cm between centers, as is indicated in Figure 3.6. Calculate the inductance per meter length of line.

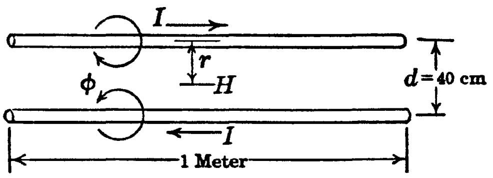


FIG. 3.6. Inductance of a transmission line.

The value of H at a distance r from a single conductor carrying a current I can be determined from equation 3.9, that is,

$$2\pi r H = I$$

or

$$H = \frac{I}{2\pi r} \text{ amperes per meter} \tag{3.17}$$

and

$$\beta = \mu\frac{I}{2\pi r} \text{ webers per square meter} \tag{3.18}$$

The total flux between the two conductors per meter length of line due to the current in one conductor is

$$\Phi = \int_{r_c}^{d} \beta\, dr = \frac{\mu I}{2\pi}\int_{r_c}^{d}\frac{dr}{r}$$

$$= \frac{\mu I}{2\pi}\log_e\frac{d}{r_c} \tag{3.19}$$

where r_c is the radius of the conductors and d is their separation between centers.

The total flux linking the circuit due to currents in both conductors is double this value and the inductance is given by the equation

$$L = \frac{\mu}{\pi} \log_e \frac{d}{r_c} \text{ henries} \tag{3.20}$$

Substituting the values of the problem

$$L = \frac{4\pi \times 10^{-7}}{\pi} \log_e 400$$

$$= 2.39 \times 10^{-6} \text{ henries per meter}$$

$$= 2.39 \text{ microhenries per meter}$$

3.14. Inductance as Flux Linkages per Ampere. From equation 3.16, self-inductance may be given as

$$L = n \frac{d\Phi}{di} \text{ henries} \tag{3.21}$$

where now it is shown to be a function of the magnetization characteristic of the circuit, Figure 3.7. This is merely part of the *B-H* curve discussed earlier in physics where now the ordinate is total flux linkage rather than flux density, and the abscissa is the corresponding magnetizing current rather than the magnetizing force. Consequently, the rate of change of flux with respect to current is the slope of the magnetization curve, and inductance may be highly variable.

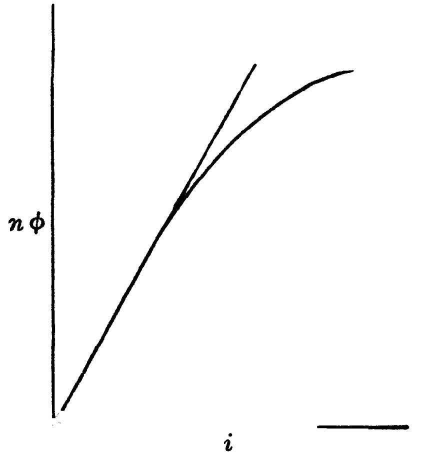


Fig. 3.7. Characteristic magnetization, or saturation, curve for a magnetic circuit.

From the origin of Figure 3.7 to the point at which saturation becomes evident, that is, where the curve begins to deviate from a straight line, inductance has a constant value. In any magnetic circuit one may use inductance as a single constant only when the current and its associated flux linkages are linearly related. Beyond this region inductance decreases, and very rapidly for many magnetic systems. Below the region of saturation, inductance may be given as the ratio of flux linkages to current, for example,

$$L = \frac{n\Phi}{i} \text{ henries} \tag{3.22}$$

Equation 3.22 is not a general relation but does serve to characterize inductance in terms of flux linkages per ampere. It is a concept useful to a degree, but it is not a fundamental definition. It must also be remembered that this expression implies, first, that all of the flux links all of the turns, and, second, that there is no saturation. If the limitations surrounding any particularized expression are well understood, then one may be justified in using it even though it is not exact. One then is content with an approximate result, knows the extent of error, and makes proper allowance in using the calculated value of the constant.

3.15. Relation Between Self- and Mutual Inductances. The voltage of mutual induction in the second of two coupled circuits is

$$e_2 = M_{12}\frac{di_1}{dt} = k_1 n_2 \frac{d\Phi_1}{dt} \tag{3.23}$$

whence

$$M_{12} = k_1 n_2 \frac{d\Phi_1}{di_1} = k_1 \frac{n_2}{n_1} L_1 \text{ henries} \tag{3.24}$$

Similarly, if circuit 2 is carrying current and producing a mutual flux,

$$M_{21} = k_2 n_1 \frac{d\Phi_2}{di_2} = k_2 \frac{n_1}{n_2} L_2 \text{ henries} \tag{3.25}$$

Note that the mutual inductances involve the magnetization characteristics of the individual magnetic circuits.

It can be shown that the mutual inductance is the same no matter which circuit produces the flux. Hence

$$M^2 = M_{12}M_{21} = k_1 k_2 L_1 L_2$$

and

$$M = k\sqrt{L_1 L_2} \text{ henries} \tag{3.26}$$

The term $k = \sqrt{k_1 k_2}$ is called the coefficient of coupling and expresses the extent to which two circuits are coupled. The maximum value the coefficient of coupling can have is unity and would be obtained if the flux completely linked both circuits.

Also, with the same qualifications as just stated for self-inductance, mutual inductance may be given as the ratio of the mutual flux linkages in a circuit to the current in the neighboring circuit which is producing that mutual flux.

$$M_{12} = \frac{n_2(k_1\Phi_1)}{i_1} = \frac{n_2\Phi_1'}{i_1} \text{ henries} \tag{3.27}$$

$$M_{21} = \frac{n_1(k_2\Phi_2)}{i_2} = \frac{n_1\Phi_2'}{i_2} \text{ henries} \tag{3.28}$$

k_1 is that percentage of the total flux created by the current i_1 which links the turns n_2, and consequently Φ_1' is a mutual flux. Φ_2' is defined similarly.

The differences of the total and mutual fluxes, that is, $\Phi_1 - \Phi_1'$ and $\Phi_2 - \Phi_2'$, are called leakage fluxes. The leakage flux of one circuit with respect to a second is simply that part of the total flux produced by the first which is not mutual to the two circuits, as is indicated in Figure 3.8. The partial linkages of a single coil, for example, exist by virtue of leakage fluxes which do not enclose all the turns. It should be pointed out that, although the mutual coefficients M_{12} and M_{21} are equal for two coils, the number of turns and the total and mutual fluxes may be quite different. Also, in general, k_1 and k_2 are not equal.

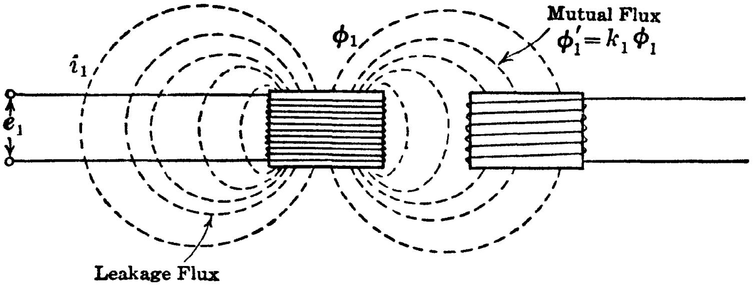


FIG. 3.8. Mutual and leakage flues in a coupled circuit with one coil carrying current.

3.16. Example on Inductance as Relating to Flux Linkages. Four coils of 500 turns each are in series addition on the same magnetic circuit, as is indicated in Figure 3.9. Each coil produces 0.03 weber of flux while carrying 10 amp, and 85 per cent of the total flux of any coil will link each of the other coils. Calculate the self-inductance of each coil, the mutual inductance between any two coils, and the over-all inductance of the entire circuit.

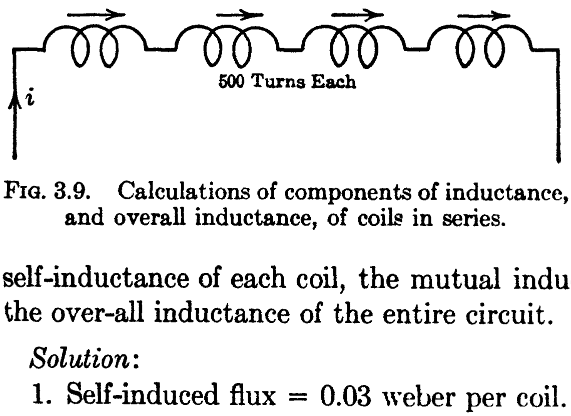


FIG. 3.9. Calculations of components of inductance, and overall inductance, of coils in series.

Solution:

1. Self-induced flux = 0.03 weber per coil.
2. Mutual flux per pair of coils = 0.85 × 0.03 = 0.0255 weber.
3. Leakage flux per coil = 0.15 × 0.03 = 0.0045 weber.
4. Total flux per coil = 0.03 + 3 × 0.85 × 0.03 = 0.1065 weber.
5. Flux common to all coils = 4 × 0.85 × 0.03 = 0.102 weber.
6. Self-inductance per coil $= \dfrac{500 \times 0.03}{10} = 1.5$ henries.
7. Mutual inductance per pair of coils $= \dfrac{500 \times 0.85 \times 0.03}{10} = 1.275$ henries
8. Over-all inductance = 4 self + 12 mutual = 6 + 15.3 = 21.3 henries.

In this special case, $k_1 = 0.85 = k_2 = k_3 = k_4$, and the coefficient of coupling between any two coils is also 85 per cent.

3.17. Losses in Inductances. All inductances have losses associated with them which at audio and radio frequencies often set the limit of performance of a circuit. On steady direct current, the only loss is that due to the d-c resistance of the winding. On alternating current, except at very low frequencies, the equivalent series resistance of an air-core coil is very much greater than the d-c resistance. The increase in resistance is due to the skin and proximity effects discussed in Section 3.7, hysteresis losses in dielectrics—the coil form for example—and eddy-current losses in metallic objects such as the chassis and supports in the vicinity of the coil. All these losses increase with frequency. If a core of magnetic material is used, the losses are further increased as a result of the eddy-current and hysteresis losses in the magnetic material. The eddy-current losses can be kept reasonably low by using thin laminations of the magnetic material. Laminations as thin as 0.002 in. have been used in inductances that have a fairly low loss at frequencies up to 1,000,000 cycles. Powdered magnetic cores have usable qualities at frequencies somewhat above a hundred megacycles. These consist of a fine powder (powder diameters as small as 10 microns are in commercial use) molded into form with an insulating binder. The molded cores are used to a considerable extent at all communication frequencies, both audio and radio, and have mainly displaced the laminated cores at frequencies where the use of very thin laminations was necessary.

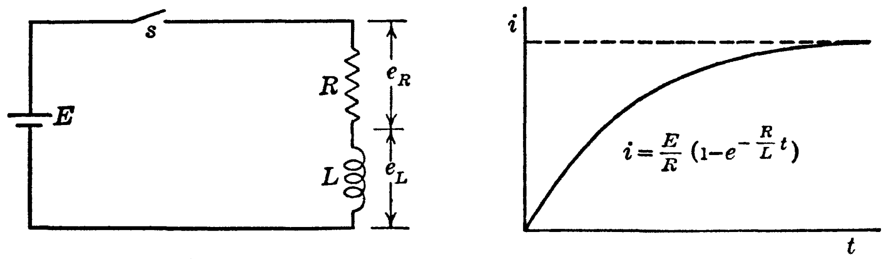
The effective series resistance R of a coil generally takes into account all the losses, its value being such that when multiplied by the square of the line current the product is equal to the total losses. The figure of merit of a coil is generally expressed by the ratio of the reactance (to be discussed later) to the resistance. This ratio is known as the Q of the coil, that is,

$$Q = \frac{\omega L}{R} \tag{3.29}$$

The Q of a coil at first increases with frequency, reaches a broad maximum, and then decreases as the frequency is further increased. A coil can generally be designed so that the maximum value of its Q occurs in the intended working frequency range of the coil. The Q of radio-frequency coils is usually in the range of from 100 to 300.

3.18. Energy Storage in Magnetic Fields. A pure inductance does not consume energy from an electrical system but stores the energy in its magnetic field. Energy is required to establish the magnetic field but not to maintain it. The energy stored in the field is similar to the kinetic energy stored in a freely moving body. It requires energy to put such a body in motion but none to maintain the motion if there are no frictional forces present. Since the field is present only when currents are flowing, the energy is directly associated with the flow of current and hence this form of energy is termed kinetic.

The expression for the energy stored in a magnetic field can be obtained by considering the energy absorbed by an inductance coil when it is connected to a d-c voltage source. Since any physical inductance coil will have resistance in the conductor, we should also consider the energy dissipated as heat in the copper of the winding. This energy lost as heat is in no way related to the energy of the magnetic field. On closing the switch s of the circuit shown in Figure 3.10a, a current will flow. There will be voltage drops across both the resistance and the inductance of the circuit. The summation of these voltage drops must equal the voltage of the supply. Expressing this relation in equation form



(a) (b)

FIG. 3.10. Rise of current in an R-L circuit.

$$E = e_R + e_L = Ri + L\frac{di}{dt} \tag{3.30}$$

In Chapter II, it was shown that the instantaneous rate of flow of energy (power) from a power source is given by the equation

$$P = Ei \text{ watts}$$

It follows that the energy taken from the supply, in t seconds after closing the switch, is expressed as

$$\begin{aligned} W &= \int_0^t Ei\,dt \\ &= \int_0^t \left[Ri^2 + Li\frac{di}{dt}\right] dt \\ &= \int_0^t Ri^2\,dt + \tfrac{1}{2}Li^2 \text{ watt-seconds} \end{aligned} \tag{3.31}$$

The first term of this last equation is the energy dissipated as heat in the resistance of the conductor. The second term is the energy stored in the magnetic field of the inductance and exists only while current is flowing. The energy stored in the magnetic field also can be expressed in terms of the field quantities β and H, and this treatment also leads to the expression given in equation 3.31. Basic considerations in electromagnetic theory show the energy stored per unit volume by a magnetic field is given by the expression

$$W \text{ per unit volume} = \int_0^{\beta} H\, d\beta \tag{3.32}$$

When the relation between β and H is linear, equation 3.32 becomes

$$W \text{ per unit volume} = \tfrac{1}{2}\beta H \text{ watt-seconds} \tag{3.33}$$

For a magnetic circuit of the type discussed in Section 3.12, the total energy stored in the volume is equal to

$$\begin{aligned} W &= \tfrac{1}{2}(A\beta)(Hl) \text{ watt-seconds} \\ &= \tfrac{1}{2}(\Phi)(ni) \text{ watt-seconds} \\ &= \tfrac{1}{2}Li^2 \text{ watt-seconds} \end{aligned} \tag{3.34}$$

where ni has been substituted for Hl from equation 3.12, and Li has been substituted for $n\Phi$ from equation 3.22.

With the exception of equation 3.32, it should be emphasized that the foregoing expressions for energy assume a constant permeability and that the inductance is not a function of current or flux. When the relation of β to H is non-linear, the energy stored in the magnetic system is given by either

$$W = \int_0^{\Phi_1} i\, d(n\Phi) \text{ watt-seconds} \tag{3.35}$$

or

$$W = Al\int_0^{\beta_1} H\, d\beta \text{ watt-seconds} \tag{3.36}$$

The integrals of equations 3.35 and 3.36 are the expressions for the shaded areas of Figures 3.11a and 3.11b, respectively.

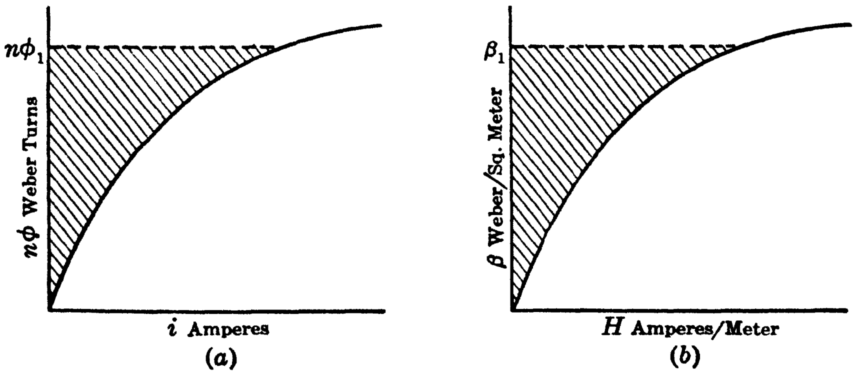


FIG. 3.11. Typical magnetization curves.

When the switch is opened, all this energy must be out of the field by the time the current reaches zero. The energy stored in the field clearly manifests itself by arcing at the switch contacts when the current in a large inductance is interrupted.

As a practical example of the phenomena of an inductive circuit, the energy stored in the magnetic fields of machines may be several hundred, or a few thousand, foot-pounds. Upon opening the field circuit, a very high voltage, dangerous to life and damaging to the insulation of the coils, may be induced. The switch also may be injured because of arcing between the blades and clips. These are the reasons that a resistance, called a discharge resistor, often is connected across the field terminals of a machine just before the field supply switch is opened. The stored energy thus is dissipated as heat in the discharge resistor and the coils and does not appear in arcing at the switch.

The solution of equation 3.30, for the initial condition that $i = 0$ at $t = 0$, is

$$i = \frac{E}{R}(1 - e^{-(R/L)t}) \text{ amperes} \tag{3.37}$$

This equation is plotted in Figure 3.10b. The quantity L/R is called the "time constant" of the circuit. It has a magnitude equal to the time in seconds required for the current to reach 63.2 per cent of its final value. When the current has reached its final value, the voltage drop across the inductance element is zero and the applied voltage is equal to the iR drop in the circuit. Although equation 3.37 indicates an infinite time for the current to reach its steady state value, actually, in any circuit this state is reached in a very short time, at most, a matter of seconds.

3.19. Importance of Inductance. Inductance has very important uses. In terms of these circuit constants are expressed the self- and mutual flux linkages of circuits, the self- and mutual induced voltages, and the energy stored in the magnetic fields. These uses are apparent from the discussions on the derivations of these quantities. In alternating-current work, a knowledge of inductance is vital to the solutions of such problems because of the associated *reactances* which are derived from the inductances. The performance of electrical machines is described almost wholly in terms of inductive, or reactive, coefficients, and a thorough understanding of the physical relations surrounding inductance readily will permit a good grasp of the principles of operation of electrical apparatus. Practically all problems involving varying currents require information at the start on the circuit inductance before calculations can be made. Mutual and self-inductances are indispensable circuit elements in radio-frequency transmitters and receivers. Inductances have important applications as "choke coils" in suppressing undesired a-c components from rectifiers supplying d-c power. They are inserted in communication cable lines, generally referred to as loading coils, to neutralize the effect of the inherent capacity of the cable and thereby make possible effective communication over greater distances. These are but a few of the many applications.

3.20. Capacitance of a Condenser. An electrical condenser is composed of two conducting materials separated by a nonconducting medium, or dielectric. A condenser possesses the property of storing charges and therefore may accumulate a quantity of electricity. The simplest type of condenser, and one quite familiar, is made of parallel plates of copper, aluminum, tin foil, etc., separated by air, mica, glass, paper, etc. Alternate layers of conducting and non-conducting material may be built up to form a multi-plate condenser, every other conductor connected to one terminal, and the alternate conductors to a second terminal. Although condensers perhaps are usually thought of as special devices constructed purposely for the storage of charge, all electrical systems exhibit condensive effects. The conductors of transmission lines, the turns of coils, the windings of machines, in fact, the wires of any circuit become plates of condensers.

When a voltage is applied to a condenser, a current will flow and a charge will be given the condenser plates. The quantity of electricity stored is in direct proportion to the applied voltage. Conversely, a charged condenser shows a potential difference between its terminals in direct ratio to the charge which it carries, this voltage being exactly equal to the voltage required in the storing process. The relation between the quantity q of the stored electricity and the voltage e is

$$q = Ce \text{ coulombs} \tag{3.38}$$

where C is the factor of proportionality and is called the *capacitance* of the condenser.

Capacitance is the ratio of the charge to the voltage required to store that charge, or the ratio of the charge on the condenser to the potential difference appearing between its terminals. It also may be defined as the quantity of electricity required to raise the potential of the condenser a given amount. In the mks system the unit of capacitance is the *farad*, and an electrical system has a capacitance of one farad when one coulomb is necessary to raise the potential one volt. Because the farad is a very large unit, the *microfarad* (10^{-6} farad) or the *micromicrofarad* (10^{-12} farad) is used in practice.

The derivation of capacitance is analogous to the derivation of the spring constant given in the symbolic statement of Hooke's law. The force required to elongate a spring is directly proportional to the elongation, the factor of proportionality being the spring constant. This constant, obtained from mechanics, and capacitance, an electrical circuit constant, bear analogous reciprocal relations to each other.

3.21. Geometric Properties of Capacitance. The basic problem in capacitance calculations is to determine the electric field distribution in the region of the conducting plates. When the field distribution can be determined by inspection of the geometry, the capacitance can be readily calculated with the aid of the following fundamental relations:

$$\int_s D_n \, ds = q \text{ coulombs} \tag{3.39}$$

$$D = \epsilon \mathcal{E} \text{ coulombs per square meter} \tag{3.40}$$

$$\mathcal{E} = -\frac{dE}{dx} \text{ volts per meter} \tag{3.41}$$

The surface integral is to be taken over a closed surface, D_n is the component of electric flux normal to the element of area ds, and q is the quantity of charge enclosed by the area. Equation 3.39 states that the electric flux diverging from a charge q is equal in magnitude to q.

The dielectric constant ϵ in equation 3.40 is defined by the relation

$$\epsilon = \epsilon_r \epsilon_0 \text{ farads per meter} \tag{3.42}$$

where ϵ_r is the dimensionless dielectric constant of the insulating material relative to free space and ϵ_0 is the dielectric constant of free space. In the rationalized mks system, ϵ_0 is given by

$$\epsilon_0 = 8.854 \times 10^{-12} \simeq \frac{1}{36\pi} \times 10^{-9} \text{ farad per meter}$$

Equation 3.41 states that the electric field intensity (the force in newtons on a unit charge) is equal to the negative of the potential gradient.

As an example of the use of these relations, consider the capacitance of a parallel-plate condenser, shown in Figure 3.12, in which the fringing of the electric flux lines in the vicinity of the edges can be neglected. Under this approximation, the flux, due to a charge q, will be uniformly distributed over the plate area A, that is,

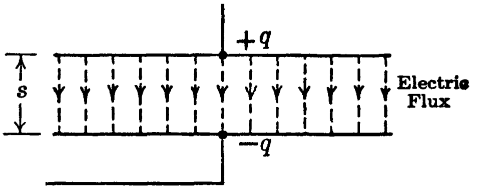


FIG. 3.12. Capacity of a parallel-plate condenser.

$$D = \frac{q}{A} \text{ coulombs per square meter}$$

and

$$\mathcal{E} = \frac{1}{\epsilon} D \text{ volts per meter}$$

$$= \frac{q}{\epsilon A}$$

Since the flux density is uniform, and hence the potential gradient $\mathcal{E}$ is constant between the plates, the voltage between the plates is given by the relation

$$E = s\mathcal{E}$$

$$= \frac{s}{\epsilon A} q \text{ volts}$$

where s is the plate separation in meters. The capacitance is

$$C = \frac{q}{E}$$

$$= \epsilon \frac{A}{s} \text{ farads} \tag{3.43}$$

all quantities being in mks units. Equations of capacitance, derived in a similar manner, for the more common configurations are available in the various engineering handbooks.

The capacitance of a condenser is measured most accurately by bridge methods, usually at a fixed frequency, for example, 1,000 cycles per second. Capacitance also may be calculated from capacitive reactance measurements at a known frequency, or it may be found by a d-c comparison test with a ballistic galvanometer.

Practically, all condensers have some energy loss in their use, although this loss may be extremely small and, in a good many cases, negligible. Whereas it is generally necessary to consider the losses in all practical inductors, condensers can be designed that approach so closely the ideal that the losses can be determined only by most careful and accurate measurements. The lead wires and the plates themselves, especially if the plates are long, possess resistance. Further, on an a-c voltage, dielectric hysteresis accompanies the reversals of voltage, and heating of the dielectric results, this being the main loss in a condenser at radio frequencies. Heating decreases the dielectric strength of the insulating material, and, if too great, may lead to breakdown of the dielectric. Losses usually are taken into account by assuming a resistance, either in parallel or in series with the condenser, of such magnitude that the copper loss in it equals the total energy loss of the condenser.

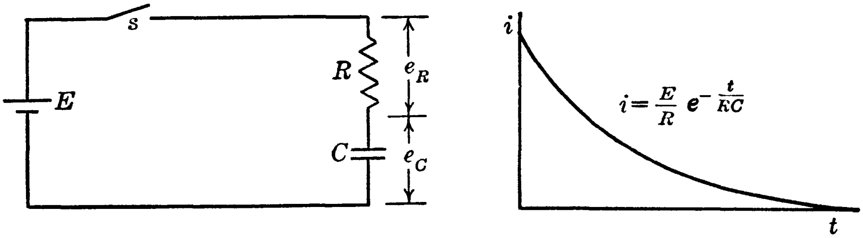
3.22. Condensers in Combination. When condensers are placed in parallel, the charge stored on each depends upon the applied voltage and is in no way affected by the adjacent condensers. Consequently, the total capacitance of the group in parallel is the sum of the capacitances of the serveral condensers. With condensers in series, and if leakage current can be neglected, the same amount of charge, but of opposite sign, is placed on each set of positive and negative plates of all condensers. Since no charge can flow through the insulation, the net charge on the inner plates of two consecutive condensers is zero, and only the outer plates carry a resultant charge. The voltage across the condensers varies inversely as the separate capacitances, and the over-all capacitance of the combination is calculated from the relation

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{3.44}$$

where C_{eq} is the equivalent capacitance of the group in series.

3.23. Energy Storage in Electric Fields. A condenser is similar in one respect to a pure inductance, in that it can store energy but does not consume it. When energy is stored in the electric field of a condenser, a voltage exists between the positive and negative plates, as given by equation 3.38. The energy in the electric field is always associated with charges on the plates and is in the form of potential energy. In this respect it is similar to the energy stored in a suspended weight.

In calculating the energy stored in a condenser, the same general method as was used in treating the inductance coil can be applied. The circuit arrangement for resistance and capacitance is shown in Figure 3.13a. The summation of the voltage drops across the resistance and across the condenser must equal the applied voltage, hence



(a) (b)

FIG. 3.13. Condenser charging current in an R-C circuit.

$$E = e_R + e_C = iR + \frac{q}{C} \text{ volts} \tag{3.45}$$

The energy taken from the supply is given by the expression

$$\begin{aligned} W &= \int_0^t Ei\,dt \\ &= \int_0^t \left[Ri^2 + \frac{q}{C} i \right] dt \\ &= \int_0^t Ri^2\,dt + \frac{1}{2}\frac{q^2}{C} \text{ watt-seconds} \end{aligned} \tag{3.46}$$

The similarity between equations 3.46 and 3.31 is evident. The second term of equation 3.46 is the energy absorbed by the condenser. This energy relation also may be written

$$\text{energy} = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} qe = \frac{1}{2} Ce^2 \text{ watt-seconds} \tag{3.47}$$

As is true of the energy stored in inductors, the energy stored in a condenser can be expressed in terms of the electric field. Experimental evidence indicates the energy stored by the electric field per unit volume is

$$W \text{ per unit volume} = \tfrac{1}{2}\varepsilon D \text{ watt-seconds} \tag{3.48}$$

The total energy stored is, therefore,

$$W = \frac{1}{2}\int_0^V \mathcal{E}D\,dv \text{ watt-seconds} \tag{3.49}$$

where the integration is to be carried out over the region occupied by the field. For the case of the parallel-plate condenser, the field is uniform, and the expression for the stored energy becomes

$$W = \frac{1}{2}(s\mathcal{E})(AD) \text{ watt-seconds} \tag{3.50}$$

which is equal to

$$W = \frac{1}{2}eq \text{ watt-seconds} \tag{3.51}$$

The solution of equation 3.45, for the initial condition that $q = 0$ (or $i = E/R$) when $t = 0$, is

$$i = \frac{E}{R}e^{-t/RC} \tag{3.52}$$

which is plotted in Figure 3.13b. The time constant for the circuit is RC and has the same interpretation as L/R of the R-L circuit.

In any actual circuit, the current reaches zero after a few seconds. At this time, the applied voltage exists wholly across the condenser. The capacitive circuit differs from that of the inductive circuit in that the energy remains stored in the condenser after the switch is opened. The existence of energy is shown by a spark when the condenser is discharged by short-circuiting its plates. The electrical energy stored in a condenser, although disconnected from any circuit, may be sufficient to endanger life. For this reason, it is always advisable to discharge a condenser after it has been in use.

PROBLEMS

1. No. 20 copper wire has a diameter of 32 mils. Calculate the resistance of 1,000 ft of this wire, assuming $K = 12$ ohms per circ.-mil ft, and compare your result with that obtained from an interpolation of the wire table for a 60°C resistance.

2. The 275-kv transmission line between Hoover Dam and the city of Los Angeles is 270 miles long. The copper conductors are in the general form of cylinders having an outside diameter of 1.400 in. and a wall thickness of approximately 0.09 in. Determine the resistance of one conductor at 20°C. For an assumed current density of 500 amp per sq in., determine the power loss in the conductor.

3. A heat test is taken on a coil. The initial current is 0.80 amp with a coil voltage of 49 volts. Fifteen minutes later the current is 0.68 amp with a coil voltage of 58 volts. What has been the average temperature rise through the coil? The initial temperature is 20°C. By what percentage has the resistance increased if, at the end of 7 min, the average temperature rise is 70°C?

4. A coil of given dimensions is designed for No. 9 wire. If the design is changed to call for No. 12 wire, the over-all dimensions remaining the same, what is the approximate ratio of resistances of the 2 coils?

5. The field coils of a shunt motor take 1.22 amp with 68 volts across them at an initial temperature of 20°C, and 1.08 amp with a drop of 70.5 volts after a load run. If

115 volts are applied to the total field circuit, what resistance is in the control rheostat? If the rheostat setting has not changed during the test, what is the final average temperature rise of the windings?

6. A 230-volt heating unit consists of two elements in parallel, each without taps. Maximum power input to the unit is 2,000 watts. What connections can be used for reduced heating, and what is the corresponding power input to the unit?

7. A test on a 50-watt, 120-volt tungsten-filament lamp shows a resistance of approximately 18 ohms at 20°C. What is the apparent current the lamp will draw just as 120 volts are applied to it? What is the percentage increase in resistance at normal operating temperature? Repeat the solution but for a 16-candlepower, 110-volt carbon-filament lamp which has a resistance of 485 ohms at 20°C and which takes a normal current of 0.413 amp at 110 volts.

8. What series resistance is required for a 25% reduction in the power taken by a 120-volt, 500-watt heating element? Assume that the resistance of the element does not change appreciably with temperature. What decrease in voltage would give the same decrease in power taken by the element?

9. A multi-conductor cable has 40 strands of No. 3 copper wire insulated from each other. What is the resistance at 55°C of 2,500 ft of cable if (a) all strands are connected in series, (b) all strands are in parallel, (c) all conductors are arranged in 5 equal and parallel paths?

10. The complete armature winding of a machine is composed of 504 coils arranged in 12 parallel paths. Each coil has one turn and is composed of 3 strands of rectangular copper placed side by side. The dimensions of a strand are 0.05 by 0.55 in. and the mean length of turn is 90 in. Calculate the over-all resistance of the winding at 60°C, and the weight of copper.

11. The armature winding of a machine is composed of 864 coils arranged in 18 parallel paths. Each coil has one turn of 0.135 by 0.55 in. conductor section and a mean length of turn of 96 in. Calculate the 60° resistance of the winding and the weight of copper.

12. In testing for the insulation resistance of a generator armature, one terminal of a 238-volt power supply is connected to the frame of the machine. The other power terminal is connected through a voltmeter to the armature winding. The voltmeter indicates 28.5 volts on the 300-volt range and has a resistance of 300,000 ohms. What is the insulation resistance of the armature winding?

13. An aluminum cable, steel reinforced, contains 30 circular conductors of aluminum of 0.1456 in. diameter, and 19 circular strands of steel conductor of 0.0874 in. diameter. Calculate the over-all resistance per mile of cable. $K = 17$ ohms per circ. mil ft for aluminum, and 95 for steel. By what percentage does the presence of the steel wire alter the resistance had the cable contained only 30 aluminum conductors?

14. A spool has the following net dimensions for a coil: 2 in. inner diameter, 6 in. outer diameter, 4 in. height. No. 27 copper wire is used which has a diameter of 0.0192 in. including insulation. On 230 volts, what will be the ampere-turns of the coil? What will be the heat dissipation in watts per square inch of total outside area of the coil?

15. A coil of 9,425 turns is wound with No. 28 copper wire. The coil dimensions are: 1.5 in. inner diameter, 4.18 in. outer diameter, and 3 in. height. What is the approximate diameter of the wire including insulation? If the maximum heat dissipation is 0.4 watts per sq in. of total outside area of the coil, what is the maximum voltage that may be applied? How many ampere-turns does the coil have on 115 volts d-c?

16. A face-plate rheostat has a rating of 125 volts, 300 ohms, 1.4–0.33 amp. What minimum resistance must be in series with this device so as not to exceed these values of current?

17. A rheostat for the field circuit of a machine has a rating of 125 volts, 250 ohms, 10–0.5 amp. Under what condition can the rheostat be placed directly across a 125-volt power supply? What minimum resistance should be in series with the rheostat so that it will not be damaged, regardless of the position of the movable arm? Over what range will the rheostat control the voltage of a 70-ohm field circuit if the supply voltage is 115 volts? For what maximum supply voltage can this rheostat be used as a potential divider? What mechanical change is required in the rheostat for this use? Sketch the circuit showing the device both as a rheostat and as a potential divider.

18. A d-c motor to drive a pump on board ship requires a feeder circuit of 200 ft, one way, from the main 240-volt bus. What size of copper wire should be installed if the motor is to deliver rated output of 25 hp, at 90% efficiency, with a voltage drop no greater than 4% for the feeder? $K = 10.8$ ohms per circ. mil ft.

19. A 35-kw d-c load is supplied over an 800-ft (one way) feeder circuit of No. 6 copper wire. For a feeder input voltage of 600 volts and a temperature of 30°C, what is the voltage at the load? What is the efficiency of transmission of electrical power for the feeder circuit?

20. A 600-volt d-c generator is to supply a 30-kw load 1,000 ft away at 550 volts over a transmission line. A second load requiring 50 amperes is to be connected to this line at its midpoint. What size of copper wire should be used for the line? $K = 10.6$ ohms per circ. mil ft. Calculate the kw of the 50 ampere load.

21. The 6 identical sections of a complete motor winding are connected in series to form a closed loop. Each section has 2 parallel electrical paths of 6 coils each. All coils have 25 turns of cylindrical copper wire, 72 mils diameter, and a mean length of turn of 35 in. If the winding is tapped between every two sections, what is the over-all resistance at 60°C between any 2 taps?

22. In making a resistance measurement by the voltmeter-ammeter method, the ammeter is placed ahead of the resistor and the voltmeter. What is the percentage error in the measurement if the resistance actually is 200 ohms and the resistance of the voltmeter is 15,000 ohms? For this arrangement of the instruments, and a given percentage permissible error, what is the maximum resistance that can be measured, expressed in terms of the resistance of the voltmeter? If the voltmeter is connected so as to read the drop across both the ammeter and the resistor, what is the minimum resistance that can be measured for a given percentage permissible error? Express the resistance in terms of the resistance of the ammeter.

23. A 500-turn coil has an inductance of 0.15 henry. What approximate number of additional turns are required to give the coil an inductance of 0.225 henry?

24. The over-all inductance of 2 coils in series is 12 henries. When the connections of one coil are reversed, the over-all inductance is 4 henries. What voltage is induced in either coil if the current in the other is reduced from 10 amp to zero in 0.1 sec? The coils remain in the same relative positions.

25. If the coils of problem 24 are alike except for the fact that one has twice the number of turns of the other, calculate their self-inductances and the coefficient of coupling.

26. The self-inductances of 2 coils are L_1 and L_2, respectively, and the mutual inductance between them is M. Show that when in series the over-all inductance of the combination is $L_1 + L_2 \pm 2M$.

27. Two coils, one of 500 turns and the other of 200 turns, are connected in series opposition. The coils are alike except for the number of turns, and 60% of the flux produced by either coil is mutual to the other. If the 500-turn coil creates 0.005 weber of flux with 6 amp, what is the over-all inductance of the combination?

28. A 1,000-turn coil produces a total flux of 0.01 weber when carrying 10 amp. Leakage flux is negligible. Calculate (a) the inductance per turn, (b) the mutual inductance between any 2 turns, (c) the total mutual inductance of the coil, and (d) the over-all coil inductance. What is the ratio of the total mutual inductance to the sum of the self-inductances of the individual turns?

29. Four coils of 400, 300, 200, and 100 turns, respectively, are all connected in series aiding. With 5 amp they produce a total mutual flux of 0.015 weber. Assuming the coils to be alike except for the number of turns, and neglecting leakage flux, calculate (a) the self-inductance of each coil, (b) the mutual inductances between the several pairs of coils, and (c) the over-all inductance of the circuit. Calculate the over-all inductance of the circuit if the connections to the 200-turn coil are reversed.

30. Two coils of 600 and 900 turns, respectively, are connected in series opposition. The coils are alike except for the number of turns, and for a current of 1.2 amp they produce a resultant mutual flux of 40,000 lines. With the coils in the same relative positions, but disconnected from each other, what rms voltage will be induced at the terminals of the 900-turn coil if the other carries a current of 0.5 sin $377t$ amp?

31. The following corresponding data are for a magnetic circuit:

NI (ampere-turns) =	50	100	150	200	300	400	600	800	1,000	1,300	1,600
ϕ (kilolines) =	150	700	2,400	4,000	5,700	6,500	7,300	7,800	8,100	8,300	8,500

The exciting coil of 100 turns carries 2 amp. If the current is increased 0.1 amp in 0.01 sec, what voltage is induced in the coil? What is the inductance of the winding for this position on the magnetization curve? For the same rate of change of current, determine the induced voltages for initial currents of 0, 1, 4, 6, 10, and 14 amp.

32. The field coils of a 6-pole d-c generator have 1,200 turns each and are in series electrically. Sketch the general configuration of the complete magnetic circuit. With a normal field current of 5 amp, the total flux per pole is 0.0334 weber of which 0.03 weber is useful (air-gap) flux mutual to the field and armature windings. Calculate (a) the inductance of the field circuit, (b) the total energy storage in watt-seconds, (c) for normal flux configuration, the value of the magnetomotive force associated with the flux that is mutual to adjacent poles. In computing the inductance from total flux linkages, have all self and mutual inductances been included? Neglect saturation.

33. The field coils of a 4-pole generator have 2,000 turns each and all coils are in series. For total flux linkages per pole of 48 weber-turns the generated voltage is 125 volts. A plot of the generated voltage as a function of the field current is known as the saturation curve of the machine. By what factor must the slope of the saturation curve be multiplied in order to obtain the curve of inductance vs. field current?

34. The field coils of a 12-pole generator have 600 turns each, are connected in series, and carry 26.75 amp. The total flux per pole is 0.1715 weber of which 83.5% crosses the air gap to the armature. What is the over-all inductance of the field circuit? What is the maximum mutual inductance between the field circuit and a single-turn armature coil if the coil spans a full pole pitch? Neglect effects of saturation.

35. A coil of R ohms resistance and L henries inductance is carrying a steady current when suddenly short-circuited. What quantity of electricity, expressed symbolically, flows through the circuit during the complete discharge of the magnetic field?

36. What is the time constant of a circuit containing 2 coils in series, with negligible mutual inductance, if their individual time constants are 6 and 14 sec? Assume, first, that $L_1 = L_2$, and, second, that $R_1 = R_2$.

37. Show that the energy stored in the mutual field of a coupled circuit is Mi_1i_2.

38. Solve the differential equation for the R-L circuit, with a d-c voltage applied, for the current. You may separate the variables and integrate or follow the general treatment for the linear differential equation.

39. A coil has a resistance of 40 ohms and a time constant of 0.125 sec. If 100 volts d-c is applied to the coil, at what rate is the current rising at $t = 0$; at $t = 0.125$ sec? What is the magnitude of the current at $t = 0.125$ sec? What time would be required for the current to reach its final value if it could continue to rise at its initial rate? What is the value of the current when the magnetic field is receiving energy at its greatest rate?

40. The air-gap flux of a 6-pole machine is 7.2×10^6 lines. Leakage flux per pole is

15% of this value. Each field coil has 620 turns, the series resistance of the coils being 7.1 ohms. Normal field current is 15 amp for 125 volts applied to the field circuit. How long does it take for the field current to reach half value after the field switch is closed? What is the energy stored in the magnetic field at full current? If a field discharge resistor is connected across the field circuit an instant before the field switch is opened, what must be the ohmic value of the resistor to limit the voltage across the circuit to 1,500? How much energy will it have to dissipate? Neglect effects of saturation in the problem.

41. A transmission line consists of two concentric cylinders having radii of 0.5 cm and 2.5 cm. Determine the inductance per mile of line.

42. Each plate of a 25-plate variable air condenser has an area of 2.5 sq in., and the separation between consecutive plates is 0.03 in. Calculate the maximum capacitance of the condenser.

43. A 0.5-μf condenser is made of parallel sheets of tin foil and paper. The paper has a thickness of 2 mils and a relative dielectric constant of 2.6. What length of each kind of material is required if the sheets are to be 4 in. wide? How many layers of paper are required if the condenser is to be made by stacking alternate layers of foil and paper, each 4 in. by 6 in.? If 2 sheets each of paper and foil, 4 in. wide, are laid alternately on each other and the whole rolled into a cylinder, what length of foil is required?

44. A condenser is made of 2 brass plates each having dimensions of 10 in. by 6 in. for the surface areas. The measured capacitance is 660 $\mu\mu$f when a glass plate 0.12 in. thick is inserted between the conducting plates. What is the relative dielectric constant of the glass as indicated by the measurements? Repeat for a bakelite plate 0.13 in. thick as the dielectric and with $C = 810$ $\mu\mu$f. What is the resultant capacitance of the condenser if both glass and bakelite are inserted together? If 500 volts are applied to the brass plates, what is the voltage across each dielectric, and what is the voltage gradient of each?

45. A transmission line consists of 2 concentric cylinders having radii of 0.5 cm and 2.5 cm. Determine the capacitance per meter length of line when the dielectric is air.

46. How many units of charge are stored on the plates of a 25-μf, 460-volt condenser? What is the stored electrostatic energy at rated voltage?

47. Three condensers of 2, 3, and 5 μf, respectively, are connected in parallel across a 1,500-volt supply. What is the stored energy in each condenser? If these condensers are connected in series across the same d-c supply, what is the voltage that exists across each condenser? Neglect leakage current.

48. A condenser bank is to be built to give a maximum capacity of approximately 30 μf to be reached in steps of one μf. What are the minimum number of condensers required and what should be the capacitance of each? Sketch the circuit arrangement.

49. A voltage of 110 volts d-c is applied to a series circuit of 70 ohms resistance and 25 μf capacitance. Derive the equation for the variation of current. At what rate is the current changing just as the switch is closed on the circuit? In what length of time would the current become zero if it could continue to decrease at this rate? What is the value of the time constant T of the circuit? What is the magnitude of the current at $t = T$ sec? Calculate the maximum value of stored electrostatic energy in the condenser. What total energy is given the circuit from the time the switch is closed until the current has died away? What energy has been lost in the resistance during the complete charging process? How does this energy compare with the stored energy?

50. A 25-μf condenser is fully charged from a 115-volt d-c supply. What is the charge on the plates and what is the stored energy? The condenser then is discharged through a resistance of such magnitude that half voltage is reached in 0.05 sec. What is the ohmic value of the resistance, and what is the current at this time?

51. A 150-volt voltmeter having 15,000 ohms resistance, a 5,000-ohm resistance, and a 25-μf condenser are connected in series. If 115 volts d-c are applied to the circuit with the condenser short-circuited, what will be the scale reading of the voltmeter? What will the voltmeter read 0.25 sec after the switch across the condenser is opened? What multiplying factor for the voltmeter scale will give corresponding currents?

52. In testing the accelerating characteristics of grades of gasoline, use has been made of a fifth wheel mounted on the running board of an automobile. The wheel drives a small d-c generator having constant flux. Connected in series across the armature of the generator are a condenser and a galvanometer. Neglecting circuit resistance, show that the galvanometer readings give a measure of the acceleration of the car.

53. A small neon glow lamp which ionizes at 70 volts is placed across a 1-μf condenser. In series with the parallel combination is a resistance R. What must be the value of R for the neon lamp to glow 20 sec after 120 volts d-c are applied to the circuit?

54. A voltage of 120 volts d-c is applied to a series circuit consisting of a 20-μf condenser and a resistor. The circuit has a time constant $T = 0.16$ sec. At what time will the voltage across the condenser be 70 volts? At what time will the voltage across the resistor be 30 volts? What resistance is required in order that the voltage across the condenser be 70 volts at the end of 0.1 sec?

55. A neon glow tube is connected across the terminals of a 1.5-μf condenser, and the condenser is charged from a 110-volt d-c source through a 1.2-megohm resistor. The tube ionizes at 69.5 volts and deionizes at 23 volts. Sketch the variation of condenser voltage as a function of time, neglecting the small time interval between ionization and deionization of the tube. What is the frequency at which the tube flashes?

56. A 25-μf condenser, charged to 120 volts, and a 10-μf condenser, charged to 460 volts, are connected in series opposition through a 5,000-ohm resistor and an open switch. What is the total initial energy stored in the condensers? Write the equation for the current after the switch is closed. What is the total energy stored in the condensers one second after the switch is closed? Account for any energy change and check your result. Neglect the leakage resistance of the condensers.

57. An electrical circuit consisting of 2 series condensers is formed when a gasoline truck discharges gasoline to an underground storage tank through a hose having a metal protective covering. The paint of the tank and the tires of the truck are the respective dielectrics of the condensers which have a common ground. The flow of gasoline produces a charge, corresponding to a current flow in this closed circuit. Assuming an equivalent current of 3×10^{-8} amp for a discharge rate of 50 gal per min, and a capacity of 850 $\mu\mu$f to ground for the truck, what electrostatic voltage is developed for a discharge of 75 gal at this rate? Assume the truck tank is not grounded.

58. With condensers connected in series across a d-c voltage source, what determines the distribution of voltage across the condensers if the leakage current of the condensers cannot be neglected?

59. Three condensers of 0.7, 1, and 1.5 μf, respectively, are connected in series across an 8,000-volt d-c supply. A 5-megohm resistor is connected across each condenser. Calculate the charge on and the voltage across each condenser. With a 5-megohm resistor connected across the 0.7 μf condenser, what values of resistance should be connected across the other two condensers in order to have the same charge on each condenser? With these new resistors, what voltage appears across each condenser?

CHAPTER IV

SINGLE-PHASE, ALTERNATING-CURRENT CIRCUITS

It is the purpose of this chapter to discuss the fundamental relations with respect to voltage, current, and power in the single-phase circuit. Specifically, interest lies in the investigation of the effects of the circuit constants upon the current and power when a sine wave of voltage is impressed on these elements, either singly or in combination. The waves of current and voltage ordinarily encountered in practice are sinusoidal and yield readily to elementary methods of analysis.

4.1. Sine Waves. From the preceding developments, a voltage and a current are sinusoidal if their periodic variations in time may be expressed by the equations

$$e = E_m \sin 2\pi ft = E_m \sin \omega t$$

$$i = I_m \sin 2\pi ft = I_m \sin \omega t$$

E_m and I_m are the maximum values of voltage and current, f is the frequency of the waves in cycles per second, and ω is the angular velocity in electrical radians per second at which the generating system produces the voltage. The equations give the instantaneous values for any arbitrary time t seconds.

The above equations for e and i definitely imply measurement of time from that point at which the waves cross the time axis and are increasing positively. In more general terms, we may write

$$e = E_m \sin (\omega t \pm \alpha_1)$$

$$i = I_m \sin (\omega t \pm \alpha_2)$$

where an arbitrary reference ordinate from which time is measured now is chosen at a *phase angle* α to the right, or to the left, of the zero point of the waves, as α is + or −. Further, α may be different in magnitude and, in general, will be for every voltage and current component in a circuit. The phase angle between any voltage or current and any other voltage or current is the algebraic difference between their respective phase angles. Specifically, it is the angle between corresponding points on the two waves, for example, between adjacent positive peaks.

4.2. Vector Representation of Sine Waves. In treating electrical circuits under steady-state conditions, it has been found much simpler to resort to the so-called vector diagram method of displaying voltages and currents rather than to retain their trigonometric forms of expression. In the vector method, the magnitude of a voltage or a current is shown by a line having a length, to scale, equal to the maximum value, or to the effective value E or I. The phase angle for the voltage or current is indicated by rotating the line through an angle $\pm\alpha$ from the horizontal axis. Normal mathematical conventions are understood to hold throughout.

In the sine-wave and vector forms, a voltage and a current

$$e = 141.4 \sin(\omega t + 50^\circ)$$

$$i = 28.3 \sin(\omega t - 35^\circ)$$

are shown in Figures 4.1a and b, respectively. The phase angle between the two waves is 85 electrical degrees, the angle by which the current lags behind the voltage. In practice, it is more usual to speak of the current lagging or leading the voltage, rather than the voltage leading or lagging the current, because the magnitude and phase of the current are determined by the load circuit, and the voltage usually is available at a standard magnitude and frequency. The arrowheads on the vectors are for the explicit purpose of fixing the angle to be considered rather than its supplement. It is understood that in the above equations radians are converted to degrees or degrees to radians before the two angles are added.

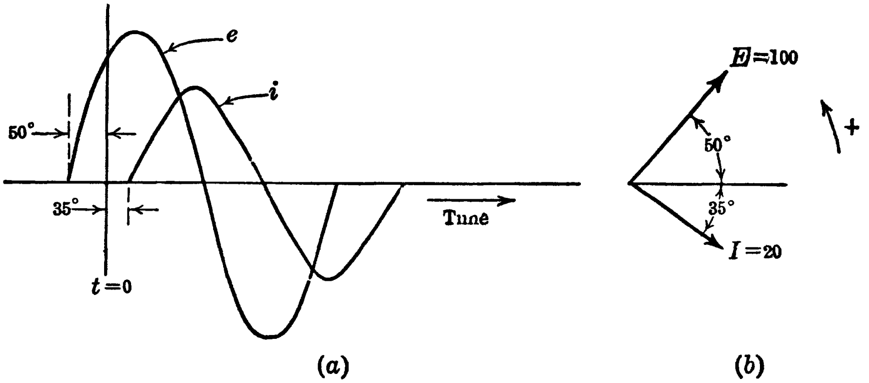
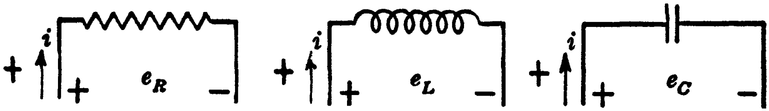


FIG. 4.1. Sine-wave and vector representations of voltage and current.

The addition or subtraction of sine waves of the same frequency yields a sine wave as the resultant. Only sine-wave quantities of the same frequency and of the same units may be added or subtracted on a vector diagram to give the corresponding magnitude and phase position of the resultant. This is all that is intended in any vector diagram and justifies its use. Such a geometric figure can be used for the addition of component voltages or component currents, or to show angular relations between a voltage and a current, or among several voltages and currents. Usually the effective magnitudes are represented.

4.3. Circuit Relations. The constants normally encountered in alternating-current analyses are the three circuit elements shown in Figure 4.2. The fundamental relations between applied voltage and resultant current for these three elements are

$$e_R = iR \qquad e_L = L\frac{di}{dt} \qquad e_C = \frac{q}{C} = \frac{1}{C}\int i\,dt.$$



Resistance, *R*. **Inductance, *L*.** **Capacitance, *C*.**

FIG. 4.2. Fundamental circuit constants.

If an alternating current $i = I_m \sin \omega t$ flows through each element, the respective voltages that must exist across them are given, from the above fundamental relations, by the following equations:

(1)

$$\begin{aligned} i &= I_m \sin \omega t \\ e_R &= I_m R \sin \omega t \\ &= E_m \sin \omega t \end{aligned}$$

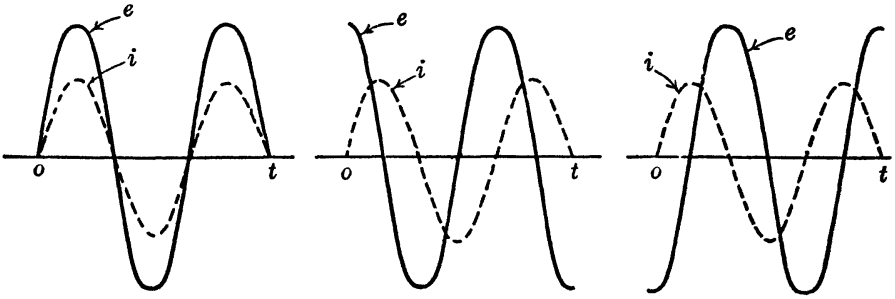
(2)

$$\begin{aligned} i &= I_m \sin \omega t \\ e_L &= \omega L I_m \cos \omega t \\ &= E_m \cos \omega t \\ &= E_m \sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned}$$

(3)

$$\begin{aligned} i &= I_m \sin \omega t \\ e_C &= -\frac{I_m}{\omega C}\cos \omega t \\ &= -E_m \cos \omega t \\ &= E_m \sin\left(\omega t - \frac{\pi}{2}\right) \end{aligned}$$

The corresponding current and voltage may be plotted to show in each case sinusoidal variation, relative magnitude of current and voltage, and the phase angle between the current and the voltage, Figure 4.3.



Resistance circuit **Inductance circuit** **Capacitance circuit**

FIG. 4.3. Current and voltage for the three fundamental circuits.

The above equations and figures show several important relations:

(1) *Resistance Circuit.*

(a) The ratio E_m to I_m, which also is the ratio of the respective effective values, is equal to R ohms.

(b) The current and voltage waves pass through their zero and maximum values at the same time and are, therefore, *in phase.*

(2) *Inductance Circuit.*

(a) $\dfrac{E_m}{I_m} = \dfrac{E}{I} = \omega L = 2\pi fL = X_L$ ohms inductive reactance. (4.1)

(b) The current reaches its maximum value 90° later than does the voltage and, therefore, is said to *lag* the impressed voltage by 90°.

(3) *Capacitance Circuit.*

(a) $\dfrac{E_m}{I_m} = \dfrac{E}{I} = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = X_C$ ohms capacity reactance. (4.2)

(b) The current reaches its maximum value 90° ahead of the voltage and, therefore, is said to *lead* the impressed voltage by 90°.

The use of the polarity marks and arrows shown on the circuit diagrams is discussed in Section 4.22.

The useful quantities with respect to voltage and current in these three elements, or in any combination of them, are:

1. The magnitude of the effective voltage E.
2. The magnitude of the effective current I.
3. The phase angle between the voltage and the current.

These three quantities are completely set forth either by the equations for e and i, by the curves of e and i as time functions, or by the more convenient vector diagram, Figure 4.4. The current, drawn horizontally and to the right, has been chosen as the reference for the component voltages. The length of each vector is drawn proportional to the effective value. The angle between the vectors shows the phase displacement, where counterclockwise rotation designates advance in phase (lead). The title of *vector* diagram applied to graphic illustrations

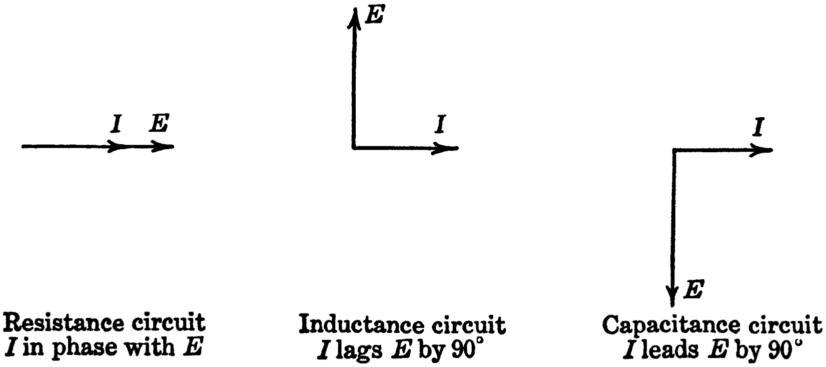


FIG. 4.4. Vector diagrams for the three fundamental circuits

of this character is justifiable since, as we shall see, similar quantities of the same frequency shown on the diagram can be added and subtracted as vectors. The results obtained from this form of geometric solution are the same as those found by trigonometric means.

4.4. *R* and *L* in Series. A circuit containing a resistance R and an inductance L, Figure 4.5, is used first in illustrating the general theory of the single-phase a-c circuit. Let a current $i = I_m \sin \omega t$ flow in the series combination, the problem being that of determining the applied voltage and its phase angle with respect to the current. The analysis will be carried out by two methods, the trigonometric equations and the vector diagram.

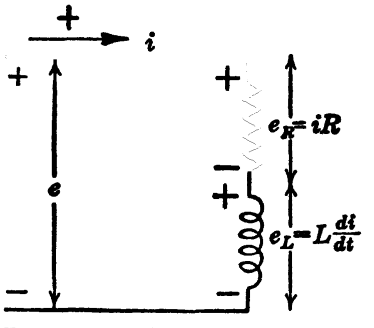


FIG. 4.5. Resistance and inductance in series.

In Figure 4.5,

e = instantaneous applied voltage

e_R = instantaneous voltage drop across the resistance

and

e_L = instantaneous voltage drop across the inductance

Since the instantaneous value of the impressed voltage must equal the sum of the instantaneous voltage components across the two circuit elements, we can write

$$\begin{aligned}
e &= e_R + e_L = Ri + L\frac{di}{dt} \\
&= RI_m \sin \omega t + \omega L I_m \cos \omega t \\
&= I_m[R \sin \omega t + X_L \cos \omega t] \\
&= I_m\sqrt{R^2 + X_L^2}\left[\frac{R}{\sqrt{R^2 + X_L^2}} \sin \omega t + \frac{X_L}{\sqrt{R^2 + X_L^2}} \cos \omega t\right] \\
&= I_m Z\left[\frac{R}{Z} \sin \omega t + \frac{X_L}{Z} \cos \omega t\right] \\
&= I_m Z[\cos \theta \cdot \sin \omega t + \sin \theta \cdot \cos \omega t] \\
&= E_m \sin (\omega t + \theta) \\
&= E_m \sin\left(\omega t + \tan^{-1} \frac{X_L}{R}\right)
\end{aligned}$$

From the definition of cos θ and sin θ in the above equations, it is evident that R, X_L, and θ are related as shown in the triangle of Figure 4.6 which is called the *impedance triangle* of the circuit.

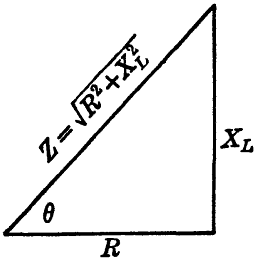


FIG. 4.6. Impedance triangle for an inductive circuit.

From the expressions for the alternating current and voltage the following observations are apparent:

1.

$$\frac{E_m}{I_m} = \frac{E\text{ (effective)}}{I\text{ (effective)}} = \sqrt{R^2 + X_L^2}$$
$$= Z \text{ ohms impedance} \qquad (4.3)$$

The impedance Z of any circuit is defined as the ratio of the effective applied voltage to the effective current which flows in the circuit. It is to be noted that the impedance of a series R-L circuit increases with the frequency of the applied voltage, that is,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2} \text{ ohms impedance} \qquad (4.4)$$

2. The phase angle between the voltage and the current is

$$\theta = \tan^{-1}\frac{X_L}{R} \qquad (4.5)$$

and depends solely upon the magnitudes of the circuit constants.

Note also that $\cos\theta = R/Z$, that $\sin\theta = X_L/Z$, that it is not necessary to refer to trigonometric tables for these functions of the phase angle, and that the phase angle can be expressed in terms of the circuit constants. The phase angle always can be obtained from the impedance diagram for the circuit.

3. The voltage reaches its maximum value θ degrees before the current reaches its maximum. The voltage in such a circuit is said to lead the current or, conversely, the current lags the voltage by the phase angle θ.

The instantaneous values of voltage and current can be represented graphically as in Figure 4.7 and can be observed in the laboratory on a 2-element oscillograph. If $t = 0$ is chosen at point (a), the circuit equations are

$$i = I_m \sin \omega t$$

$$e = E_m \sin (\omega t + \theta)$$

If point (b) is chosen for the time reference, then

$$e = E_m \sin \omega t$$

$$i = \frac{E_m}{Z} \sin (\omega t - \theta)$$

Any other reference point for zero time would be shown by the corresponding time angle $\pm\alpha$. For example, with reference to point (c),

$$i = I_m \sin(\omega t + \alpha)$$

$$e = E_m \sin(\omega t + \alpha + \theta)$$

It is evident that either set of equations states the same physical concepts. The ratio of voltage to current (the impedance of the circuit) and the phase relation between current and voltage are strictly constants of the circuit and must not vary with our system of representation.

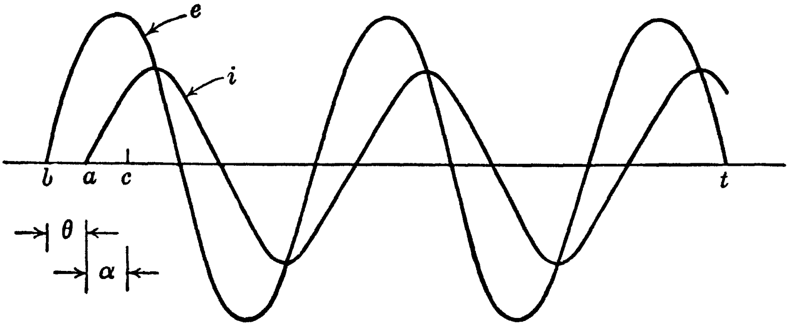


Fig. 4.7. Resistance and inductance in series.

The equations previously developed, namely,

$$\begin{aligned} e &= e_R + e_L \\ &= I_m R \sin \omega t + I_m X_L \cos \omega t \\ &= I_m\sqrt{R^2 + X_L^2}\sin(\omega t + \theta) \end{aligned}$$

and

$$i = I_m \sin \omega t$$

show the applied voltage to consist of two components. One is required to drive the current through the resistance R, is in phase with the current, and has an effective value $E_R = I_m R/\sqrt{2} = IR$. The other component is required to drive the current through the inductive reactance X_L, leads the current by 90 degrees, and has an effective value $E_L = I_m X_L/\sqrt{2} = IX_L$. The current, voltage components, and applied voltage can be shown in their proper phase relations by the vector diagram of Figure 4.8. The current vector is chosen as the reference because it is common to both circuit elements.

The effective applied voltage E is

$$E = \sqrt{E_R^2 + E_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2} = IZ \quad (4.6)$$

The complete interpretation of equation 4.6 also shows that for a given applied voltage and known circuit impedance elements the current can be calculated immediately as the ratio of that voltage to the circuit impedance.

In the foregoing discussion, the fundamental methods used in analyzing series a-c circuits have been presented. The impedance and phase angle are obtained from the circuit constants, the effective value either of voltage or of current then calculated when the other quantity is known. The reader is to carry through for himself the similar steps in the treatment for the *R-C* series circuit.

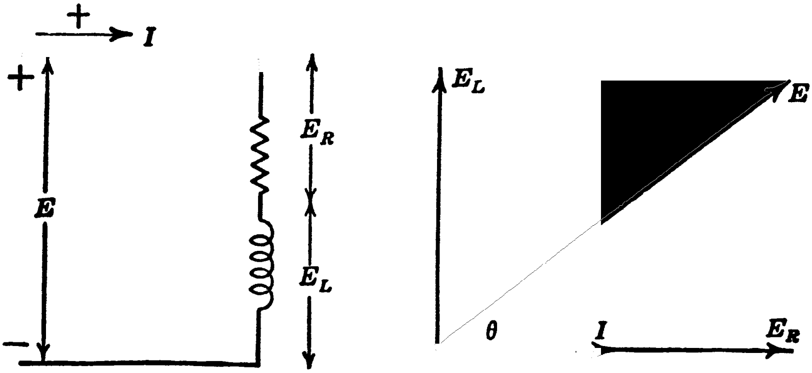


FIG. 4.8. Vector diagram for an inductive circuit.

4.5. *R*, *L*, and *C* in Series. The general methods of analysis now will be applied to the series *R-L-C* circuit. The following equations should be readily understood since the treatment parallels that of the *R-L* circuit. Let

e = applied instantaneous voltage

$i = I_m \sin \omega t$ = current through each element

Then

$$e = e_R + e_L + e_C = iR + L\frac{di}{dt} + \frac{\int i\,dt}{C}$$

$$= I_mR \sin \omega t + I_m\omega L \cos \omega t - I_m \frac{1}{\omega C} \cos \omega t$$

$$= I_m \left[R \sin \omega t + \left(\omega L - \frac{1}{\omega C} \right) \cos \omega t \right]$$

$$= I_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \sin (\omega t + \theta)$$

$$= I_m \sqrt{R^2 + (X_L - X_C)^2} \sin (\omega t + \theta) \tag{4.7}$$

The impedance of the circuit now is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \frac{E \text{ (effective applied voltage)}}{I \text{ (effective current)}} \tag{4.8}$$

and the phase angle between current and applied voltage is given by the expression

$$\tan \theta = \frac{X_L - X_C}{R} \tag{4.9}$$

The term $(X_L - X_C)$ is the net reactance of the circuit. If X_L is greater than X_C the reactance of the circuit is predominantly inductive, θ is positive, and the current lags the applied voltage. If X_C is greater than X_L the net reactance of the circuit is capacitive, θ is negative, and the current leads the applied voltage. Since $E_L = IX_L$, and $E_C = IX_C$, the voltage drop across the inductance is greater than the drop across the capacitance if the net reactance is inductive.

The vector diagram for the *R-L-C* circuit, showing all components of the applied voltage, is given by Figure 4.9. In this diagram the voltage drop across the capacitance is greater than the drop across the inductance. The net reactance is, therefore, capacitive.

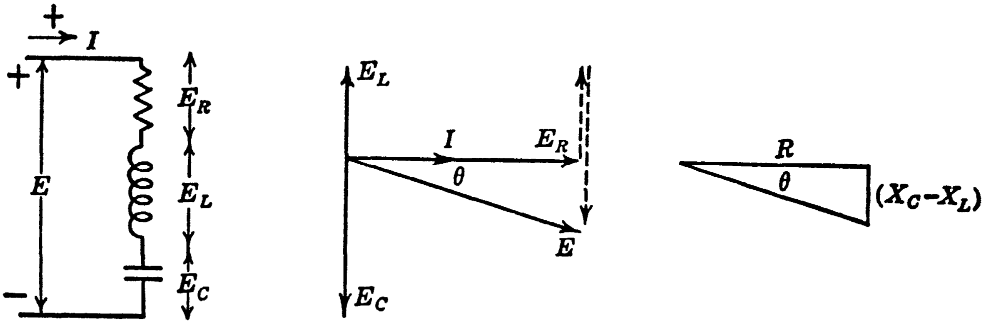


FIG. 4.9. Vector diagram and impedance triangle for a circuit containing resistance, inductance, and capacitance, all in series.

In a series circuit having any number of elements of any character, the component impedance quantities are added geometrically to obtain the over-all impedance. As an example, consider a series circuit consisting of 4 ohms resistance, 3 ohms inductive reactance, 18 ohms capacitive reactance, 6 ohms inductive reactance, and 10 ohms resistance. The over-all impedance consists of 14 ohms resistance and 9 ohms equivalent capacitive reactance, and has a magnitude of 16.6 ohms.

4.6. Resonance in a Series Circuit. The equation for the current in the *R-L-C* circuit

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \tag{4.10}$$

shows that the current will be a maximum when the net reactance $(X_L - X_C)$

is zero. For this condition, the phase angle is zero, the circuit impedance reduces to a pure resistance, and the circuit is said to be in *resonance.* For series resonance, then,

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{4.11}$$

For any fixed frequency there is an infinite number of combinations of L and C that will give resonance. If the circuit constants are fixed, there is one frequency that will give resonance. If the value of the applied voltage is maintained constant and the frequency of the voltage source varied, the current variation with frequency will be as shown by the curve of Figure 4.10. Frequency-selective circuits of this type are of particular importance in the field of radio communication. By them one is able to tune his radio receiver to any desired broadcast station.

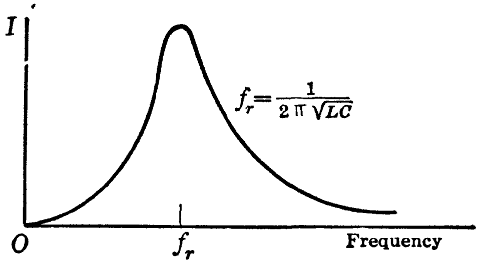


FIG. 4.10. Resonance curve of a series circuit.

4.7. Power in a Single-phase Circuit. The instantaneous power input to any single-phase circuit always is the direct product of the instantaneous applied voltage and the instantaneous input current. Thus,

$$p = ei = \text{instantaneous power in watts}$$

In the a-c circuit the power is pulsating, and it is the *average power* that determines the energy consumed over a given period of time. Wattmeters, therefore, are calibrated to indicate this average power. Taken over a complete cycle,

$$P = \text{average power}$$

$$= \frac{1}{T}\int_0^T ei\,dt \text{ watts} \tag{4.12}$$

where T is the period, the time to complete one cycle of either voltage or current. Substituting for the instantaneous values of voltage and current, changing the variable of integration to angle, and altering the limits of integration to correspond, we have, for sinusoidal voltage and current,

$$P = \frac{1}{2\pi}\int_0^{2\pi} [E_m \sin \omega t][I_m \sin (\omega t \pm \theta)]\, d(\omega t)$$

$$= \frac{E_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta$$

$$= EI \cos \theta \text{ watts} \tag{4.13}$$

The average power in a single-phase circuit is, therefore, equal to the product of the effective values of the applied voltage and the line current and the cosine of the phase angle between the voltage and current. Whether the current leads or lags the voltage is immaterial. Equation 4.13 is a special case of the general power equation, and average power is defined in this way because of the particular definitions of effective values for voltage and current. It should be apparent that equation 4.13 holds only for sine-wave quantities of the same frequency and for the steady-state d-c circuit, which is but a special zero-frequency case of the general a-c circuit. In the d-c circuit, effective a-c values are replaced by the d-c voltage and current, and $\theta = 0$.

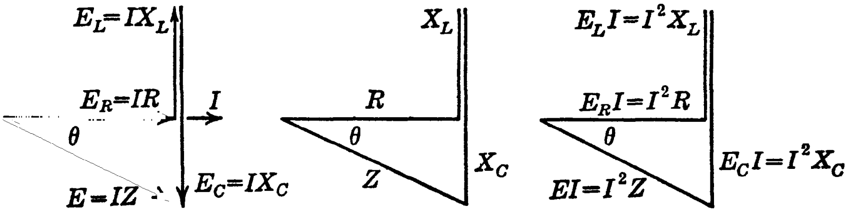


FIG. 4.11. Voltage, impedance, and power diagrams for the series *R-L-C* circuit.

Since the current flowing in a pure inductance or capacitance is 90 degrees out of phase with the voltage drop across these elements, the average power consumed by either an inductance or a capacitance is zero. All the power consumed by the circuit must be accounted for by the resistance elements. These properties already have been pointed out in the discussion on circuit constants. The component of the applied voltage required to drive the current through the resistance is

$$E_R = IR = E \cos \theta$$

Therefore, the average power expended in the circuit also is

$$P = IE_R = I^2R \text{ watts} \tag{4.14}$$

Each component of resistance in the circuit takes its proportionate share of the total power. In Figure 4.11, $E \cos \theta$ is the component of voltage in phase with the current. Hence, power can be defined as the product of the *in-phase com-*

ponents of voltage and current, and either vector may be projected upon the other.

Figure 4.11, which is simply an extension of Figure 4.9, shows the geometrically similar polygons of voltage, impedance, and power. Defined for the single-phase circuit,

$$EI = I^2Z = \text{volt-amperes } \textit{apparent power} \tag{4.15}$$

$$E_RI = I^2R = \text{watts } \textit{active power} \tag{4.16}$$

$$-E_LI = -I^2X_L = \text{volt-amperes } \textit{reactive power} \tag{4.17}$$

$$E_CI = I^2X_C = \text{volt-amperes } \textit{reactive power} \tag{4.18}$$

$$-I^2(X_L - X_C) = \textit{net} \text{ volt-amperes } \textit{reactive power} \tag{4.19}$$

The sign of the reactive power is considered positive when the current leads the voltage. This is the convention adopted by the American Institute of Electrical Engineers.

By definition, the *power factor* (pf) for any circuit is given by the relation

$$\text{power factor} = \frac{\text{watts}}{\text{volts} \times \text{amperes}} = \frac{\text{active power}}{\text{apparent power}} \tag{4.20}$$

Obtained from instrument readings, it is the ratio of watts power indicated by a wattmeter to the product of the effective voltage and current as shown on a voltmeter and an ammeter. For the special case of sinusoidal voltages and currents,

$$\text{power factor} = \cos\theta = \frac{R}{Z} \tag{4.21}$$

as indicated in the several diagrams of Figure 4.11.

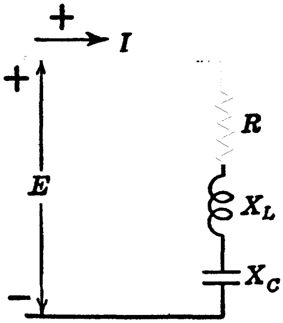


Fig. 4.12. Series circuit of R, L, and C.

4.8. Numerical Example. Let an alternating voltage of 110 volts at a frequency of 60 cycles per second be impressed on a series circuit of 8.66 ohms resistance, 0.106 henry inductance, and 75.8 μf capacitance, Figure 4.12. The problem is to determine (a) the current and its phase angle with the impressed voltage; (b) the voltage drop across each element of the circuit; and (c) the power input to the circuit.

Solution:

The inductive reactance is

$$\begin{aligned} X_L &= 2\pi fL \\ &= 377(0.106) \\ &= 40 \text{ ohms} \end{aligned}$$

The capacitive reactance is

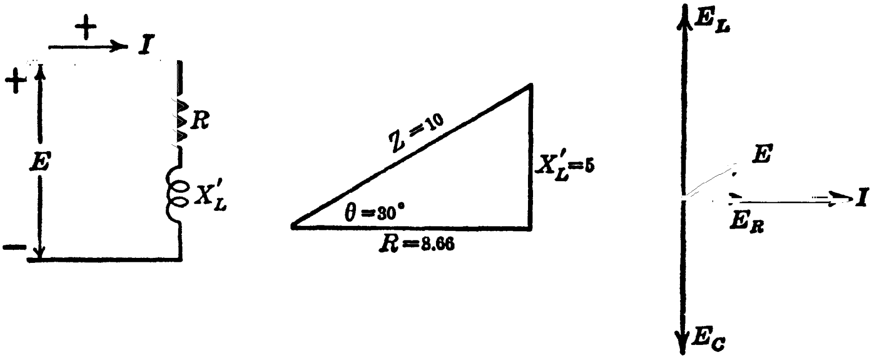
$$X_C = \frac{1}{2\pi f C} = \frac{1}{377(75.8)10^{-6}}$$

$$= 35 \text{ ohms}$$

The net reactance of the circuit is

$$X_L' = (X_L - X_C) = 5 \text{ ohms}$$

and is inductive. The actual circuit, therefore, can be reduced to that shown in Figure 4.13a, which has the impedance triangle of Figure 4.13b. The complete vector diagram for the actual circuit is given by Figure 4.13c in which $E_L - E_C$ is the voltage drop across the net reactance X_L'.



(a) (b) (c)

FIG. 4.13. Equivalent circuit, impedance triangle, and vector diagram of circuit of Fig. 4.12.

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{8.66^2 + 5^2}$$

$$= 10 \text{ ohms}$$

and the current is

$$I = \frac{E}{Z} = \frac{110}{10} = 11 \text{ amperes}$$

The phase angle is

$$\theta = \tan^{-1}\frac{5}{8.66} = 30°$$

whence the power factor is

$$\cos\theta = \frac{8.66}{10} = 0.866$$

The voltage drop across the resistance is

$$E_R = IR = 11 \times 8.66 = 95.2 \text{ volts}$$

the voltage drop across the inductance is

$$E_L = IX_L = 11 \times 40 = 440 \text{ volts}$$

and the voltage drop across the capacitance is

$$E_C = IX_C = 11 \times 35 = 385 \text{ volts}$$

It can readily be shown that the *vector sum*—not the algebraic sum—of these three component voltages is equal to the applied voltage of 110 volts.

If a voltmeter were connected so as to read the voltage drop across the series combination of L and C, it would indicate $(440 - 385) = 55$ volts, since the voltage drop across the inductance is directly opposite in polarity to that across the capacitance. The fact that voltage components greater in magnitude than the applied voltage exist in the circuit should not disconcert us. These voltages actually are present in the circuit and can be measured with an ordinary voltmeter. Analogous physical conditions often are met in mechanical systems

The power input to the circuit is

$$\begin{aligned} P &= EI \cos \theta \\ &= 110 \times 11 \times 0.866 \\ &= 1{,}050 \text{ watts} \end{aligned}$$

This must also equal

$$\begin{aligned} P &= I^2R \\ &= (11)^2 \times 8.66 = 1{,}050 \text{ watts} \end{aligned}$$

Seldom is it necessary in electrical circuit work to know phase angles in degrees. The angles are completely specified, of course, through their trigonometric functions, and it is only these quantities which generally are used in calculations. It is sufficient to obtain the trigonometric functions unless a solution requires a phase angle in degrees.

4.9. Summary of the Series Circuit. In summarizing the analysis of the series circuit, the major relations are:

1. The current is common to all the elements of the circuit. This is the definition of the series circuit.
2. The vector sum of all the voltage drops must equal the applied voltage.
   (a) The voltage drop across a resistance is in phase with the current and is equal in magnitude to IR.
   (b) The voltage drop across an inductance leads the current by 90 degrees and is equal in magnitude to IX_L.
   (c) The voltage drop across a capacitance lags the current by 90 degrees and is equal in magnitude to IX_C.
3. The circuit can be reduced, for any one frequency, to an equivalent circuit consisting of one resistance and one reactance in series.

4. From the impedance triangle of the circuit the ratio of applied voltage to line current and the phase angle between them can be determined.
5. The magnitudes of quantities and their phase relations can be shown conveniently by means of the vector diagram.
6. The power expended in the circuit can be determined by the product of the in-phase components of voltage and current. This power also must be equal to the total I^2R of the circuit.

4.10. The Parallel Circuit. All the fundamental theory and methods required to solve any single-phase circuit have been given in the discussion on the series circuit. It is a simple matter to apply these concepts to the parallel circuit or to combinations of series and parallel circuits.

A parallel circuit is defined as one in which the voltage is common to all branches of the circuit. The solution of a parallel circuit consists in finding the branch currents and combining them vectorially to obtain the line current.

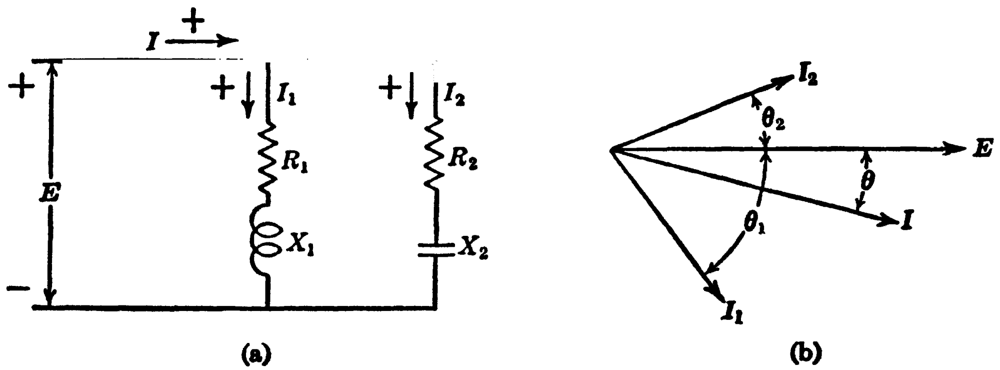


FIG. 4.14. Parallel circuit.

Figure 4.14a shows a two-branch circuit. In drawing the vector diagram, Figure 4.14b, for this circuit, the voltage is taken as the reference vector since it is common to the two branches.

The branch currents can be obtained from the equations

$$I_1 = \frac{E}{Z_1} = \frac{E}{\sqrt{R_1^2 + X_1^2}} \quad \text{and} \quad \theta_1 = \tan^{-1}\frac{X_1}{R_1}$$

and

$$I_2 = \frac{E}{Z_2} = \frac{E}{\sqrt{R_2^2 + X_2^2}} \quad \text{and} \quad \theta_2 = \tan^{-1}\frac{X_2}{R_2}$$

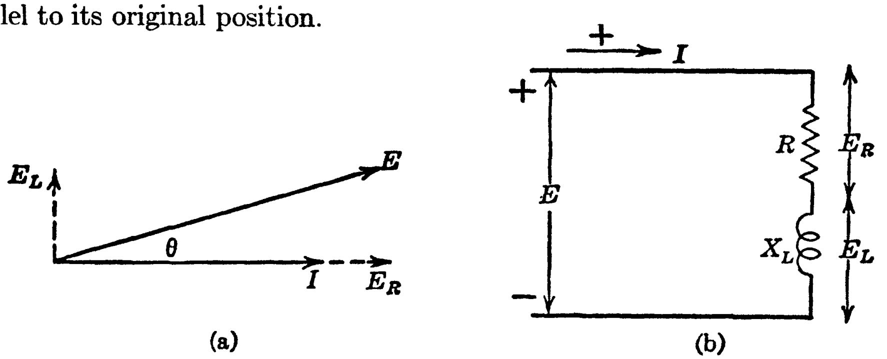
The line current I is the vector sum of the two branch currents. The summation can be made graphically on the vector diagram. An alternate method is to resolve each current vector into two components, one component in phase with the voltage and another 90 degrees out of phase with the voltage. The in-phase component of line current is the algebraic sum of the in-phase components of the branch currents. The out-of-phase component of line current is the algebraic sum of the out-of-phase components of the branch currents.

The current components shown in Figure 4.14 are such that their resultant I lags the applied voltage by the angle θ. The power taken by the parallel circuit is

$$\begin{aligned} P &= I_1^2R_1 + I_2^2R_2 \\ &= E(I_1 \cos\theta_1 + I_2 \cos\theta_2) \\ &= EI \cos\theta \text{ watts} \end{aligned}$$

4.11. Equivalent Circuits. Any parallel circuit can be replaced, for any particular frequency, by an equivalent series circuit. The solution of the equivalent series circuit must yield the same line current in the same phase relation as that for the parallel circuit. This condition is realized if the vector diagrams, showing line current and applied voltage, are identical.

The vector diagram, showing the line current and applied voltage, for the circuit of Figure 4.14a is redrawn in Figure 4.15a. Although the vectors of E and I have been rotated from their positions in Figure 4.14b, the same magnitudes and phase angles have been maintained. Since a vector diagram can show only relative magnitudes and phase relations, a vector diagram is not changed by the rotation of all vectors through the same angle or by moving any vector parallel to its original position.



(a) (b)

FIG. 4.15. Vector diagram and equivalent series circuit of the parallel circuit of Fig. 4.14(a).

From previous discussion it is evident that the vector diagram shown in Figure 4.15a is also the vector diagram of the series circuit of Figure 4.15b. The voltage E can be resolved, as shown by the dotted vectors, into two components, one in phase with the current and the other leading the current by 90 degrees. The values of the equivalent circuit constants R and X_L are found from the relations

$$IR = E_R = E\cos\theta$$

$$R = \frac{E}{I}\cos\theta = Z\cos\theta$$

$$IX_L = E_L = E\sin\theta$$

$$X_L = \frac{E}{I}\sin\theta = Z\sin\theta$$

Since E, I, Z, and θ are known from the solution of the parallel circuit, the constants of the equivalent series circuit can readily be determined.

Viewed from the input terminals and for a particular frequency, the equivalent series circuit is identical in all respects to the parallel circuit which it is to represent. Any measurements made at the input terminals of either circuit in the laboratory would not yield information to allow us to distinguish between the two circuits. This certainly is sufficient justification for replacing a parallel circuit by an equivalent series circuit if we so desire. In many problems the replacement of a parallel circuit by its series equivalent greatly simplifies the analysis.

It also follows that any series system can be replaced by an equivalent parallel circuit, one branch to contain resistance only and the other branch but a single reactance. The resistance must account for the active power taken by the circuit ($R = E^2$/watts), and the reactance for the net reactive volt-amperes ($X = E^2$/reactive volt-amperes). Equivalent parallel circuits often are found of advantage, for example, in treating electrical machines.

Note that, although there is an infinite number of equivalent circuits, either series or parallel, that can represent the terminal relations of any system on a fixed-frequency supply, we are discussing here only the simplest. These involve but one resistance and one reactance. The character of the reactance is inductive or capacitive, conforming to lagging or leading current in the initial system.

4.12. Numerical Example. A voltage of 100 volts, 60 cycles per second, is applied to the circuit of Figure 4.16a. The system elements are given by their ohmic values. In the analysis of this circuit, the impedance of the first branch is

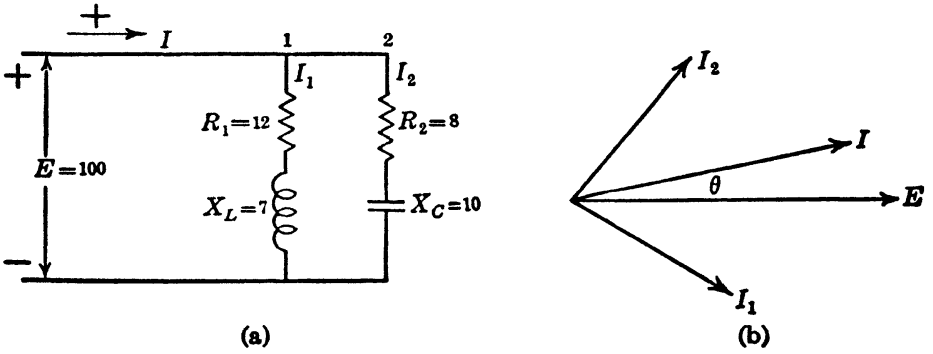


FIG. 4.16. Parallel circuit.

$$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{(12)^2 + (7)^2} = 13.89 \text{ ohms}$$

whence

$$I_1 = \frac{E}{Z_1} = \frac{100}{13.89} = 7.20 \text{ amperes}$$

The current I_1 lags the voltage by an angle θ_1, where

$$\cos\theta_1 = \frac{R_1}{Z_1} = 0.864$$

and

$$\sin\theta_1 = \frac{X_1}{Z_1} = 0.504$$

The current I_2 is

$$I_2 = \frac{E}{Z_2} = \frac{E}{\sqrt{R_2^2 + X_2^2}} = \frac{100}{12.8} = 7.81 \text{ amperes}$$

and leads the voltage by an angle θ_2, where

$$\cos\theta_2 = \frac{R_2}{Z_2} = 0.625$$

$$\sin\theta_2 = \frac{X_2}{Z_2} = 0.781$$

The line current I is the vector sum of the branch currents and can be expressed in terms of two components. From the vector diagram, Figure 4.16b, it is evident that

$$\begin{aligned} I \text{ in phase with } E &= I_1 \cos\theta_1 + I_2 \cos\theta_2 \\ &= 6.22 + 4.88 = 11.10 \text{ amperes} \end{aligned}$$

and

$$\begin{aligned} I \text{ out of phase with } E &= -I_1 \sin\theta_1 + I_2 \sin\theta_2 \\ &= -3.63 + 6.10 = 2.47 \text{ amperes} \end{aligned}$$

It should be noted that the proper algebraic sign must be taken in the summation. This is evident from the vector diagram. The magnitude of I is

$$\begin{aligned} I &= \sqrt{(11.10)^2 + (2.47)^2} \\ &= 11.37 \text{ amperes} \end{aligned}$$

The trigonometric functions of the angle θ between line current and voltage are

$$\begin{aligned} \cos\theta &= \frac{I \text{ in phase with } E}{I} \\ &= \frac{11.10}{11.37} = 0.976 \end{aligned}$$

and

$$\begin{aligned} \sin\theta &= \frac{I \text{ out of phase with } E}{I} \\ &= \frac{2.47}{11.37} = 0.217 \end{aligned}$$

The power input to the circuit is

$$\begin{aligned} P &= EI\cos\theta \\ &= 100 \times 11.37 \times 0.976 \\ &= 1{,}110 \text{ watts} \end{aligned}$$

and is equal also to

$$\begin{aligned} P &= I_1^2R_1 + I_2^2R_2 \\ &= (7.20)^2 \times 12 + (7.81)^2 \times 8 \\ &= 1{,}110 \text{ watts} \end{aligned}$$

Since the line current leads the voltage by an angle θ, the equivalent serie circuit must consist of a capacitive reactance, X_C, and a resistance, R. From line voltage, current, and phase angle,

$$Z = \frac{E}{I} = \frac{100}{11.37} = 8.80 \text{ ohms}$$

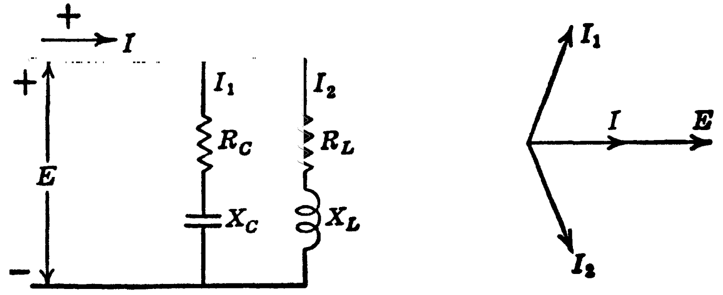
$$R = Z\cos\theta = 8.80 \times 0.976 = 8.58 \text{ ohms}$$

$$X_C = Z\sin\theta = 8.80 \times 0.217 = 1.91 \text{ ohms}$$

and

$$\begin{aligned} \text{power input} &= I^2R = (11.37)^2 \times 8.58 \\ &= 1{,}110 \text{ watts} \end{aligned}$$

4.13. Resonance in a Parallel Circuit. In a parallel circuit consisting of an inductive branch in parallel with a capacitive branch, the out-of-phase line current is the algebraic difference of the out-of-phase components of the two branch currents. If the out-of-phase line current is zero, the circuit is said to be in resonance (sometimes called anti-resonance to distinguish it from resonance in a series circuit).



(a) (b)

FIG. 4.17. Parallel resonant circuit.

The condition for parallel resonance can be obtained by equating the out-of-phase components of the two branch currents of the circuit shown in Figure 4.17a. The vector diagram is shown in Figure 4.17b. The out-of-phase components of the two branches are given by the equations

$$I_1 \text{ out of phase} = E\frac{X_C}{R_C^2 + X_C^2}$$

$$I_2 \text{ out of phase} = E\frac{X_L}{R_L^2 + X_L^2}$$

whence for resonance

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2} \tag{4.22}$$

When the reactances of the circuit are high in comparison with the resistances, as is often the case in resonant circuits, the expression for resonance can be simplified. When this is done, equation 4.22 becomes

$$X_C = X_L$$

from which

$$\omega = \frac{1}{\sqrt{LC}} \tag{4.23}$$

This is the same condition required for resonance in a series circuit. If only the resistance in the capacitance branch is negligible, a condition often met in practice, equation 4.22 becomes

$$\omega C = \frac{\omega L}{R_L^2 + \overline{\omega L}^2}$$

or

$$\omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \tag{4.24}$$

For resonance, the line current is in phase with the applied voltage and the impedance of the circuit is a pure resistance. The variation of line current with frequency has the general form shown in Figure 4.18. It will be observed that this frequency response curve is quite different from that for the series resonant circuit. At the resonant frequency, the impedance of the parallel circuit is high whereas the impedance of the series circuit is a minimum. Parallel, as well as series, resonant circuits are widely used in the field of communication engineering.

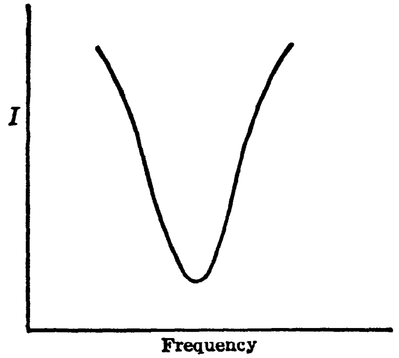


FIG. 4.18. Resonance curve of a parallel circuit.

4.14. Combination of a Series and Parallel Circuit. A series-parallel combination is shown in the circuit of Figure 4.19. The ohmic values of the circuit elements are indicated on the diagram. No new concepts are involved in the analysis of this problem and the steps in the solution need only be pointed out. This can best be done by actually solving a particular problem.

The first step is to reduce the parallel circuit to an equivalent series circuit. In solving for the equivalent series circuit, any value of voltage can be assumed to exist across the parallel circuit. If a voltage of 100 volts is assumed as the voltage across the parallel circuit, the steps in the determination of the constants of the equivalent series circuit are as explained in preceding sections. It should be noted that these circuit constants are independent of the assumed voltage.

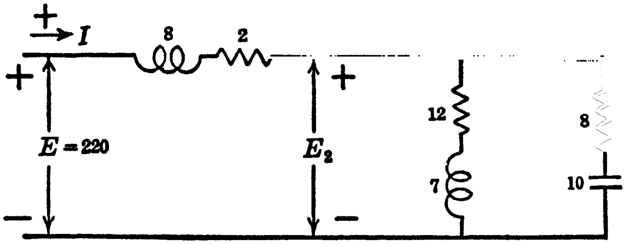


FIG. 4.19. Combination of a series and parallel circuit.

Replacing the parallel branches of Figure 4.19 by their equivalent series values, shown in Figure 4.20a, we obtain for the initial system the simple and equivalent series circuit of 10.58 ohms resistance and 6.09 ohms inductive reactance, Figure 4.20b. The impedance of the initial series-parallel circuit is, therefore,

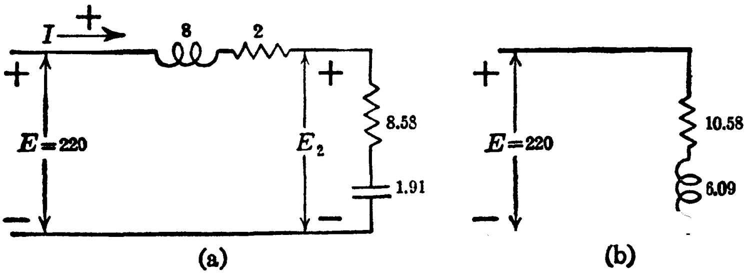


FIG. 4.20. Equivalent series circuit of Fig. 4.17.

$$Z = \sqrt{(10.58)^2 + (6.09)^2} = 12.2 \text{ ohms}$$

and the current is

$$I = \frac{E}{Z} = \frac{220}{12.2} = 18.0 \text{ amperes}$$

lagging by the phase angle

$$\theta = \tan^{-1}\frac{6.09}{10.58}$$

The voltage drop across the parallel circuit (or its equivalent series circuit) will be found to be 158.3 volts lagging the current by an angle of 12.6 degrees. The total circuit power is

$$P = (18.0)^2 \times 10.58 = 3{,}428 \text{ watts}$$

4.15. Power Diagrams. As we have seen, the solution of a parallel circuit of any number of branches reduces to an equivalent series circuit containing a single resistance and reactance. The resulting line current, with its in-phase and out-of-phase components, may be leading or lagging the applied voltage, depending upon the predominance of capacitive or inductive reactance in the system. Figure 4.21 represents the vector diagram for the equivalent circuit of a group of parallel loads, the line current and its components being multiplied by the magnitude of the line voltage E. We now have a power diagram in which

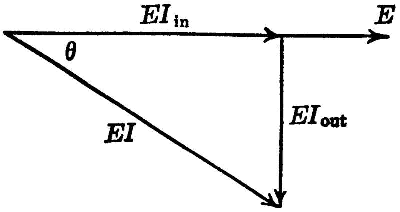


FIG. 4.21. Power diagram.

$$EI_{\text{in}} = \text{watts } \textit{active power}$$

$$EI_{\text{out}} = \text{volt-amperes } \textit{reactive power}$$

$$EI = \text{volt-amperes } \textit{apparent power}$$

and

$$\cos\theta = \frac{\text{active power}}{\text{apparent power}}$$

The active power of any circuit always is given in watts or kilowatts, whereas the reactive power and apparent power are expressed in volt-amperes or kilovolt-amperes. The reactive power may be lagging or leading, in conformity with that component of current which it represents.

It is evident that for unity power factor the active power is equal to the apparent power, whereas for a power factor less than unity the apparent power is greater. Generating capacity of 1,000 kva will supply a 1,000-kw load at 1.0 pf, but a 1,667-kva supply is needed if the power factor is 60 per cent. This means a much higher investment in generating, transmission, and distribution equipment than is necessary. In addition, the copper losses are greater because the current is 167 per cent of the required minimum. The power factor of any circuit, plant, or system is determined by conditions at the load and not at the generating end. In practice, load currents usually are lagging and sometimes highly so because of inductive characteristics of the circuits and the connected apparatus. The addition of capacitance at load centers, either in the form of static condensers or equivalent effects obtained by synchronous machines, hence improves the over-all power factor. It is usual also to find power-factor clauses in power company rates such that a penalty is added for a power factor below a given minimum and a bonus awarded for a better power factor. Such power-factor clauses will continue to exist until rate sheets show separate costs for watts power and for reactive-volt-amperes.

It is possible to illustrate many problems, single-phase or polyphase, by means of power diagrams. They are of particular advantage, for example, when studying plant and system loads, perhaps with a view to power-factor correction. As an illustration, the various loads on a plant may be given in terms of any two of the three quantities: kw, kva, and pf. Vector addition of these loads, taking into account their respective power factors, gives the total active, reactive, and apparent power for the plant and also the resultant power factor. Tabulation of the kw and kvar (reactive kilovolt-ampere) components for the individual loads also yields the same result. The total plant kw is the sum of the kw of the individual loads, and the total kvar is the sum of the respective kvar of the loads, due attention paid to the algebraic sign before these components since some may be leading and others lagging. From these summations,

$$\text{plant kva} = \sqrt{(\text{plant kw})^2 + (\text{plant kvar})^2}$$

$$\text{over-all pf} = \frac{\text{plant kw}}{\text{plant kva}} \quad \text{(lagging or leading)}$$

Plant studies usually involve readings of energy rather than power, the readings being in kilowatt-hours (kwhr) and reactive kilovolt-ampere-hours (kvarhr) over a time interval of perhaps a month. The resulting kva and pf so obtained are merely representative values for the time interval chosen. If the power factor is lagging and too low, it can be improved by adding leading reactive kva. The power diagram will show graphically what this kva must be in order to raise the power factor to any desired value. Again, among other things, the diagram will show the added kw a plant can carry for a given power factor improvement and a fixed supply kva.

4.16. The Direct-current Circuit. The d-c circuit under steady-state conditions is but a special case of the steady-state a-c system in which the frequency is zero. All inductive reactances become zero and capacitive reactances infinite, or open circuits. Only resistances remain to impede the flow of current. The resistances in those paths which carry current can be combined according to methods developed in elementary physics so as to give but a single equivalent resistance which will represent the over-all effects of the initial system.

In series-parallel circuits, each combination of parallel branches first is reduced to a single equivalent resistance R, where

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n} \tag{4.25}$$

R_1, R_2, $\cdots$ R_n are the resistances of the n parallel branches. The equivalent resistances of the parallel branches then are added to adjacent series resistances to give in turn further single net resistances of paths. The scheme employed in reducing a complicated series-parallel system is similar to that for the single-phase circuit but is simpler in calculation. Power factor now is not introduced explicitly, although effectually it is unity, and the circuit power is

$$P = EI = \frac{E^2}{R} = I^2R \text{ watts}$$

where E and I are line voltage and current and R is the single equivalent resistance for the entire network. Once R and I are found, it is relatively easy to calculate the current, voltage, and power for any particular resistance in the system.

As an example, consider the network of Figure 4.22 in which 200 volts d-c is applied to the arrangement of resistances given by their ohmic values. The equivalent resistance of the three right-hand branches in parallel is 10 ohms, whence 25 ohms is obtained when added to the two adjacent series resistances.

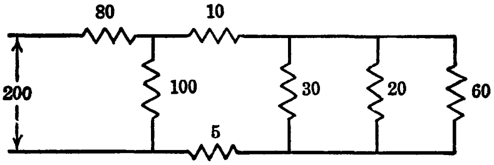


FIG. 4.22. Network of resistances.

The equivalent resistance of the new parallel combination of 100 and 25 ohms then is 20 ohms. The net resistance of the entire system thus is 100 ohms, the line current is 2 amp, and the circuit power is 400 watts.

From this point on, the complete voltage and current distribution for the network is readily available. The 80-ohm resistance consumes 160 volts, whence 40 volts exist across the remainder of the circuit. The current through the 10- and 5-ohm resistances is 1.6 amp and the voltage drop across the three right-hand branches then is 16 volts. The reader may complete the solution from these indicated steps. Note that in the end the sum of the I^2R values for all the resistances will equal 400 watts. It should be apparent, also, that the solution of the above circuit is the same for an a-c applied voltage of 200 volts effective as it is for 200 volts d-c.

4.17. Analysis by Complex Numbers.[1] In the preceding treatment on electrical circuits, it was often found expedient to resolve a voltage or a current into two components at right angles to each other. The components generally were chosen along the ordinary X and Y axes. In effect, the magnitudes and positions of voltages and currents then were given on the vector diagram in terms of their respective rectangular coordinates. Since the scheme is identical to the graphical representation of a complex number, we therefore observe the opportunity of handling a-c circuit problems by means of the algebra of complex numbers.

Analysis by means of complex-quantity algebra resolves each vector into two components. The X axis is chosen arbitrarily as a reference line, and those components perpendicular to it are so designated by the prefix $\pm j$. Figure 4.23 shows a vector diagram of voltage and current with the rectangular axes included. Voltage and current are expressed as complex numbers and are referred to as *vector* volts and *vector* amperes, the term *vector* being used in the same sense as complex number.

---

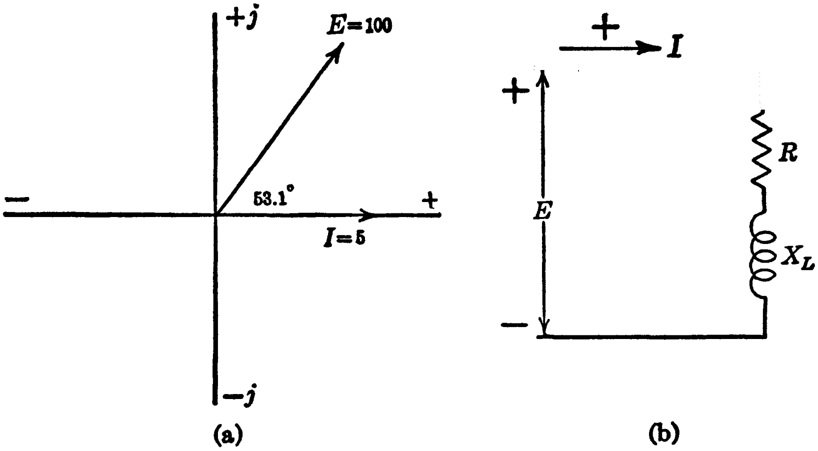
[1] A brief treatment of complex numbers is presented in Appendix II.

The voltage and current shown in Figure 4.23a can be expressed as

$$E = 60 + j80$$

$$I = 5 + j0$$

The symbol j is an operator, which by definition rotates a vector through 90 degrees counterclockwise when multiplied by it. Thus, jE leads E by 90 degrees on the vector diagram. Multiplying a vector by $-j$ rotates the vector clockwise 90 degrees. It follows that multiplying a vector by $+j^2$ rotates a vector through 180 degrees or is the equivalent of multiplying the vector by -1. Therefore,



(a) (b)

FIG. 4.23. Vector diagram of a simple series circuit.

$$j^2 = -1$$

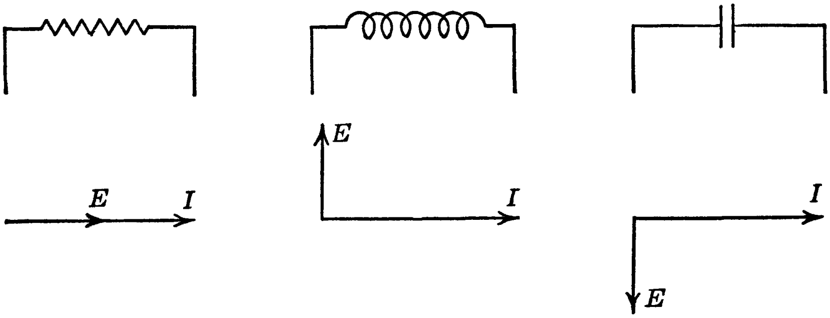
and

$$(+j)(-j) = +1$$

When complex algebra is used in the solution of circuit problems, impedance is defined as the ratio of voltage to current when both the last two quantities are given as complex numbers. Circuit impedance now becomes a *vector* quantity that relates the vector voltage to the vector current by the equation

$$E = IZ \tag{4.26}$$

The vector impedance can be calculated from the values of the parameters and the frequency of the circuit. Equation 4.26 is written for each of the basic circuit elements shown in Figure 4.24 immediately below their respective vector diagrams. The vector impedances consistent with these equations are written below the respective voltage equations. When voltages, currents, and impedances are expressed as complex numbers, the process of analyzing a circuit is reduced to one of relatively simple algebraic operations.



$E = IR$
$Z = R$
Resistance

$E = jX_L I$
$Z = jX_L$
Inductance

$E = -jX_C I$
$Z = -jX_C$
Capacity

Fig. 4.24. Impedances, expressed in complex form, of the three fundamental circuit elements.

For the series circuit shown in Figure 4.9, the vector voltage is given by the equation

$$
\begin{aligned}
E &= E_R + jE_L - jE_C && (4.27)\\
&= IR + jX_L I - jX_C I\\
&= I[R + jX_L - jX_C] && (4.28)\\
&= IZ && (4.29)
\end{aligned}
$$

Hence, the vector impedance of a series circuit is the sum of the vector impedances of the individual circuit elements.

If vector quantities are used in computing the branch currents of a group of parallel circuits, the currents will be expressed as complex numbers, and the sum of these currents gives the vector line current. Expressed in equation form,

$$
\begin{aligned}
I &= I_1 + I_2 + I_3 + \text{etc.} && (4.30)\\
&= \frac{E}{Z_1} + \frac{E}{Z_2} + \frac{E}{Z_3} + \text{etc.}\\
&= E\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \text{etc.}\right] && (4.31)\\
&= \frac{E}{Z} && (4.32)
\end{aligned}
$$

where Z_1, Z_2, Z_3, etc., are the vector impedances of the several branches. From equations 4.31 and 4.32, it is apparent that the equivalent vector impedance Z of several parallel branches is found from the relation

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \text{etc.} \tag{4.33}$$

where all impedances are expressed as complex numbers.

4.18. Determination of Power Using Complex Notation. If the current and voltage are expressed in complex form

$$I = I_1 + jI_2 \tag{4.34}$$

$$E = E_1 + jE_2 \tag{4.35}$$

the power is equal to

$$P = E_1I_1 + E_2I_2 \tag{4.36}$$

Each component of current and voltage must be taken with its proper algebraic sign. The j is not included in the multiplication. This method of determining power is consistent with our previous definitions of power. It is the sum of the products of the in-phase components of voltage and current. This is evident from the vector diagram shown in Figure 4.25. In this diagram E_2 and I_2 are in phase and E_1 and I_1 are in phase.

Since the power is

$$P = E_1I_1 + E_2I_2$$

it can be written as

$$\begin{aligned} P &= E\cos\theta_E \cdot I\cos\theta_I + E\sin\theta_E \cdot I\sin\theta_I \\ &= EI[\cos\theta_E \cdot \cos\theta_I + \sin\theta_E \cdot \sin\theta_I] \\ &= EI\cos(\theta_I - \theta_E) \end{aligned}$$

The last equation is the one developed in the first part of this chapter for power expended in a circuit.

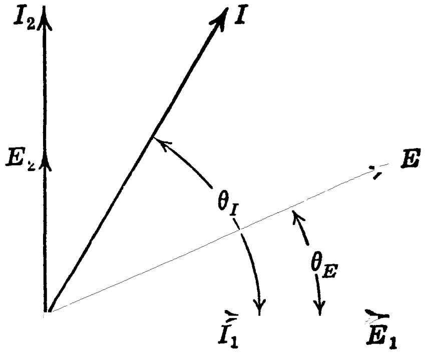


FIG. 4.25. Vector components of current and voltage.

It should be stressed that power *cannot* be obtained from the product of vector current and vector voltage—the product having no physical significance. However, a useful result follows from the product of I and the conjugate of E. The real part of this latter product is the active power, while the j term gives the reactive power with its proper sign, that is,

$$I\overset{*}{E} = \text{active power} + j\ \text{reactive power} \tag{4.37}$$

where $\overset{*}{E}$ is the conjugate of E. For equations 4.34 and 4.35,

$$\begin{aligned} I\overset{*}{E} &= (I_1 + jI_2)(E_1 - jE_2) \\ &= I_1E_1 + I_2E_2 + j(I_2E_1 - I_1E_2) \end{aligned}$$

It is well to point out that complete solutions of electrical networks can be obtained without the use of complex algebra. In general, a clearer understanding of the problem is obtained if the methods discussed in the previous sections are employed. For this reason, it is suggested that the use of complex-algebraic methods be avoided until the performance of electrical circuits is well understood. However, after one has grasped the underlying fundamentals of electrical theory, the application of complex-algebraic methods will often simplify the work.

A few comments should be made on the usual methods of handling alternating-current quantities. General practice, which also is followed throughout this work, is to represent voltages and currents by arrows called vectors. The lengths of the lines give respective magnitudes, and the arrowheads are for the purpose of defining completely the angles between vectors in order to fix the correct algebraic sign before the trigonometric functions of the angles. In this manner of treating elementary a-c theory, however, the methods developed for true vectors in the fields of mathematics and mathematical physics are now followed. Our vectors only are vectors so-called, and, although advanced studies on circuits may employ vector analysis as such, our approach is based purely on the algebra of complex numbers. If these points are recognized, there should be no misunderstanding on the terminology and the methods of solution.

4.19. Numerical Example. A voltage of $104 + j60$ volts is applied to a circuit consisting of 2 parallel branches. One branch contains 6 ohms resistance, 10 ohms inductive reactance, and 5 ohms capacitive reactance. The second branch contains 8 ohms resistance and 3 ohms capacitive reactance. Using complex quantities, determine the branch currents, the line current, the equivalent impedance, the power input to each branch, and the total power input.

Solving for the current in the first branch,

$$Z_1 = 6 + j10 - j5 = 6 + j5$$

and

$$\begin{aligned} I_1 &= \frac{E}{Z_1} = \frac{104 + j60}{6 + j5} = \frac{104 + j60}{6 + j5} \times \frac{6 - j5}{6 - j5} \\ &= \frac{924 - j160}{36 + 25} \\ &= 15.1 - j2.62 \\ &= 15.3 \text{ amperes in magnitude} \\ I_1\overset{*}{E} &= (15.1 - j2.62)(104 - j60) \\ &= (1{,}570 - 157) - j(272 + 906) \\ &= 1{,}413 \text{ watts} - j1{,}178 \text{ volt-amperes} \end{aligned}$$

Therefore, the power input to Z_1 is 1,413 watts, which is also equal to $(15.3)^2R_1$, and the reactive power taken by Z_1 is $-1{,}178$ volt-amperes. The second branch can be solved in a similar manner, giving

$$Z_2 = 8 - j3$$

and

$$\begin{aligned} I_2 &= \frac{E}{Z_2} = \frac{104 + j60}{8 - j3} \\ &= 8.93 + j10.85 \\ &= 14.05 \text{ amperes in magnitude} \\ I_2\overset{*}{E} &= (8.93 + j10.85)(104 - j60) \\ &= 1{,}580 \text{ watts} + j592 \text{ volt-amperes} \end{aligned}$$

The input to Z_2 is 1,580 watts power and $+592$ volt-amperes reactive power.

The line current is the vector sum of the two branch currents, that is,

$$\begin{aligned} I &= I_1 + I_2 \\ &= (15.1 - j2.62) + (8.93 + j10.85) \\ &= 24.03 + j8.23 \\ &= 25.4 \text{ amperes in magnitude} \end{aligned}$$

The power input and reactive power input are equal to the respective sums of the power inputs and the reactive power inputs to the 2 branches. Therefore,

$$\begin{aligned} \text{power input} &= 1{,}413 + 1{,}580 \\ &= 2{,}993 \text{ watts} \end{aligned}$$

and

$$\begin{aligned} \text{reactive power} &= -1{,}178 + 592 \\ &= -586 \text{ volt-amperes} \end{aligned}$$

The values of power and reactive power are also given by

$$\begin{aligned} I\overset{*}{E} &= (24.03 + j8.23)(104 - j60) \\ &= 2{,}993 \text{ watts} - j586 \text{ volt-amperes} \end{aligned}$$

The input impedance is equal to

$$\begin{aligned} Z &= \frac{E}{I} = \frac{104 + j60}{24.03 + j8.23} \\ &= 4.63 + j0.91 \text{ ohms} \end{aligned}$$

The impedance can also be determined from equation 4.33. The numerical calculation is left as an exercise for the student.

4.20. Three-wire Systems. Electrical power, usually at relatively low voltages, very often is distributed by means of the 3-wire system, either a-c or d-c. The method is commonly used for applications of combined lighting and power loads where the latter are of small horsepower. A third lead, called the neutral or mid-wire, carries a voltage between it and either of the outer wires of one-half the voltage between the main leads. In this way, power at voltages having a 2:1 ratio, for example 240/120 volts, can be supplied lamps and low-voltage appliances connected between either main wire and the neutral, while at the same time motors or other loads requiring the higher voltage can take power from the two outer wires. The primary advantage is the saving in copper over that for two separate circuits, each having its own voltage.

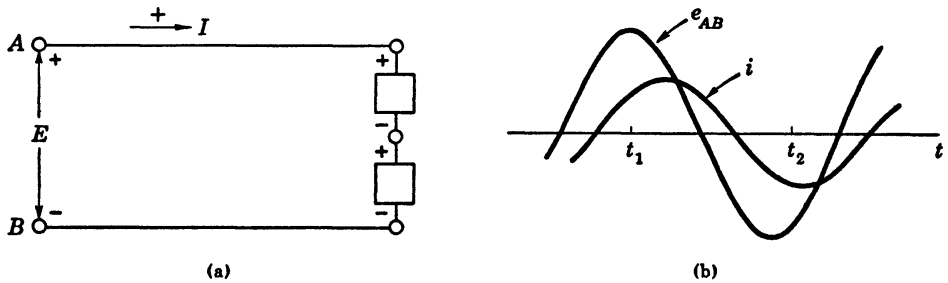
The loads at the lower voltage are distributed as evenly as possible between the two halves of the system. With complete load balance, no current flows in the mid-wire, the total power being transmitted effectually at the higher voltage. In general, some unbalance in current between the two halves of the system always occurs, the difference between the currents in the outer lines being that which flows in the neutral. It is found desirable generally to limit this difference, or unbalanced, current to 10, 15, or perhaps 25 per cent at most of the normal current of the main lines. The mid-wire hence can be of much smaller size than the other two. The 3-wire system is a special case of the 2-mesh network, and the manner of handling it is given in the following discussions on general circuits.

4.21. General Networks. There are many instances in which one may wish to know the current or voltage of only one branch of a series-parallel network without determining completely the distribution of these quantities over the entire system. Again, it is common to find voltages introduced at several points in a network, the current through any branch then being the result of the combined effects of all the voltages. Many distribution systems are of this character. Also, in some d-c circuits there is the case of the main supply augmented by a battery in parallel with it, the battery charging at light loads and discharging at heavy loads. Generator, battery, and load currents may be desired.

In the several branches of electrical engineering, a variety of electrical circuits are used as transmission networks. In the power field, a transmission network may consist of an array of interconnected transmission lines, transformers, generator stations, and consumers' loads. A good part of the analysis of such a system consists in solving the transmission characteristics of particular links of the system. Although the links may be in themselves complex, they often can be reduced to elementary equivalent circuits having two input and two output terminals. Four-terminal networks of this type are also of great importance to the communication engineer. In the transmission of intelligence from one pair of terminals to another, he must be concerned with many types of networks, as, for example, amplifiers, long transmission lines, electric wave filters, attenuators, and equalizers for correcting the defects in the transmission-frequency characteristics of some other part of the system. These networks are most satisfactorily handled by being reduced to simpler equivalent circuits or by making use of certain theorems. From the theorems studied in the following sections of this chapter, the student will be able to recognize that much time and labor can be saved through their use. The rather brief presentation of these general theorems will serve as an introduction to some of the methods used in solving network problems with which both the power and communication engineer should have some familiarity. Also, many of the relations can be extended to include mechanical and acoustical networks.

4.22. Notation for Voltage and Current. In systems containing several sources of voltage and several currents, it is essential to adopt a systematic method of notation for designating the current and voltage vectors. Once having established a convention, its use must be continued without deviation throughout the analysis. If this is done, the treatment will be straightforward and errors will be avoided.

The direction of positive current flow can be indicated by means of an arrow on the circuit diagram, as shown in Figure 4.26a. The arrow, $\xrightarrow{+}$, indicates the chosen direction of *positive* current flow. For example, at time t_1, Figure 4.26b, the current is positive and the flow of current in the circuit would be, therefore, in the direction indicated by the arrow, while at time t_2 the flow would be in the opposite direction. It is apparent that the choice of positive direction of current flow is somewhat arbitrary in a-c circuit analysis.



(a) (b)

FIG. 4.26. The use of polarity marks and arrows on a-c circuits.

The polarity markings + and − between terminals indicate the actual polarity at the time when the instantaneous voltage is positive. For example, at time t_1, when the voltage is positive, the polarity between terminals is as indicated by the polarity markings, while at time t_2, when the voltage is negative, the actual polarity of the terminals is the reverse of that shown on the circuit diagram. The polarity markings also indicate the direction of the voltage drop, from + to −, through the load.

Although it appears that the assignment of voltage polarities and current directions on the circuit diagram is somewhat arbitrary, it is very essential that their relative directions within the circuit be consistent with what can be found from laboratory experiments, i.e., *voltage drop through an impedance must always be taken in the direction of positive current flow.* These same concepts and conventions are used when voltages and currents are treated as vector quantities.

Voltage drops can be designated also by the use of double subscripts. The notation adopted in this book is that the symbol E_{AB} always stands for the *voltage drop* in the direction of the subscripts, and when going through an impedance it is to be taken in the direction of positive current flow. The symbolism also can be interpreted as designating the potential of terminal A with respect to B.

In a similar manner, double subscripts can be used to designate the direction of current flow. Thus, in Figure 4.26a, the current through the load can be designated as I_{AB}, the flow being in the direction of the subscripts.

It is evident that

$$E_{AB} = -E_{BA} \tag{4.38}$$

and that

$$I_{AB} = -I_{BA} \tag{4.39}$$

In this text, either the double-subscript notation, the polarity marks and arrows, or a combination of the two is used, depending upon which seems more suited to the problem.

4.23. The Solution of General Networks. The current and voltage distribution for a network can be found by solving simultaneously a set of equations involving the mesh currents. A mesh is any single closed opening in a network. The method is applicable to a-c and d-c systems under either steady-state or transient conditions, and a particular voltage or current may be obtained without specific knowledge of the others. The scheme employs only Kirchhoff's law of voltages for a closed path, which can be stated as follows: If any closed path is traversed, the vector sum of the voltage drops across the passive elements (impedances) is equal to the total rise in voltage across the active elements (voltage sources), both taken in the same direction of traversal. For steady-state conditions the analyses are algebraic only, the method of attack for the a-c circuit being carried out by means of complex quantities.

As an illustration, consider the 2-mesh network of Figure 4.27 in which the steady-state currents are desired. Let the voltages be alternating, their complex-quantity magnitudes being known. The impedances of the arms may represent series combinations of R, L, and C, or the equivalent impedances of other configurations. These also are expressed in complex-quantity form.

The voltage equations for the loops are

$$\begin{aligned} I_1Z_1 + I_1Z_2 - I_2Z_2 &= E_1 \\ -I_1Z_2 + I_2Z_2 + I_2Z_3 &= E_2 \end{aligned} \tag{4.40}$$

In general, the actual direction of the current flow is not known and may be alternating, the assumed positive direction, as indicated by an arrow on the circuit diagram, being wholly arbitrary. In any case, the solution of the equations will give the correct phase of the current relative to the initially assumed direction of positive current flow. Voltage drops for the passive elements are positive when going with the current, and negative when going against a current. In accordance with the statement of Kirchhoff's law, the voltage of a source should be stated as the voltage of the terminal from which the current leaves the source, relative to the terminal by which it enters. Thus, E_1 is actually E_{ab} and E_2 is E_{fe}.

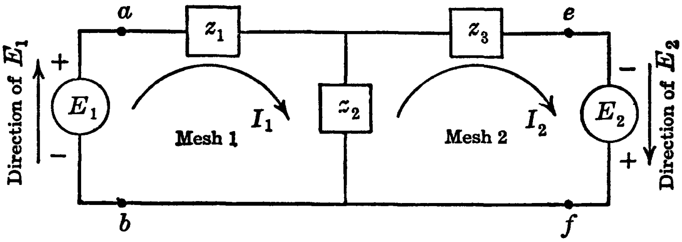


FIG. 4.27. A two-mesh network with two voltage-sources.

Equations 4.40, in a more generalized form, can be written

$$\begin{aligned} I_1Z_{11} + I_2Z_{12} &= E_1 \\ I_1Z_{21} + I_2Z_{22} &= E_2 \end{aligned} \tag{4.41}$$

from which I_1 and I_2, as obtained by simultaneous solution, are

$$\begin{aligned} I_1 &= \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}^2}E_1 - \frac{Z_{12}}{Z_{11}Z_{22} - Z_{12}^2}E_2 \\ I_2 &= \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}^2}E_2 - \frac{Z_{21}}{Z_{11}Z_{22} - Z_{12}^2}E_1 \end{aligned} \tag{4.42}$$

In these equations, Z_{11} is the total impedance offered by mesh 1 to current I_1, with mesh 2 open-circuited. In Figure 4.27, $Z_{11} = Z_1 + Z_2$. In general, Z_{nn} is the impedance of the nth mesh, with all other meshes open-circuited. The mutual impedance, Z_{12} or Z_{21}, between the two meshes, is defined as the vector ratio of the voltage produced in one mesh to the current flowing in the second mesh, with all meshes but the second open-circuited. In general, Z_{jk} or Z_{kj} is the mutual impedance of, or the impedance common to, meshes j and k. When the mutual impedances will transmit energy equally in either direction, they are said to be bilateral, and $Z_{jk} = Z_{kj}$. Mutual and self-inductance, resistance, and capacitance are bilateral, whereas vacuum tubes are not. For Figure 4.27, and with the direction of positive current flow as indicated, the mutual impedance $Z_{12} = Z_{21} = -Z_2$. The sign of the mutual impedance depends upon the positive directions of currents that are arbitrarily assigned to the various meshes and takes into account the sign of its voltage component. When the arrows representing the directions of positive current flow pass through the impedance that is common to the two meshes in the same direction, the common impedance is a positive mutual impedance; if the arrows are in opposite directions, the mutual impedance is the negative of this common impedance.

The preceding discussion can be extended to a circuit containing any number of meshes. For a network having n meshes, it is evident that n voltage equations can be written as follows:

$$\begin{aligned} I_1Z_{11} + I_2Z_{12} + \cdots + I_nZ_{1n} &= E_1 \\ I_1Z_{21} + I_2Z_{22} + \cdots + I_nZ_{2n} &= E_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ I_1Z_{j1} + I_2Z_{j2} + \cdots + I_nZ_{jn} &= E_j \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ I_1Z_{n1} + I_2Z_{n2} + \cdots + I_nZ_{nn} &= E_n \end{aligned} \tag{4.43}$$

These equations can be solved for the currents most easily by the use of determinants. The solution for the current in mesh k is given by the equation

$$I_k = \frac{M_{1k}}{D}E_1 + \frac{M_{2k}}{D}E_2 + \cdots + \frac{M_{jk}}{D}E_j + \cdots \frac{M_{nk}}{D}E_n \tag{4.44}$$

in which D is the determinant of the impedances and can be written

$$D = \begin{vmatrix} Z_{11} & Z_{12} & \cdots & Z_{1j} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2j} & \cdots & Z_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{j1} & Z_{j2} & \cdots & Z_{jj} & \cdots & Z_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{n1} & Z_{n2} & \cdots & Z_{nj} & \cdots & Z_{nn} \end{vmatrix} \tag{4.45}$$

The quantity M_{jk} in equation 4.44 is the principal minor of D and is formed by eliminating the jth row and kth column from D. In evaluating M_{jk}, it must be multiplied by $(-1)^{j+k}$.

Superposition Theorem. Equation 4.44 is a proof of the superposition theorem, which states that in a linear system each applied stimulus produces a response independent of the response due to any other stimulus, and that the total response is the sum of the individual responses. This theorem applies to all linear systems, including electrical, mechanical, acoustical, or any combination of these systems. A linear system is one composed of circuit elements that show a direct proportionality between the current and the voltage or between the mechanical deformation and the applied force. Electrical circuits containing resistances, air-core inductances, capacitances, some iron-core inductances, and some tube circuits are linear. In non-linear systems, notably vacuum-tube circuits and saturated magnetic circuits, the application of the superposition principle, even for obtaining a partial answer to the problem, should be made only when the limitations involved are clearly understood.

When several sources are present in a circuit problem, the solution is often advantageously obtained by applying the principle of superposition and considering one generator at a time. Another application of the principle is the determination of the response of a system to a stimulus having a periodic but complex wave form. A periodic complex wave can be analyzed into a Fourier series of sinusoidal terms consisting of a fundamental and a series of harmonics. The response to each component of the wave can then be determined and the resultant response calculated by adding the individual responses.

The Reciprocity Theorem. Equation 4.44 also contains the proof of the reciprocity theorem. In deducing this important theorem, we note that the current in any mesh k caused by a voltage applied in any other loop j is

$$I_k = \frac{M_{jk}}{D} E_j \tag{4.46}$$

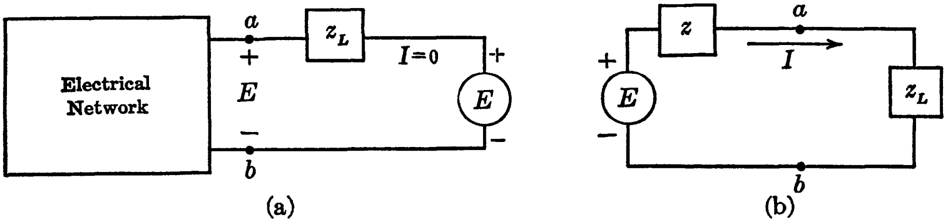
and that the current in mesh j caused by a voltage applied in loop k is

$$I_j = \frac{M_{kj}}{D} E_k \tag{4.47}$$

When the mutual impedances are bilateral, then Z_{jk} and Z_{kj} are identical and it can be shown that M_{jk} and M_{kj} are equal. This equality becomes evident when the two determinants are written out and it is noted that the only difference between them is that a column and a row of one determinant are interchanged with the corresponding row and column of the other. This interchange does not affect the value of the determinant. From these considerations, it follows that if E_j acts alone, the current in mesh k is the same as the current in mesh j when E_k acts alone. In other words, a pure voltage source in any mesh and a perfect current-measuring instrument (one having zero impedance) in any other mesh can be interchanged without changing the ratio of voltage to current. This ratio is known as the transfer impedance from one mesh to the other. Briefly summarized, the reciprocity theorem states that in a system composed of linear bilateral elements, the transfer impedance of one mesh to any other is the same in both directions.

An important conclusion from this theorem is that a network of linear bilateral elements transmits with equal effectiveness in both directions. In addition to electric circuit problems, the reciprocity principle applies to problems in acoustics and electromagnetic radiation. In this latter field, its application frequently simplifies the treatment of antenna problems, since, through its use, it is possible to express the receiving characteristics of an antenna system in terms of those it possesses when it is being used for transmitting.

Thévenin's Theorem. An important deduction can be made from an extension of the superposition theorem. This is the theorem of Thévenin which can be stated as follows: Any network of linear elements and impedanceless generators (in an actual generator the impedance can be considered external to it), when viewed from any two terminals of the network, can be treated as a simple generator of voltage E and impedance Z; E is the voltage appearing between the terminals when no load impedance is connected to them, and Z is the impedance looking back into the network when all generators are short-circuited. In proof of this theorem, consider the two terminals of the network closed through a load impedance Z_L, and a hypothetical generator having a voltage equal to E but acting in the opposite direction around the circuit, as is shown in Figure 4.28a. Under these conditions, no current will flow in the external loop. This zero current, in accordance with the principle of superposition, may be regarded as the sum of the current due to the network in the absence of the hypothetical generator, and the current due to the hypothetical generator acting when all other sources are short-circuited. When Z_L alone is connected to the terminals, the current in the external branch is the negative of that driven by the hypothetical generator, acting alone, through Z and Z_L in series. This allows the network, with its connected load, to be represented as shown in Figure 4.28b.



(a) (b)

FIG. 4.28. Equivalent circuit derivation by Thévenin's theorem.

Thévenin's theorem offers a very effective means of simplifying complicated network problems. It is used extensively in determining the performance of communication networks. It can be conveniently used to calculate the changes introduced by connecting measuring instruments to high-impedance networks.

Four-terminal Coupling Networks—T and π Networks. In both the power and the communications fields, a source of electrical power is often connected to its load through a rather complex coupling network which may contain long transmission lines, several transformers, and various other circuit elements. In some cases, the circuit constants of the coupling network may be unknown, it being represented only as a box with two input and two output terminals. Irrespective of how complicated such a 4-terminal network is, its transmission characteristics at any one frequency can be completely expressed in terms of just three circuit parameters, provided that the network consists only of linear bilateral elements and contains no internal power sources. These three parameters can be determined from three measurements made at the terminals of the network, and even though the details of the complete internal structure may be unknown, it can be represented, insofar as the two pairs of terminals are concerned, by three circuit elements.

The 4-terminal network can be shown as in Figure 4.29, with the voltages and currents associated with the terminals, as indicated on the diagram. By the use of the superposition theorem, the currents can be expressed by the equations

$$I_1 = \frac{E_1}{Z_{11}} + \frac{E_2}{Z_{21}}$$
$$I_2 = \frac{E_1}{Z_{12}} + \frac{E_2}{Z_{22}} \tag{4.48}$$

The equations, themselves, should make clear the definitions of the various impedances which appear in them. When terminals 2–2 are short-circuited, E_2 is zero, and equations 4.48 give

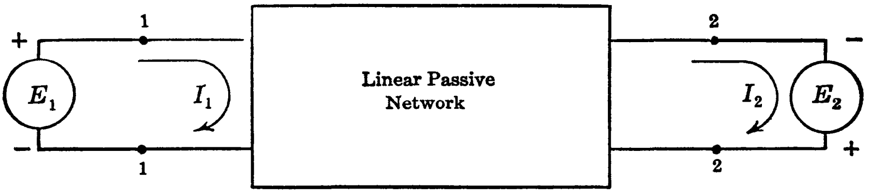


FIG. 4.29. Four-terminal network.

$$Z_{11} = \frac{E_1}{I_1} \tag{4.49}$$

and

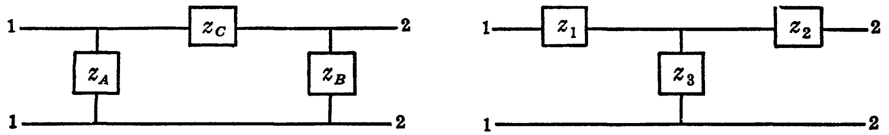
$$Z_{12} = \frac{E_1}{I_2} \tag{4.50}$$

It is seen that Z_{11} is the impedance looking into terminals 1–1 with terminals 2–2 short-circuited. Similarly, if terminals 1–1 are short-circuited, the impedance looking into terminals 2–2 is Z_{22}. The impedance Z_{12} is called the short-circuit transfer impedance and is equal to the vector ratio of the applied voltage at terminals 1–1 to current at the short-circuited terminals 2–2. Because of the reciprocity theorem, which applies to linear bilateral networks, Z_{12} and Z_{21} are identical, and therefore only three parameters are necessary to specify the network uniquely.

The voltage E_2 need not be a voltage source but may be the voltage drop across a load impedance Z_L. Consistent with the chosen positive directions of current and voltage, the value of E_2 to be used in the equations is given by the relation

$$E_2 = -I_2 Z_L \tag{4.51}$$

It now has been shown that the characteristics of a linear bilateral network can be expressed in terms of the three parameters Z_{11}, Z_{22}, and Z_{12}. These parameters can be calculated from the constants of the network if these constants are known, or they can be obtained from measurements on the network. After the three parameters have been determined, the network can be represented by either one of two configurations, each containing three circuit elements, as is shown in Figure 4.30. Figure 4.30a shows a π network, so called because of its configuration, and, for the same reason, Figure 4.30b is called a T network. When these same networks are used in 3-phase work, they are called delta (Δ) and wye (Y) connections, respectively. For the π network, the following relations are left for the student to prove:



(a) π or Δ network (b) *T* or *Y* network

Fig. 4.30. π and *T* networks.

$$Z_C = Z_{12}$$

$$Z_A = \frac{Z_{11}Z_{12}}{Z_{12} - Z_{11}} \tag{4.52}$$

$$Z_B = \frac{Z_{22}Z_{12}}{Z_{12} - Z_{22}}$$

Similar relations can be developed for expressing the network in terms of an equivalent T network. In fact, at any single frequency, any π network can be transformed into an equivalent T network, and vice versa. The equations of transformation are identical to those of the Δ to Y transformation which are given in Section 24 of Chapter 6. In some problems, the calculated constants of an equivalent circuit are not physically realizable. The calculations may show the resistance component of one impedance to have a negative value. Although the presence of this negative resistance in the circuit constants would present difficulties in setting up the physical circuit in the laboratory, the computations of the circuit performance based upon it are not affected.

It should be emphasized that the present treatment provides but one of several ways of handling 4-terminal networks. This treatment, however, is typical and points out the general type of analysis with which both the power and communications engineer should be familiar.

Problems

1. A 2-turn coil on a 12-pole generator produces 12.8 volts effective. The time for a conductor to pass over one pole pitch is 0.005 sec. Write the general equation for the instantaneous voltage induced in a single conductor.

2. A single-phase generator sends 10 amp average through a test circuit for 80 volts average impressed on it. At a frequency of 25 cycles per second, and a 40° lag in the current with respect to the voltage, write the general equation for the instantaneous voltage and current. Show these quantities on a vector diagram with the voltage along the horizontal reference axis. Repeat but with the current displaced $-105°$ from the reference $\omega t = 0°$.

3. A 25-ohm resistor has 720 kw-sec of heat produced in it in 20 min under the application of a 50-cycle voltage. Write the equations for the instantaneous voltage and current. Show these quantities on a vector diagram with the voltage displaced $+55°$ from the reference $\omega t = 0°$.

4. A 15-ohm resistor carries a current $i = 10 \cos (\omega t + \alpha) + 10 \sin (\omega t + \alpha - 30°)$. Write the equation for the instantaneous impressed voltage. Show the current and voltage components, and the resultants, on a vector diagram. Calculate the power consumed by the resistor. Show the resultant current and voltage on a vector diagram for $\alpha = +75°$.

5. By trigonometric expansion add the 3 currents $i_1 = 8 \sin \omega t$, $i_2 = 10 \sin (\omega t + 45°)$, and $i_3 = 6 \sin (\omega t + 60°)$. Determine the phase relation of the sum with respect to i_1. Check the summation graphically by the vector addition of the 3 currents. If these currents are carried simultaneously by a single 5-ohm resistor, write the equations for the voltage components and the resultant impressed voltage.

6. Carry through the development for the *R-C* circuit as has been done for the *R-L* circuit in the textbook.

7. Plot $e = 100 \sin (\omega t + \alpha - 30°)$ and $i = 20 \cos (\omega t + \alpha - 60°)$ for $\alpha = 0°$, $+25°$, and $-80°$. Show these quantities on a vector diagram. Calculate Z, R, and X for the circuit implied by these equations, assuming R and X to be in series. State the character of the reactance X. What power is taken by the load impedance?

8. A current $i = 3 \sin (377t - 15°)$ flows in an *R-L* circuit as a result of a voltage $e = 156 \sin (377t + 45°)$. What are the resistance and the inductance of the circuit? What power is taken by the load impedance?

9. An a-c current of 10 amp effective value, 60 cycles, flows in a series circuit of 10 ohms resistance and 0.05 henry inductance. Draw the complete vector diagram showing all component voltages and the phase angle between line voltage and current. What power is taken by the load impedance?

10. A voltage $e = 141.4 \sin (377t + 60°)$ sends a current $i = 7.07 \sin (377t - 20°)$ through an impedance. What are the magnitudes of the constants of the simplest circuit represented by these equations? If the circuit contains an inductance of 0.1 henry, what is the simplest circuit the equations imply, and what are the values of the circuit elements? What value of capacitance, for the same voltage, will force the current to lead the voltage by 25°?

11. A series circuit contains 8 ohms resistance, 127.3 μf capacitance, and 98.7 mh inductance. The equation for the voltage across the condenser is

$$e_c = 353.55 \sin (314.16t - 20°)$$

Write the equation for the voltage impressed on the circuit.

12. Three identical coils of 200 turns each are in series addition on the same magnetic circuit. Assume unity coefficient of coupling and neglect saturation. If each coil contains 15 ohms resistance, and the circuit takes 359 watts power on 230 volts rms at 60 cycles, what flux will one coil produce with a current of 2 amp?

13. A 60-cycle voltage of 220 volts produces a current of 32 amp in a series circuit of resistance and a reactance. When the frequency is doubled, the voltage remaining the same, the current increases to 40 amp. Evaluate the circuit constants.

14. A condenser has a rating 2 kva, 460 volts, 60 cycles. Calculate the capacitance of the condenser and the maximum value of current it will draw at this voltage. What will be the rating of the condenser on 220 volts, 60 cycles? On 460 volts, 25 cycles? If the loss in watts within the capacitor is $\frac{1}{6}$ of 1% of its kva rating, what is the phase angle of the capacitor?

15. A coil takes 20 amp on 115 volts d-c and 7 amp on 115 volts a-c, 60 cycles. What are the resistance and inductance of the coil? What minimum series capacitance will allow 15 amp to flow on 115 volts a-c?

16. Two coils are connected in series on a 110-volt supply. $R_1 = 3$ ohms and $X_1 = 4$ ohms inductive reactance, $R_2 = 8$ ohms and $X_2 = 6$ ohms inductive reactance. What voltage is measured across each coil? What is the current, and what is the phase angle between the current and the circuit voltage?

17. A voltage of 115 volts at 60 cycles is applied to a 2-kva, 460-volt, 60-cycle capacitor. The capacitor current is 1.08 amp. What is the impedance of the capacitor? How closely do the instrument readings agree with the rating of the capacitor at reduced voltage? A low-power-factor wattmeter having a rating of 2.5/5 amp//75/150 volts, 75 watts full scale, is connected between the capacitor and the voltmeter. The instrument contains 3,130 ohms resistance in its potential circuit and has an accuracy within $\frac{1}{4}$ of 1% of full scale at any point on the scale. It indicates 5.4 watts when uncompensated and 1.0 watts when compensated. What are the limits of the calculated loss of the capacitor?

18. Two inductive coils in series on 220 volts carry 5 amp and each shows 120 volts between its terminals. One of the coils draws 20 amp on 120 volts d-c. What are the resistances and reactances of the coils?

19. A current of 10 amp, 60 cycles, flows in a series circuit containing $R = 10$ ohms, $L = 0.3$ henry, and $C = 20\ \mu$f. Determine the impressed voltage and the voltage across each element. Draw the complete vector diagram.

20. An 8-pole, single-phase generator operates over a wide speed range and has a series circuit of $R = 5$ ohms, $L = 0.157$ henry, and $C = 54.8\ \mu$f connected across its terminals. What will be the generator voltage when the output current is 0.75 amp and the speed is 450 rpm? When the speed is 1,800 rpm?

21. A voltage of 100 volts produces 20 amp in a series circuit of $R = 3$ ohms and a condenser of unknown capacitance, 60 cycles. How much must the capacitance be changed to allow a current of 20 amp at 120 volts, 30 cycles, to flow?

22. A voltage of 110 volts, 60 cycles, produces a 60° leading current of 1 amp in a series circuit of R ohms, 0.146 henry, and C microfarads. What are the values of the circuit constants?

23. A voltage of 115 volts, 60 cycles, is impressed on a circuit of $R = 70$ ohms, $L = 0.3$ henry, and $C = 25\ \mu$f. What change in either L or C will make the circuit resonant? What will be the maximum value of the current at resonance?

24. A 115-volt, 24-pole, 2,500-rpm alternator has an inductance of 0.03 henry and a condenser of 25-μf capacitance in series across its terminals. What must be the speed of the machine to give resonance?

25. Derive, for a series resonant circuit, the expression

$$\frac{I_f}{I_{f_r}} = \frac{\text{actual current}}{\text{current at resonance}} = \frac{1}{\sqrt{1 + \left[\frac{\omega_r L}{R}\left(\frac{f}{f_r} - \frac{f_r}{f}\right)\right]^2}}$$

where f = frequency and f_r = resonant frequency.

26. A coil has an inductance of 200 μh and a resistance of 25 ohms. Determine the capacitance necessary for series resonance at 1,000,000 cycles, and sketch the shape of the resonance curve. How is the shape of the resonance curve changed if the resistance is doubled? What range of capacity is required for tuning throughout the broadcast band?

27. A voltage of 100 volts at 1,000,000 cycles produces 3 amp in a series resonant circuit. For the same voltage, 10,000 cycles from resonance, the current is 2.1 amp. Determine R, L, and C of the circuit.

28. What average power is taken by a 10-ohm resistor carrying a current $i = 25 \sin (\omega t + 30°) + 15 \sin (3\omega t - 45°)$? By a 70-ohm resistor carrying a current $i = 1.3 + 1.6 \sin \omega t$?

29. Measurements on an inductance coil show 2 amp and 20 watts for 115 volts at 60 cycles impressed on it. Calculate the coil constants.

30. An inductance coil takes 2 amp and 160 watts from a 110-volt, 60-cycle source. If the frequency is changed to 50 cycles, the voltage remaining constant, what power will the coil take?

31. A 35-μf condenser is connected in series with a 110-volt, 150-watt lamp. A 60-cycle voltage impressed on the circuit is adjusted until the lamp burns at normal brilliancy. Determine: (a) the impressed voltage; (b) the power factor; and (c) the power input.

32. A rectangular voltage wave of 100 volts maximum value produces a triangular current wave of 50 amp maximum, symmetrical about the 90° axis. Both waves always cross the axis at the same time and in the same direction. What power is represented by these waves?

33. A single-phase motor draws 4 kw, 0.8-pf lagging, at 220 volts. The motor is supplied over a transmission line $\frac{1}{4}$ mile long having a resistance of 1 ohm and an inductive reactance of 3 ohms. What are the input voltage and power of the line? What is the efficiency of transmission?

34. A single-phase load of 5.5 kva at 0.85 pf lagging draws its power at 220 volts. It is fed through a short transmission line having a resistance of 0.6 ohm and a series capacitor. If the input voltage to the line also is 220 volts, what is the net reactive effect of the line in ohms? What is the equivalent series circuit for the line and load, the power input to the line, and the power factor?

35. A 5-hp motor, 0.75 pf lagging, and 78% efficiency, is in parallel with a lamp load of 2.5 kw. What must be the supply voltage if the load voltage is 230 volts and the power is drawn through a feeder having a resistance of 0.5 ohm and an inductive reactance of 1.4 ohms?

36. One branch of a parallel circuit takes 4 kw at 0.6 pf lagging and the second takes 2 kva at 0.8 pf leading. The supply voltage is 220 volts. Calculate the line current and power factor, and the equivalent resistance and reactance of the second branch.

37. A series circuit of 5 ohms resistance and 9 ohms capacitive reactance is connected across a 220-volt supply. A 2.5-hp induction motor having an efficiency of 85% and a power factor of 0.8 lagging is connected across the same supply. Calculate the line current, power input and the equivalent circuit constants of the combined load.

38. A resistance and a pure reactance are in parallel. The line current is 11.2 amp at 100 volts, 60 cycles, and is 14.1 amp at 100 volts, 120 cycles. Determine the circuit constants.

39. A coil of 10 ohms inductive reactance and 20 ohms resistance is connected in parallel with a capacitive reactance of 10 ohms. The line current is 8 amp for a 60-cycle impressed voltage. Determine the voltage, the line power factor, and the power input to the circuit.

40. An inductive load at 60% pf lagging consumes 400 watts from a 110-volt source. For the same load power, what is the value of the circuit element which connected in series with the load will give a pf 60% leading? Suppose the element were connected in parallel with the load. How big should it be?

41. Two single-phase induction motors are connected in parallel to a 220-volt line. Motor A develops 5 hp at 82% efficiency and 0.8 pf. Motor B develops 3 hp at 78% efficiency and 75% pf. Determine the line current, power factor, and power input to the combination.

42. The first of two parallel branches contains 6 ohms resistance and 8 ohms inductive reactance. The second branch is an unknown impedance. The circuit voltage is 120 volts and the total power is 1,342 watts at a pf of 0.895 lagging. What are the values of the circuit elements in the second branch? Calculate the equivalent series and parallel circuits that can represent the combination. What value of capacitive reactance connected across the supply lines would bring the power factor of the combination to unity? Would this value connected in series also result in unity pf?

43. A transformer has a rating of 25 kva, 2400/240 volts, 60 cycles. An open-circuit test made on the 240-volt winding at rated voltage shows 2.1 amp and 230 watts. Calculate the equivalent parallel circuit that can represent these readings. Express the open-circuit current as a per cent of the rated current of the 240-volt winding.

44. A 2-kva, 460-volt, 60-cycle capacitor has a loss in watts of $\frac{1}{6}$ of 1% of its kva rating when used under nameplate conditions. Calculate the equivalent series and parallel circuits which at rated voltage and frequency may represent this capacitor when losses are to be considered.

45. In a test on a 3-phase induction motor at standstill, the following readings were taken for each phase winding: 55 volts, 84 amp, 2.2 kw. Calculate the constants for the equivalent series circuit of each winding.

46. A single-phase induction motor has the following nameplate rating. 1.5 hp, 220 volts, 8.9 amp, 1,720 rpm, 60 cycles. On test, the full-load motor efficiency is found to be 78%. What is the power factor of the motor, and what are the constants of the equivalent series circuit that could represent the motor when running at full load?

47. A pure resistance R and a pure reactance X are in parallel. In terms of R and X, what are the magnitudes of the 3 sides of the impedance triangle representing the equivalent series circuit?

48. Two impedances, Z_1 and Z_2, are in parallel, carrying currents i_1 and i_2, respectively. $i_1 = 10 \sin (\omega t - 15°)$, $i_2 = 20 \sin (\omega t + 30°)$, and the applied voltage is $e = 311 \sin (\omega t + 15°)$. Calculate the magnitudes of the component elements of Z_1 and Z_2, the equivalent series and parallel impedances for the system, and the power taken by the complete circuit.

49. One branch of a parallel circuit contains 4 ohms resistance and 5 ohms inductive reactance. The second branch contains 5 ohms resistance and 8 ohms capacitive reactance. What is the frequency at which the net reactance of the circuit is zero, if the above constants were determined on a 110-volt, 60-cycle test?

50. A coil having a resistance of 5 ohms and an inductive reactance of 80 ohms is connected to a 110-volt, 60-cycle source. What value of capacitance should be placed in parallel with the coil to give 1.0 pf? Under these conditions, what is the power input? Plot: (a) the input reactance and resistance; (b) the input current; and (c) the power input all as a function of frequency.

51. The first branch of a 2-branch parallel circuit contains R ohms and C farads. The second branch contains R ohms and L henries. If R in each branch has the magnitude $\sqrt{L/C}$, what is the equivalent impedance of the circuit? Does frequency affect this impedance in any way?

52. The 3 branches of a parallel circuit contain, respectively: 10 ohms resistance; 10 ohms resistance and 10 ohms inductive reactance; and 10 ohms resistance and 10 ohms capacitive reactance. In series with this circuit is 15 ohms resistance, 8 ohms inductive reactance, and 8 ohms capacitive reactance. Draw the equivalent circuit diagram. Determine the circuit power for 200 volts impressed.

53. A parallel circuit consists of 20 ohms resistance in parallel with an unknown impedance. In series with this combination is 5 ohms resistance, 5.03 ohms inductive reactance, and 13.69 ohms capacitive reactance. The applied voltage, the voltage across the series branch, and the voltage across the parallel branch all are equal and are 100 volts. Determine the unknown impedance, all currents, and the circuit power.

54. The first branch of a parallel circuit contains 4 ohms resistance and 3 ohms inductive reactance. The second branch contains 5 ohms resistance and 5 ohms inductive reactance. In series with the parallel circuit is 6 ohms inductive reactance and R ohms resistance. A voltage of 120 volts causes a 12-amp line current to flow. Draw the equivalent circuit and determine the circuit power.

55. The electrical load of a town is 400 kw, 0.9 pf lagging, at 12,000 volts. The load is supplied by a generating station over 2 transmission lines in parallel. Line No. 1 is 50 miles long and has a resistance of 0.26 ohm per mile of line and a reactance of 0.64 ohm per mile of line. Line No. 2 is 80 miles long and has a resistance of 0.14 ohm per mile of line and a reactance of 0.45 ohm per mile of line. Determine the generator current, each feeder current, and the efficiency of transmission.

56. A transmission line, having the constants of line No. 2 of problem 55, supplies a load of 300 kw, at 0.8 pf leading, and at 12,000 volts. The load is 100 miles from the generating station. A second load 50 miles from the generating station is 150 kw, 0.9 pf lagging. Determine the voltage at the second load and the voltage at the generating station.

57. A 40-kw load at 0.8 pf lagging receives its power at 440 volts. Power is supplied over a parallel feeder system in which one feeder has 0.1 ohm resistance and 0.2 ohm inductive reactance while the second feeder has 0.03 ohm resistance and 0.12 ohm reactance. Calculate the kva and kw carried by each feeder, the impedance drop across each feeder, and the voltage and power at the supply terminals.

58. 120 volts, 5 amp, and 400 watts are read at the input terminals of a series-parallel system. The first branch of the 2-branch parallel combination has 6 ohms R and 2 ohms X_C. The second branch contains 4 ohms R and 3 ohms X_L. What are the constants in the series arm of the network, assuming it has but one reactance?

59. A 440-volt, single-phase factory load is 300 hp. Assuming 80% efficiency for the total load, and a line pf of 70% lagging, what kva of capacitance is required to raise the plant power factor to 95%?

60. An industrial plant has the following loads:

50 kw at 1.0 pf

100 kw at 0.75 pf lagging

200 kva at 0.8 pf lagging

150 hp at 0.8 pf leading, 92% efficiency

What kva is supplied the plant, what is the power taken by the plant, and what kva in static capacitors is required to correct the plant power factor to 100%?

61. A plant load has the following components: 200 kva at 0.707 pf lagging; 120 kw at 0.8 pf lagging; 40 kw at 1.0 pf; and 150 hp, 93% efficiency, 0.8 pf lagging. Calculate the line current if the supply voltage is 440 volts. At $8 per kva of capacitance, what is the cost to correct the plant power factor to 90% lagging?

62. A small industrial plant of 25-hp connected load consumes 3,000 kwhr of energy per month. The service charge on the monthly power bill is $1 per hp of connected load. Energy costs 3.1 cents per kwhr for the first 50 kwhr per hp of connected load, 2 cents per kwhr for the second energy block of 50 kwhr per hp, and 1.1 cents per kwhr for the third energy block of 150 kwhr per hp. Calculate the monthly charge.

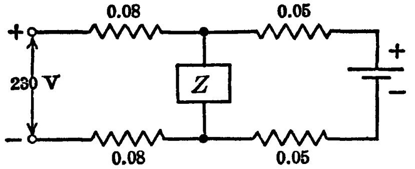
63. A power company adds a penalty of $\frac{1}{2}$% of the energy bill for each 1% decrease in pf below 85%, and subtracts a bonus of $\frac{1}{4}$% of the energy bill for each 1% increase in pf above 85%. Assuming an average rate of energy charge of 1.8 cents per kwhr, what is the yearly saving per million kilowatt-hours for a consumer who corrects his power factor from 70% to 90%?

64. Compute the yearly operating charge per corrective kva for capacitors costing \$15 per kva and having a loss of $\frac{1}{6}$ of 1%. Assume 1.6 cents per kwhr energy charge, 2,500 hours service per year, and 15% fixed charges on the investment.

65. In a laboratory test on a dry cell, a 170-ohm voltmeter was placed across a variable load resistor. When the resistance was set at 100 ohms the voltmeter indicated 1.055 volts, and for 500 ohms the instrument showed 1.265 volts. Write the numerical equation for the terminal voltage of the battery as a function of battery current. Under what condition will a cell or battery deliver maximum power to a load?

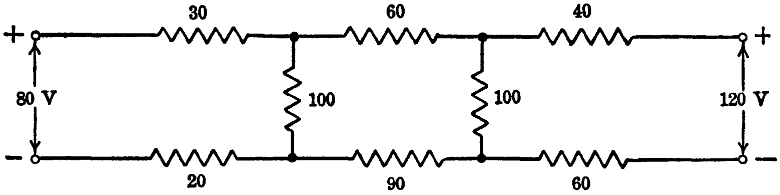
66. A d-c generator supplies 7.5 kw at 115 volts to a load 200 ft distant. A second load, 100 ft farther, takes 5 kw. If No. 3 gage copper conductor is used between the generator and the first load, and No. 8 conductor between the two loads, what is the current and voltage distribution for the system, and what is the kw output of the generator?

67. Calculate the load resistance shown in the following circuit. The load current is 200 amp. The battery has an internal resistance of 0.18 ohm and a no-load voltage of 214 volts.

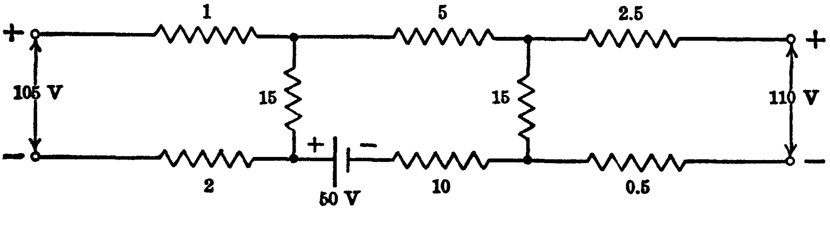


68. A battery having an emf of 240 volts and a total resistance of 0.2 ohm is connected across the lines between a 250-volt d-c generator and its load. The resistance of each line from generator to battery is 0.1 ohm, and from battery to load is 0.2 ohm. Determine the current and voltage distribution for the system when the load current is 100 amp; when the load current is 30 amp.

69. Solve for the current in each arm of the following circuit.



70. Compute the current in each arm of the following circuit.



71. A voltage 156 cos ωt produces a current 10 sin $(\omega t + 30)$ in a series circuit. Show these quantities on a complex plane. Express them and the impedance in complex form. Determine the power taken by the circuit and do so by using 3 different methods.

72. The branch currents of a line are 14.1 sin $(\omega t + 45)$, 7.07 sin $(\omega t - 30)$, and 21.2 cos $(\omega t - 10)$. Express each current in complex form. Determine the line current using the complex notation method.

73. An impedance $8 + j6$ is connected in parallel with an impedance $4 - j8$. Express the impedance of the combination in complex form. If the voltage $100 + j50$ is applied to the circuit, determine the current and power input.

74. A voltage $100 + j0$ drives a current $8 - j6$ through an impedance. Calculate the components of the impedance. Show that a voltage $80 + j60$ will drive the same rms current through the same impedance. Repeat for a voltage $-70.7 - j70.7$. Calculate the voltage for a current of $-5.27 + j8.50$ amp flowing through this impedance.

75. A voltage $90 + j70$ drives a current $5 + j12$ through an impedance. Calculate the components of this impedance for the equivalent series circuit; for the equivalent parallel circuit. What power is taken by the circuit?

76. The first branch of a parallel circuit is an impedance $Z_1 = 6 + j4$ ohms. The second branch is an impedance $Z_2 = 5 + j8$ ohms. If the current to the combination is 15 amp, calculate the branch currents, the circuit voltage, the total power, and the constants of the equivalent series and parallel circuits.

77. Calculate the constants for the equivalent series and parallel circuits to represent a 3-branch parallel system in which $Z_1 = 4 - j3$ ohms, $Z_2 = 5 + j2$ ohms, and $Z_3 = 3 - j6$ ohms. If the total current to the system is 8 amp, calculate the branch currents, the circuit voltage, and the power taken by each branch.

78. A pure resistance, a pure inductance, and a pure capacitance are in parallel. If the magnitude of each is 10 ohms, and the input current is $-j4.23$ amp, calculate the equivalent impedance of the circuit, the branch currents, and the impressed voltage.

79. A 3-branch parallel circuit contains the following impedances: $Z_1 = 8.66 + j5$ ohms, $Z_2 = 5 - j8.66$ ohms, and $Z_3 = 10 + j0$ ohms. If the line current is $-5 - j8.66$ amp, calculate the equivalent series impedance for the system, the branch currents, and the impressed voltage. Show the currents and voltage on the complex plane.

80. The following loads are connected to a 3-wire system: (a) 115 volts, 0.8 kw, unity pf, connected between line A and neutral; (b) 115 volts, 0.8 kw, 0.866 pf lagging, connected between line B and neutral; (c) 230 volts, 2 kw, 0.9 pf lagging. Calculate the 3 line currents.

81. In a 230/115-volt, 3-wire system, a load impedance Z_1 is connected from line A to neutral, and a load impedance Z_2 is connected from line B to neutral. Lines A and B each contain an impedance $0.08 + j0.2$, and the neutral line contains an impedance $0.02 + j0.06$. Calculate all currents and the 2 load voltages when Z_1 and Z_2 are equal to $3 + j1$ and $5 - j3$, respectively.

82. For the circuit of problem 81, calculate the 2 load voltages for d-c line currents in A and B having the following respective values: (a) 50 and 0 amp; (b) 50 and 20 amp; (c) 50 and 50 amp. What are the load resistances?

83. A network composed of pure resistances and voltage sources of 1,000-cycle frequency has 2 output terminals. On open circuit, the terminal voltage is 80 volts and, on short circuit, a current of 8 ma flows. By use of Thévenin's theorem, determine the current when a series combination of 5,000 ohms resistance and 1 henry inductance is connected to the terminals.

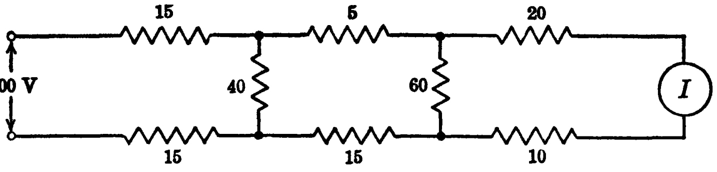
84. A certain resistance-coupled amplifier circuit can be considered as a voltage applied to a 100,000-ohm resistor and a 200,000-ohm load resistor in series, the output voltage being taken across the 200,000-ohm resistor. By the use of Thévenin's theorem, determine the frequency at which the output voltage will be decreased to 70.7% of its former value when a capacitance of 250 $\mu\mu$f is connected across the 200,000-ohm resistor.

85. A voltage, $e = 25 \sin 2\pi 60t + 10 \sin 2\pi 180t$, is applied to a series combination of 2,500 ohms resistance and 1 μf capacitance. Plot the wave forms of voltage and current.

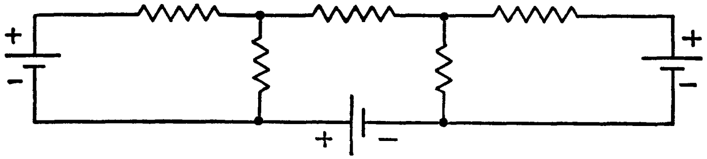
86. In Figure 4.27, $e_1 = 100 \sin \omega t$ and $e_2 = -50 \sin \omega t$. For $Z_2 = \infty$, show that equations 4.40 and 4.41 give a solution that is correct in every detail.

87. Refer to Figure 4.27. Let $Z_1 = 1 + j1$, $Z_2 = 3 - j4$, $Z_3 = 7 + j4$, $E_1 = 80 + j60$, and $E_2 = 100 + j0$. Calculate the current through Z_2.

88. Calculate the ammeter reading in the following circuit. If the voltage source and the ammeter are interchanged, what will be the ammeter reading? Check your calculations.

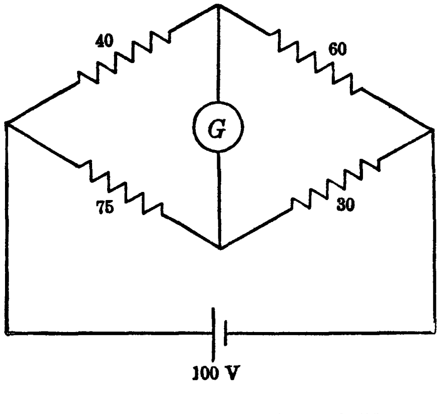


89. In the following circuit, all voltages and resistances are equal to 100 volts and 10 ohms, respectively. By means of the Superposition Theorem, calculate the current in each resistor.

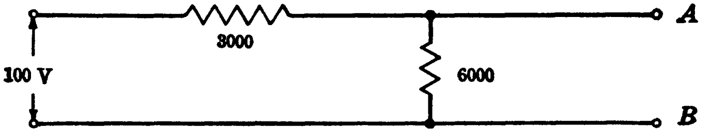


90. The following were obtained from measurements on a 4-terminal network: $Z_{11} = 5 + j10$; $Z_{22} = 15 + j5$; $Z_{12} = 10 + j20$. Calculate the equivalent π and T networks. If a voltage of 100 volts is connected to terminals 1–1, calculate the current through a 50-ohm resistor connected to terminals 2–2. Show that the ratio of voltage to load current remains the same when the voltage source and load ammeter are interchanged.

91. By the use of Thévenin's theorem, calculate the current through the 100-ohm galvanometer of the Wheatstone bridge.



92. By Thévenin's theorem, calculate the value of X_c, connected across A–B, that would support 30 volts across it.



CHAPTER V

THE SINGLE-PHASE TRANSFORMER

5.1. Introduction. The transformer is a stationary electrical machine that transforms alternating-current energy, usually from one voltage to another, the frequency remaining constant. The treatment of the transformer is introduced at this point, ahead of the general treatment on machines, since only single-phase circuit theory is involved and the transformer is finally reduced to an equivalent circuit of that type.

The transformer consists of two electrical circuits or windings insulated from each other and having a common magnetic path. The windings normally are composed of groups of coils having many turns, the total number of turns of one winding being different from that of the other. The transformer is, of course, a coupled circuit. The magnetic path is made of laminated iron to reduce losses, and the windings are so arranged with respect to each other (they may be interleaved) that their mutual effects are very great.

That winding which receives energy is the *primary,* and that from which energy is taken is the *secondary.* With a different number of turns in primary and secondary, the voltage and current at the output terminals will be different from the voltage and current at the input terminals. The winding having the higher voltage is the *high-voltage* or *high-tension* winding, the other the *low-voltage* or *low-tension* winding. If the secondary voltage is *higher* than that of the primary, the device is called a *step-up* transformer; if the secondary voltage is *lower* than that of the primary, it is called a *step-down* transformer. Obviously, energy may be transmitted either way through the transformer.

Standard voltages range through a long series from 110 volts to 250,000 volts, or more. A common voltage at generating stations is 13,200 volts. This may be raised at the station to normal transmission line voltages of magnitudes from 66,000 to 250,000 volts, with correspondingly decreased currents, and the energy transmitted over long distances. At the load end of the lines the voltage is stepped down and distributed at various low values for industrial use and for home lighting and appliances.

The transformer, which has existed since about 1880, has been a primary cause of the widespread use of alternating current. Inability to build high-voltage generators and motors gives to the transformer immediate recognition as a highly flexible link between the generation and distribution of electrical energy. Its cost is lower than that of rotating machines of the same kva rating, and its efficiencies are much higher. Single-phase transformers have been built with a rating of 133,000 kva, and 3-phase units with a rating up to 400,000 kva.

Absence of rotating parts decreases losses, and the efficiency of a large transformer may be as high as 99.7 per cent, very close to complete energy conversion. Further, the device allows transformation from one number of phases to another, and it also affords a tie between systems of the same frequency.

5.2. Fundamental Relations. The essential elements of the transformer are shown in Figure 5.1. For the time being, the resistances and leakage fluxes of the windings, saturation, and all losses will be neglected. The leakage flux of a winding is that created by current in the winding and which links with that coil alone. The effect of this leakage flux is to cause the output voltage of the transformer to vary slightly with load. Neglecting these small quantities, we have the "ideal transformer." It should be pointed out at the start that the ordinary commercial transformer comes fairly close to meeting the specifications of the ideal transformer. Under full load, the losses are of the order of 2 per cent and may be less than $\frac{1}{2}$ of one per cent for the larger sizes. The change in the value of the secondary terminal voltage from rated load to no load (regulation) is of the order of only 2 or 3 per cent for a commercial transformer.

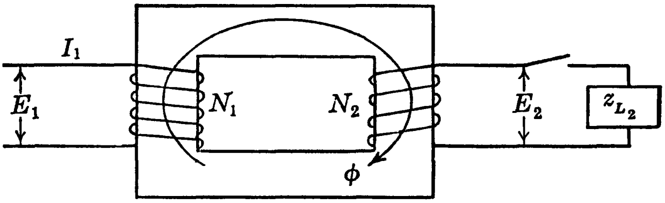


FIG. 5.1. Essential elements of the transformer.

With the secondary circuit open, the ideal transformer reduces to a pure inductive reactance connected to the power source. A no-load current I_0 flows, which lags the applied voltage by 90 degrees. This no-load current, or transformer *exciting current* as it is called, is small since the magnetic circuit, with its iron path of very low reluctance, gives rise to a high inductive reactance for the winding. The flux Φ created by the exciting current I_0 varies sinusoidally in time and induces voltages in both primary and secondary coils of turns N_1 and N_2.

The induced voltage in each turn of any winding on the transformer is fixed by the flux and the frequency, and the voltage of any coil is then directly dependent upon the number of turns in that winding; that is,

$$\begin{aligned} E_1 &= 4.44 N_1 f \Phi_m \text{ volts} \\ E_2 &= 4.44 N_2 f \Phi_m \text{ volts} \end{aligned} \tag{5.1}$$

from which

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{5.2}$$

Since the voltage induced in the primary coil must always equal the applied voltage, the relation given by the first of equations 5.1 must hold for all conditions of load. This means that as the transformer is loaded under constant impressed voltage the maximum value of the flux remains constant and is equal to the value at no load. It follows, therefore, that the net magnetomotive force (mmf) acting on the magnetic circuit does not change with the loading of the transformer.

The ratio N_1/N_2 is the *turn ratio* of the transformer and, while immaterial, it usually is given by the ratio of the larger number of turns to the smaller number. Although the actual number of turns of the two windings may not be known, the turn ratio can be found from the ratio of the terminal voltages measured at no load.

If the secondary circuit is closed, the voltage E_2 causes a load current I_2 to flow, the phase angle between E_2 and I_2 depending solely upon the impedance characteristics of the load. This secondary current results in a magnetomotive force, proportional to N_2I_2, that acts on the magnetic circuit and tends to change the magnitude of the flux. However, if the magnitude of the flux changes, the voltage induced in the primary circuit will no longer equal the applied voltage —an impossible condition. As the flux tends to change, the primary current increases with the secondary and maintains the initial value of the flux. The ampere-turns due to this load component I_1' of the primary current just balance the ampere-turns due to the secondary load current, that is,

$$N_1I_1' = N_2I_2 \tag{5.3}$$

whence

$$\frac{I_1'}{I_2} = \frac{N_2}{N_1}$$

The primary current I_1 is the vector sum of I_0 and I_1'. The exciting current is usually small in comparison with the load component of current I_1' and adds to it in general at a very large angle. For this reason, I_1 and I_1' are usually considered equal in value. With this approximation, equations 5.2 and 5.3 yield the relation

$$E_1I_1 = E_2I_2 \tag{5.4}$$

and

$$\frac{E_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{E_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_{L2} \tag{5.5}$$

where Z_{L2} is the impedance of the load on the secondary.

From equation 5.4 it is seen that the kva input to the transformer is equal to the kva output. Also, neglecting losses, the input and output power for the transformer are equal. Equation 5.5 states that the power factor measured at the primary of the transformer is equal to the power factor of the secondary load and that the magnitude of the impedance, as viewed from the primary terminals, has been changed by the square of the turn ratio. This last characteristic is often made use of in communication circuits for matching impedances.

The relations of equations 5.1 to 5.5, inclusive, are fundamental to the transformer. Although developed for the ideal transformer, they are very nearly correct for any modern iron-core transformer.

5.3. Nameplate Ratings of Transformers. Important information given on the nameplate of a transformer includes the normal kva which the machine is to transfer through it, the normal operating voltages of the input and output windings, the normal frequency of the voltage variation, and perhaps statements containing data on the very low resistance and impedance within the transformer itself. Since the transformer is built to pass rated kva through it, the primary and secondary windings both have the same kva rating, which is that of the entire machine. Normal currents for the two sets of coils therefore are implied through this kva rating and the respective voltages. Power factor is not specified because it depends solely upon load conditions over which the transformer has no control. The transformer may be called upon to transfer power with phase angles between voltage and current ranging from practically 90 degrees lagging to 90 degrees leading. For this reason, they are not rated in kw, and the load on a transformer usually is given in terms of the kva it is carrying.

Many transformers have more than one high-voltage and one low-voltage coil. In such instances the voltage rating of the transformer implies the number of separate windings for the input and output circuits and the rated voltage of each winding. For example, a transformer rating of 25 kva, 2,400:1,200::240:120 volts means that the device has two high-voltage windings of 1,200 volts each. They are to work on 2,400 volts when connected in series, and on 1,200 volts when in parallel. Information on the low-voltage windings is interpreted similarly. Connected in series, the low-voltage coils could supply a 3-wire system. Note that the normal kva of each coil is 12.5, nameplate kva being divided equally among the several coils on either side of any transformer, provided that they have the same voltage rating.

5.4. Losses in a Commercial Transformer. Because of the periodic variations of flux within the iron of the transformer, there will be a loss in the magnetic circuit. This loss, called the iron or core loss, has two components, one of eddy currents and the other of hysteresis. So long as the impressed voltage and frequency are held constant, the core loss is assumed to remain constant over the entire load range of the transformer. Since the windings of the transformer have a certain amount of resistance, there will be an I^2R, or copper, loss in each set of coils. Any additional eddy-current loss which appears because of load currents is assigned to the copper loss, whether it exists in the conductors or is a small increase in the eddy-current loss in the iron. As previously mentioned, the sum of all the losses of the transformer under rated loading is usually less than 3 per cent of the rating of the machine expressed in watts.

5.5. Iron Losses. It will be remembered from earlier studies in physics that when iron is subjected to periodic reversals of flux there is an associated loss of energy called the *hysteresis loss* which appears as heat in the iron. For a given machine, the hysteresis loss varies with the frequency of the flux reversals and to a power of the maximum flux density. It is usually expressed by

$$P_h = K_h f \beta_m{}^{1.6} \text{ watts} \tag{5.6}$$

The constant K_h depends upon the kind of iron and the units of measurement, and the exponent of the flux density is merely a representative value frequently used.

Also, when iron undergoes flux reversals within it, there are losses caused by currents circulating in the iron. The iron acts as a great number of transformer secondaries, each having a resistance load. Voltages are induced in the iron which lead to heat losses by virtue of the I^2R of these many eddy currents. The *eddy-current losses* for a given machine vary with the square of both frequency and flux density, that is,

$$P_e = K_e f^2 \beta_m{}^2 \text{ watts} \tag{5.7}$$

In practice, the combined hysteresis and eddy-current losses of a machine are calculated from the watts loss per pound of material obtained by test for known frequencies and flux densities. When it is desired to separate the component losses for a stated value of maximum flux density, measurements are taken at two frequencies in the vicinity of the operating frequency. The different manner of variation of the individual losses with frequency makes possible the evaluation of each component.

The hysteresis and eddy-current losses, combined, give the total losses appearing as heat in the iron, hence the term "iron losses." Iron losses exist in all rotating machines and transformers, in fact in all magnetic circuits containing iron carrying a periodic flux. It will be recalled from physics, though, that eddy current losses are not confined to iron alone. Copper conductors, for example, carrying alternating currents have eddy-current losses within them, thus giving rise to an apparently greater resistance, called the "effective resistance," than that measured by direct current.

The core loss is practically a constant for all loads for a given applied voltage, E_1. However, since the flux density is proportional to the applied voltage, the core loss increases nearly as the square of the applied voltage. Thus, if a 110-volt transformer is used on 125 volts, the core loss will be increased by approximately 29 per cent.

The core loss requires that the no-load current I_0 have a component in phase with the applied voltage E_1. This is a refinement for the actual transformer over the ideal machine. The power factor of I_0 is small, a representative value being about 25 per cent, corresponding to a phase angle of 75 degrees.

With a sinusoidal voltage applied to the primary, a sinusoidal flux variation will follow in order to produce the required induced voltage. Because of saturation, however, the information on which is contained in the non-linear *B-H* curve for the transformer, the exciting current will not be sinusoidal. It will consist of a fundamental with a series of harmonics of decreasing magnitudes, the most pronounced of which is the third harmonic. The no-load current will show this distortion when viewed on an oscillograph. At rated voltage, the third-harmonic current and the third-harmonic voltage associated with it usually are appreciable, and these rise very rapidly with overvoltage. Their effects are of most concern in transformer connections for 3-phase systems.

The no-load, or open-circuit, test on a transformer is usually carried out over a voltage range from zero to perhaps 150 per cent of rated voltage. For various applied voltages the exciting current and the core loss are obtained directly through ammeter and wattmeter readings. Since the exciting current is small, the copper loss due to it may be neglected. At normal impressed voltage, the ratio of the exciting current to rated current for that side of the transformer on which the test is made is the percentage exciting current. Similarly, the ratio of the core loss at rated voltage to the transformer rating is the percentage core loss. Because of convenience in the selection of measuring equipment, the open-circuit test usually is made on the low-voltage side of the transformer. The iron loss and percentage I_0 are the same in either case. Note that the actual turn ratio of the transformer can be found from the ratios of voltages in the no-load test since impedance drops within the windings are negligible. The turn ratio may or may not be equal to the nameplate voltage ratio since the latter is given for full kva, in which case small impedance drops within the transformer alter the no-load voltage ratio slightly.

5.6. Copper Loss. Both windings of the transformer have resistance, but because these circuit elements do not enter into transformer action they are taken as lumped constants and, in the circuit diagram, Figure 5.2, are placed outside the windings. The resistances are relatively small, hence their I^2R losses are very low, thus leading to high transformer efficiency. The resistances can be calculated or measured approximately by a d-c test, although the a-c, or effective, resistances are higher by several per cent because of eddy-current losses in the copper. In general, the I^2R losses in the two windings are approximately equal.

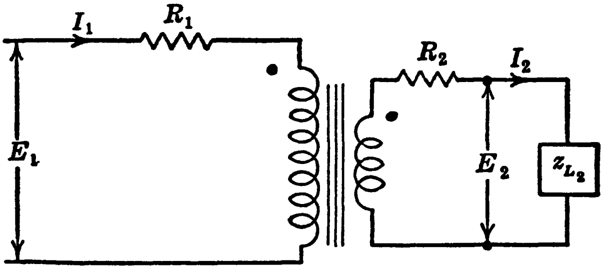


FIG. 5.2. Transformer circuit with resistances of the windings included.

The copper loss in the transformer is expressed by the equation

$$\text{copper loss} = I_1^2R_1 + I_2^2R_2 \text{ watts}$$

Substituting for I_2 from equation 5.3, this equation can be written in the form

$$\text{copper loss} = I_1^2\left[R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2\right] = I_1^2R_{1e}$$

or

$$\text{copper loss} = I_2^2\left[R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1\right] = I_2^2R_{2e} \tag{5.8}$$

where R_{1e} is the equivalent resistance of the transformer referred to the primary, and R_{2e} is the equivalent resistance referred to the secondary. From equations 5.8 it is evident that R_{1e} and R_{2e} are related by the square of the turn ratio, that is,

$$R_{1e} = \left(\frac{N_1}{N_2}\right)^2 R_{2e} \tag{5.9}$$

It is thus seen that the total copper loss in a transformer is equal to the equivalent resistance of the transformer multiplied by the square of the current in the side to which the resistance is referred.

If measurements are taken at the primary terminals of the transformer with the secondary short-circuited, the power input read on a wattmeter will be

$$\text{power input} = I_1^2R_1 + I_2^2R_2 = I_1^2R_{1e} \text{ watts} \tag{5.10}$$

Of special interest are the voltage and power for rated primary and secondary currents. The required voltage will be only about 5 per cent of rated voltage of the winding. With a correspondingly low value of flux, the core loss can be neglected for this test.

Under short circuit, the entire applied voltage is consumed in driving the current through the equivalent impedance of the transformer (the equivalent impedance is discussed in Section 5.8). Thus, the ratio of primary voltage to current is

$$\frac{E_1}{I_1} = Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \text{ ohms} \tag{5.11}$$

from which

$$X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2} \text{ ohms} \tag{5.12}$$

The constants can be converted to those for the secondary from the relations

$$R_{2e} = \left(\frac{N_2}{N_1}\right)^2 R_{1e} \quad \text{and} \quad X_{2e} = \left(\frac{N_2}{N_1}\right)^2 X_{1e} \tag{5.13}$$

Although the short-circuit test can be carried out on either side of the transformer, usually it is conducted on the high-voltage side with the low-voltage coils short-circuited. This connection permits use of a lower-range ammeter and a better reading of the impedance voltage. The current in both sides is known if an ammeter is placed in either side. The wattmeter and ammeter readings allow the calculation of R_{1e} from equation 5.10. The impedance Z_{1e} is determined from the ammeter and voltmeter readings when substituted in equation 5.11. The test taken on the low-voltage side will give corresponding values of R_{2e} and Z_{2e}. The values from either test can always be referred to the other winding by use of equations 5.13.

The ratio of the wattmeter reading of the short-circuit test, with rated current in the windings, to the kva rating of the transformer gives the normal percentage copper loss. This same percentage is also the per cent resistance, $\%R$, of the transformer. Similarly, the ratio of the voltmeter reading of the short-circuit test, with rated current, to the rated voltage of the winding is the per cent impedance, $\%Z$, of the transformer. These percentage values, which are based upon rated quantities, are discussed in Section 5.9.

5.7. Efficiency. The efficiency of the transformer, as with all machines, is the ratio of the power output to the power input. The efficiency of a transformer is, in general, very high and it is found more accurately by calculations involving the losses. Thus,

$$\text{per cent efficiency} = \frac{E_2 I_2 \cos\theta_2}{E_2 I_2 \cos\theta_2 + \text{iron losses} + I_2^2 R_{2e}} \times 100 \qquad (5.14)$$

The iron losses are substantially constant regardless of the current loading. It is a general observation in engineering that maximum efficiency for a machine is obtained when the variable losses equal the constant losses. The student is asked to demonstrate this principle for the transformer from the above efficiency equation. For each power factor there will be a maximum efficiency, the highest attainable being that at unity power factor.

5.8. Voltage Regulation. The windings of the actual transformer have *leakage reactance* as well as resistance. Consequently, as a result of voltage drops in these impedances, the output voltage will change with the magnitude of the load current and with the power factor. Expressed as a percentage of the rated full-load voltage, the change in secondary voltage from no load to full load is defined as the voltage regulation of a transformer. The regulation may be positive or negative, depending upon the load power factor. It is positive when the secondary voltage falls with increasing load.

Leakage reactances exist in a transformer since, in practice, it is impossible to obtain complete inductive coupling of two circuits. This, stated in another way, means that the entire flux is not common to the two windings of the transformer, and that the currents in each winding produce fluxes that link only with their respective turns and in no way become a part of the mutual flux. These leakage fluxes yield simple inductances in each circuit which can be represented by equivalent reactances in the primary and secondary, as is shown in Figure 5.3. Because they do not enter into transformer action, they are given lumped values and moved outside the windings as were the resistances. The coupling coefficient of an actual transformer is of the order of 99.9 per cent. It is evident, therefore, that the leakage reactances X_1 and X_2 are but very small fractions of one per cent of the total (open-circuit) reactance of the windings.

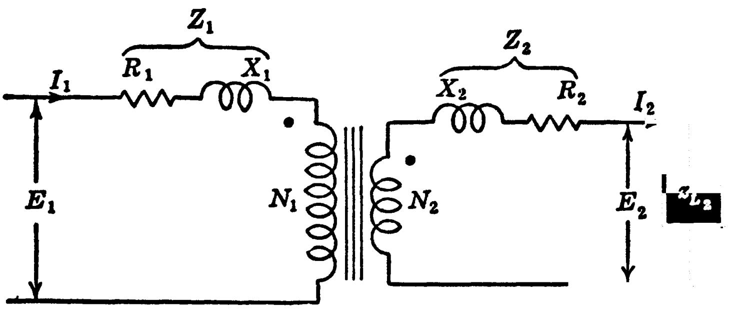


Fig. 5.3. Actual circuit of the transformer.

The relation between the primary and secondary voltages can be obtained from a direct analysis of the circuit shown in Figure 5.3, and is given by the vector equation

$$\begin{aligned} E_1 &= \frac{N_1}{N_2}(E_2 + I_2Z_2) + I_1Z_1 \\ &= \frac{N_1}{N_2}E_2 + I_2\frac{N_1}{N_2}Z_2 + I_1Z_1 \qquad (5.15) \end{aligned}$$

where Z_1 and Z_2 are the impedances of the primary and secondary windings, respectively. The quantities in equation 5.15 are, of course, vector quantities and must be added as vectors.

Substituting for I_2 from equation 5.3, equation 5.15 can be written in the form

$$\begin{aligned} E_1 &= \frac{N_1}{N_2}E_2 + I_1\left[Z_1 + \left(\frac{N_1}{N_2}\right)^2 Z_2\right] \\ &= \frac{N_1}{N_2}E_2 + I_1Z_{1e} \qquad (5.16) \end{aligned}$$

where

$$Z_{1e} = \left[R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2\right] + j\left[X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2\right] \qquad (5.17)$$

and is called the equivalent impedance of the transformer referred to the primary.

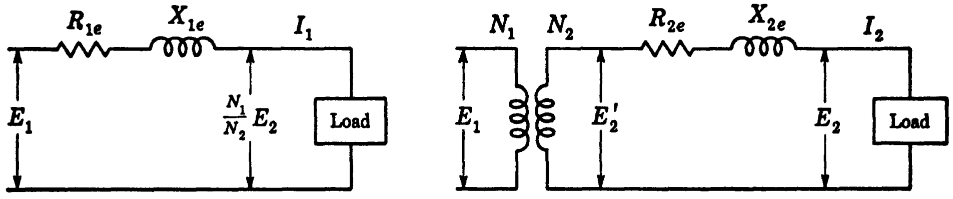
Equation 5.15 can also be written in the form

$$\frac{N_2}{N_1}E_1 = E_2 + I_2\left[Z_2 + \left(\frac{N_2}{N_1}\right)^2 Z_1\right] = E_2 + I_2Z_{2e} \qquad (5.18)$$

where Z_{2e} is the equivalent impedance referred to the secondary, and is given by the expression

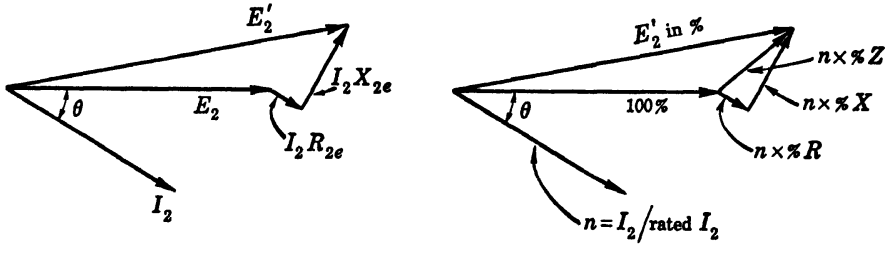
$$Z_{2e} = \left[R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1\right] + j\left[X_2 + \left(\frac{N_2}{N_1}\right)^2 X_1\right] \tag{5.19}$$

Interpretation of the foregoing expressions suggests the analysis of the transformer by way of the equivalent circuits of Figure 5.4. The transformer thus is reduced to a simple series circuit in which the major problem is that of determining the voltage at the supply terminals for specified load conditions, or the load voltage and current for a given load impedance. Such simplified circuits represent the entire transformer rather faithfully even though the no-load current I_0 is neglected.



(a) Viewed from primary **(b) Viewed from secondary**

FIG. 5.4. Simplified equivalent circuits of the transformer.



(a) In terms of secondary voltages **(b) In terms of percentage quantities**

FIG. 5.5. *Vector diagrams of Figure 5.4b for a lagging power factor load.*

It might be remarked at this point that the reduction of complex circuits and of machines to simple series-circuit equivalents is fundamental to the philosophy of the electrical engineer in the analysis of his systems. We already have observed similar steps in the treatments on single-phase circuits. Other divisions of engineering follow the same procedure whenever possible.

For a given load, expressed either in ohms impedance or by kw and kva, at a fixed power factor and voltage, the change in secondary voltage between full load and no load as a percentage of full-load voltage is the voltage regulation of the transformer; that is, from Figure 5.4b,

$$\frac{|E_2'| - |E_2|}{|E_2|} \times 100 = \text{per cent regulation} \tag{5.20}$$

where, as is indicated, magnitudes only are to be used.

As previously stated, the regulation may be positive or negative, depending upon the load power factor and the magnitude of the current. The vector diagrams of Figure 5.5 indicate a direct method of determining the value of E_2'.

5.9. Percentage Quantities. Calculations on regulation are carried out easily by using percentage quantities. Rated primary and secondary voltages and currents are 100 per cent (1.00 on unit basis), and the equivalent circuit constants are defined by ratios of their respective voltage drops at rated current to rated voltage. For example,

$$\%Z = \frac{Z_{2e}}{\text{rated } (E_2/I_2)} \times 100 = \frac{\text{rated } I_2 \cdot Z_{2e}}{\text{rated } E_2} \times 100 \tag{5.21}$$

Per cent R and X are defined similarly, and

$$\%Z = \sqrt{(\%R)^2 + (\%X)^2} \tag{5.22}$$

The per cent R and Z for a transformer were discussed in Section 5.6 with respect to the manner in which they are obtained from test results. The student can satisfy himself that any constant, expressed as a percentage, is the same whether calculated from primary or secondary quantities.

The impedance of a transformer usually is but a few per cent. A 4 per cent impedance means that the equivalent IZ drop of the transformer at rated current is 4 per cent of either E_1 or E_2, depending upon that side chosen as reference. In calculating regulation secondary quantities normally are used, and magnitudes in the vector diagram then are given in terms of percentages (or units) of rated secondary voltage. As is brought out in Figure 5.5, the use of percentage quantities in the vector diagram can be interpreted as merely changing the scale of all voltage vectors so that the reference voltage, rated E_2, becomes 100 units. Obviously, if I_2 differs from rated value by the factor n, the percentage IR and IX drops are changed by this same factor. Using percentage quantities, the regulation is simply $E_2' - 100$. It is evident that E_2' also gives the required primary voltage directly in per cent.

Transformer nameplates often state the percentage impedance or perhaps both the percentage impedance and resistance. The per cent impedance is helpful in giving information quickly on the approximate regulation for a given load condition, and on the magnitude of short-circuit current in case of trouble. The per cent resistance assists in defining the impedance angle of the transformer, in evaluating copper losses for a given kva load, and in calculating transformer regulation.

The use of percentages is of advantage in comparing similar quantities for machines of different ratings in that it places them all on the same basis. Furthermore, calculations based on percentages are usually the simplest that can be made and results are obtained with an ease and directness not possible with most other methods. Practical electrical engineering makes much use of percentage methods.

5.10. Example. A 15-kva, 2,200/220-volt, 60-cycle transformer shows the following test data:

Open-circuit test:

$$E_2 = 220 \text{ volts}, \quad I_2 = 1.92 \text{ amp}, \quad W_2 = 116 \text{ watts}$$

Short-circuit test:

$$E_1 = 106 \text{ volts}, \quad I_1 = 6.82 \text{ amp}, \quad W_1 = 186 \text{ watts}$$

Determine the efficiency and regulation of the transformer for 90 per cent rated kva at 80 per cent pf lagging.

Solution:

The output of the transformer for this load is 10.8 kw. The iron loss is 116 watts (open-circuit test), and the copper loss is the rated value of 186 watts from the short-circuit test reduced in the ratio (actual kva/rated kva)^2, or 151 watts. Hence,

$$\text{efficiency} = \frac{10{,}800}{10{,}800 + 116 + 151} \times 100 = 97.7\%$$

Maximum efficiency occurs for a load of 11.85 kw (when the copper and iron losses are equal) and has a value of 98.2 per cent.

From the short-circuit test data,

$$\%Z = \frac{106}{2{,}200} \times 100 = 4.82$$

$$\%R = \frac{186}{15{,}000} \times 100 = 1.24$$

$$\%X = 4.66$$

At 13.5 kva, then, the percentage IR and IX drops are 1.12 per cent and 4.19 per cent, and we have the vector diagram shown in Figure 5.6. Note that, in

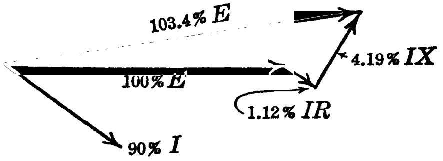


FIG. 5.6. Transformer vector diagram in percentage.

percentages, the diagram does not have specific reference to either side of the transformer. The regulation is 3.4 per cent.

Solving for the regulation by using equivalent constants of the transformer, we have, from the short-circuit test,

$$Z_{1e} = \frac{106}{6.82} = 15.57 \text{ ohms, whence } Z_{2e} = 0.156 \text{ ohm}$$

$$R_{1e} = \frac{186}{(6.82)^2} = 4.00 \text{ ohms, whence } R_{2e} = 0.04 \text{ ohm}$$

and

$$X_{2e} = 0.150 \text{ ohm}$$

The vector diagram in terms of secondary voltages is shown by Figure 5.7, whence the regulation is

$$\text{voltage regulation} = \frac{227.5 - 220}{220} \times 100 = 3.4\%$$

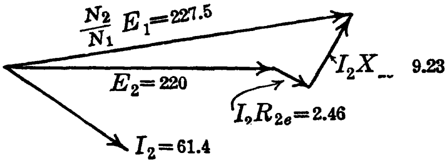


FIG. 5.7. Transformer vector diagram in terms of secondary voltages.

5.11. Polarity. The induced voltages in all windings of a single-phase transformer are due to the same time-varying flux. Depending upon which terminal of any pair is taken as a reference, therefore, the induced voltages between terminals of any pair of coils are either in phase or 180 degrees out of phase. It is necessary to know the relative polarities when two or more windings of a transformer are to be connected in parallel or in series, and when separate transformers are to be paralleled. If the windings are improperly connected, they may essentially short-circuit the power supply.

By convention, high-voltage terminals are given the notation H, and low-voltage terminals are designated by X. Numerical subscripts indicate particular leads of the several coils; that is, H_1 and H_2, H_3 and H_4, etc., designate, respectively, the two terminals of each of several high-voltage coils. Similarly, X_1 and X_2, etc., apply to terminals of low-voltage coils. No transformer will have two terminals marked alike. The order of the subscripts for all coils on the same transformer denotes the same relative direction of induced voltage at any instant. Figure 5.8 shows polarity markings for a transformer having two high-voltage and two low-voltage windings.

Polarity markings may be placed on the terminals either by the manufacturer or by the customer. Further, as discussed in Section 5.16, there may be variations in markings for certain applications. Specifically, polarity markings on the usual power transformer mean that if voltage is applied to the terminals of a winding, such as to H_1 and H_2, the voltages $E_{H_1H_2}$, $E_{H_3H_4}$, $E_{X_1X_2}$, etc., in this exact order of subscripts, are in phase with each other. Again, these markings mean that if H_2 and X_2 are connected together, and a voltage is applied either to the high-voltage winding or to the low-voltage winding, a voltmeter connected between H_1 and X_1 will read the difference between the high- and low-side voltages.

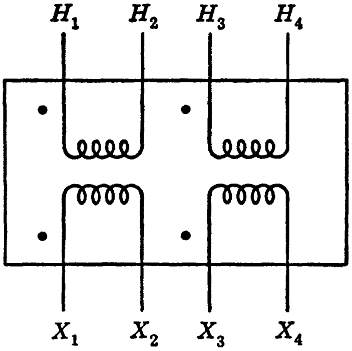


FIG. 5.8. Transformer polarity markings.

In certain instances the leads of a transformer may not be marked. It then may be necessary in sequence: (a) to ascertain which two leads are connected to a given winding (continuity test), (b) to ascertain which are the high-voltage and which are the low-voltage windings (voltage ratio test), and (c) to assign polarity marks to all leads. This is done by marking a terminal of any high- voltage coil H_1 arbitrarily, or assigning X_1 arbitrarily to any low-voltage terminal. From this point on, in conformance with the statements in the preceding paragraph, all other terminals can be marked correctly.

5.12. Parallel Operation of Transformers. For best performance, transformers to be paralleled should have:

1. The same turn ratios. In general, the transformers would be expected to have approximately the same high-voltage and the same low-voltage ratings.
2. The same percentage impedances. The equivalent impedances in ohms should therefore be inversely proportional to the respective current, or kva, ratings.
3. The same ratio of percentage, or equivalent, R to X.

The first requirement results in zero circulating current among the secondaries at no load. The second permits of the same percentage load increase on each transformer for a given increase in total output. The third condition results in the same power factor for each transformer and is that of the load. All transformer terminals having the same instantaneous polarity markings should be connected to the same line. It is understood that only transformers of the same frequency rating are considered.

The problem of calculating the load division among transformers in parallel is exactly that of determining the current division for any single-phase parallel circuit. With transformers, of course, the impedance of each branch will be given by the equivalent resistance and reactance of the transformer which it represents. Figure 5.9 shows the schematic circuit for two transformers, A and B.

The equivalent constants of Figure 5.9 may be replaced by their corresponding percentages and the parallel circuit treated in the same way. If this method is used, it is necessary first to place the percentage quantities of all transformers on the same kva base which often is the lowest common multiple of the kva ratings. The new percentage *IR*, *IX*, and *IZ* drops for each transformer then are the rated percentages times this multiple and are taken with respect to the same current. These percentages now are in the same ratios as their respective ohmic values.

As an example, $Z_{2e} = 0.242$ ohm for a 10-kva, 2,200/220-volt transformer having 5 per cent impedance, and $Z_{2e} = 0.194$ ohm for a 15-kva, 2,200/220-volt transformer having 6 per cent impedance. On a common base of 30 kva, the percentage impedances are 15 and 12, respectively, and are in the same ratio as the ohmic values of the equivalent impedances. Many problems in electrical engineering are handled by percentages in this manner.

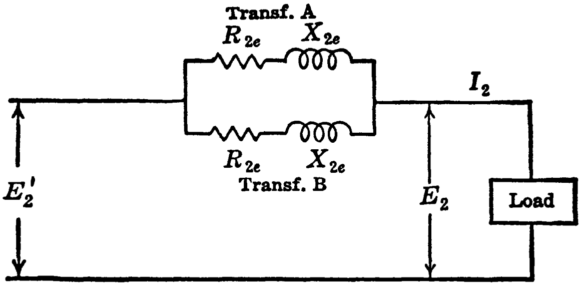


FIG. 5.9. Equivalent circuit for parallel operation.

5.13. Autotransformers. The autotransformer has a single winding part of which is common to both input and output circuits. If the supply voltage is to be stepped down, the supply voltage would be applied to the entire winding and the output voltage obtained between one input terminal and a tap some place along the winding. The ratio of voltage transformation is the ratio of these two voltages. Only part of the power is transferred by transformer action, the remainder flowing conductively from the input to the output circuit. The lower the ratio of transformation, the less will be the power transferred electromagnetically. In practice, the ratio of transformation seldom is greater than 2:1.

The autotransformer may step down or step up the voltage, in the latter case the supply being impressed on but part of the winding. In the same sense, an ordinary transformer can be used as an autotransformer by connecting the primary and secondary coils in series. Advantages of autotransformers are smaller size, lower cost, higher efficiency, and lower voltage regulation. A major disadvantage lies in the fact that the input and output circuits cannot be insulated from each other as with the normal transformer.

5.14. Induction Regulators. In order to maintain the load voltage constant over a wide range of current and power factor, it is necessary that the primary voltage be adjusted for each load condition. Practically, the load voltage usually is controlled by induction regulators in the feeder circuits or by tap-changing transformers which cut out or cut in a few turns of the primary winding so as to increase or decrease the voltage ratio.

Briefly, the induction regulator consists of a primary winding mounted on a cylindrical core and free to rotate about 180 degrees within the secondary winding which surrounds it and is stationary. The rotor may be moved either by hand or under automatic control. The induction regulator is essentially an autotransformer that has, within limits, a smooth variation in turn ratio. The circuit diagram for it is shown in Figure 5.10. The essential difference between the regulator and the transformer is that the former permits a varying rms voltage in the secondary because of the change in magnetic coupling between the two windings as the rotor is moved. In one extreme position the induced secondary voltage adds to the primary line voltage, and in the opposite position the secondary voltage subtracts from that of the primary. The line voltage therefore can be controlled within these limits, usually 10 per cent boost or buck.

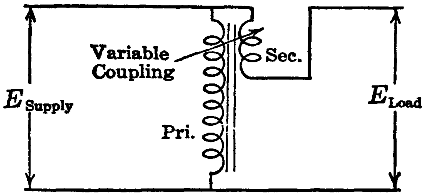


FIG. 5.10. Circuit diagram for induction regulator.

5.15. Transformers in Communication Circuits. In communication work, the transformer is generally considered as an impedance-matching network rather than as a device for changing the circuit voltage. Electronic amplifiers, for example, give optimum performance when operating into a definite value of load impedance. In practice, however, it is generally not feasible to choose arbitrarily the value of impedance for a particular type of load. A typical problem of this sort is that in which a loud-speaker, having an input impedance of 20 ohms, is to be driven from a vacuum tube requiring an optimum load impedance of 8,000 ohms. If the tube were connected directly to the load, a simple calculation shows a very inefficient power transfer from tube to speaker. In order to correct this, a step-down transformer can be inserted between the tube and speaker. Equation 5.5 shows that the impedance, as viewed from the primary winding, is changed by the square of the ratio of primary to secondary turns, that is,

$$Z_1 = \frac{E_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{5.23}$$

For the problem under consideration, a step-down transformer, having a turn ratio of 20, would make the speaker load of 20 ohms appear to the tube as the desired 8,000 ohms. For a given input signal level to the amplifier, the power delivered to the speaker would be increased 100 times over that of the direct connection. A few other problems involving impedance matching are connecting the low impedance of a communication line, microphone, or phonograph pick-up to the high-input impedance of an amplifier, and matching impedances between stages of an amplifier.

In communication work, the transformer generally has to work efficiently over a wide range of frequencies. For audio-frequency applications, the required range is approximately from 30 to 15,000 cycles. The frequency response of a transformer is measured under operating circuit conditions and is expressed by a graph of output voltage against frequency for a constant magnitude of signal voltage. The frequency response can be also expressed as the frequency range over which the output voltage does not vary by more than ±12 per cent (±1 decibel). The response at the high audio frequencies falls off because of the shunting effects of the distributed capacity of the windings. The response decreases at low frequencies because the inductive reactance of the primary winding becomes low compared with the impedance of the primary source of voltage. The required frequency range imposes design features quite different from those of power transformers. Frequency response and freedom from signal distortion, due to the iron characteristics, are of more importance than very high efficiency. The efficiency of audio-frequency transformers generally does not exceed 95 per cent and in some cases may be less than 90 per cent.

Communication transformers are commercially available for practically all applications. The manufacturer usually gives complete information on each transformer, stating the purpose for which it was intended, the power-handling capacity, the impedances between which the transformer was designed to operate, and the frequency response.

5.16. Instrument Transformers. Instrument potential transformers are built similar to small power transformers and are designed to step down the voltage of high-voltage lines to magnitudes which are suitable for the usual voltmeter, wattmeter potential circuit, power-factor meter, etc., or for relays for voltage regulation or protective devices. At the same time, the transformer insulates the measuring circuit from the high-voltage system, thereby providing protection to life and equipment. The primary windings are built for standard voltages ranging at least as high as 110 kv. The secondary coils are designed for a nominal voltage of 110 to 120 volts for use with 150-volt instruments. The nameplate on the transformer gives the primary and secondary voltages and usually the ratio of transformation as well, for example, 23,000:115 volts, ratio 200:1.

A constant ratio of transformation and phase angles as small as possible, are desired, hence these transformers are designed with a very low impedance, less than one per cent. The loads primarily are resistance since they are the potential circuits of instruments, this fact leading to a more constant ratio of transformation than with lagging-power-factor loads. Flux densities are kept low so as to prevent harmonics influencing voltage readings. The volt-ampere ratings are low, from a few watts required to operate an instrument to a few hundred watts which may be required for relay service.

Current transformers step down high main-line currents to small magnitudes which are carried to ammeters, wattmeter current coils, or current relays. The primary winding of the current transformer may be a single copper bar built into the device and carried through the iron core. This bar is inserted in series with the line, carries the line current, and is equivalent to a 1-turn coil. Again, the iron core may be in the form of a hollow square or ring so that it can be slipped over the main line or over the bushing of a power transformer or circuit breaker. The line becomes a 1-turn primary winding. For line currents up to 1,000 amp the primary winding usually has more than one turn. A few low-range portable instruments are designed with the core made in two parts and hinged so that they can be clamped around the line. One such instrument contains an iron-vane movement placed in a field of high-flux density in the core circuit and will read on either a-c or d-c lines. As with potential transformers, current transformers also are designed for constant ratios, small phase angles, and low volt-ampere ratings. Further, they are insulated for the voltage of the system to which they are connected, this information appearing on the nameplate.

The usual current transformer has a 5-amp secondary winding wrapped around the iron core, the number of turns depending upon the ratio of transformation desired. The nameplate gives the primary and secondary currents as well as the ratio, or ratios, of transformation, for example, 800:5 amperes, ratio 160:1 for a single-range transformer, or 25/50/100:5 amperes, ratios 5/10/20:1 for a multi-range portable transformer. Primary currents of many thousand amperes may be measured. Although the secondaries are designed for 5-amp instruments in permanent installations, such as on switchboards, the instrument scales usually are graduated to read directly the main line current, thus including the ratio of transformation. This is true of switchboard-type voltmeters and wattmeters as well, the scales of the latter including the ratios of both the current and the voltage transformers.

All secondary windings of instrument transformers must be grounded so as to avoid danger to life or equipment due to any high induced voltages which may appear in the secondaries. In addition, the cases of instruments used with instrument transformers also should be grounded. Current transformers are, or should be, provided with a short-circuiting switch across the secondary terminals which must be closed before the instruments are disconnected from the circuit. If the secondary of a current transformer is open, the primary current becomes magnetizing current only, with no opposing secondary ampere turns. Consequently, a very high flux density is reached. This leads to a high and dangerous secondary voltage, a core loss that may damage the transformer, and a resulting magnetic condition of the iron that may require demagnetizing and calibration of the transformer before it can be used again. For these reasons, the secondary always must be closed directly upon itself or through its normal instrument load circuit.

Instrument transformers are given polarity designations by the manufacturer through marks on one terminal each of the primary and secondary windings. These marks may be the usual H_1 and X_1 stamped adjacent to those terminals or on metal bands placed around them, a painted section of the leads, painted dots adjacent to the terminals, etc. The indications mean that current is leaving the marked secondary terminal at the same time current is entering the marked primary terminal, and circuit continuity hence is from one marked terminal to the other through the transformer and on to the instrument. Such markings are helpful in connecting instruments and relays properly. Figure 5.11 shows the arrangement of instruments and their transformers for measuring voltage, current, and power in a high-voltage, single-phase circuit.

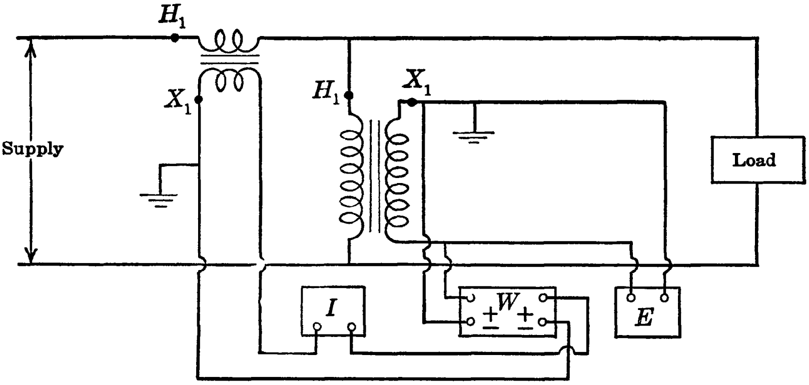


FIG. 5.11. Connection diagram for measuring current, power, and voltage of a single-phase system by use of instrument transformers.

PROBLEMS

1. A 50-kva, 22,500/230-volt, power transformer has windings in which 5.8 volts effective are induced per turn. The maximum flux density is 70,000 lines per square inch. The core material is Alleghany transformer "A" grade steel having the following characteristics:

Max. kilolines per sq in.....	10	20	30	60	70	90	100	120
Rms ampere-turns per in.....	0.45	0.7	0.95	1.95	2.8	13	40	200

Determine the turns in both windings, as well as the core area, if built for 60 cycles; for 50 cycles. If the transformer is built for 60 cycles, what will be the approximate percentage change in exciting current if it is used on a 50-cycle system of rated voltage? What is the effect on I_0 of applying rated voltage at 25 cycles to the transformer? What change in voltage would permit the 60-cycle transformer to be used on 25 cycles? What conclusions do you draw with respect to a change of frequency on a transformer?

2. A 15-kva, 2,300:230-volt, 60-cycle distribution transformer draws an exciting current of 1.2 amp at rated voltage on the low-voltage winding. What would be the exciting current if the measurement were taken on the high-voltage coil? What is the per cent exciting current? If a wattmeter reads 85 watts for rated open-circuit voltage, what is the per cent core loss? What equivalent parallel circuit would represent these readings taken from the low-voltage side of the transformer? If taken from the high-voltage side? How would the position of the voltmeter influence the reading of core loss? The position of the potential coil of the wattmeter?

3. A transformer for general low-voltage testing is rated 5 kva, 460/230//230/115 volts, 60 cycles. Sketch the connections of the coils for all possible voltage ratios. What are the rated currents for the coils, and the normal line currents for the possible connections of the coils?

4. A distribution transformer has a rating 15 kva, 4,000//500/250/125 volts, 60 cycles. The low-voltage windings consist of four separate 125-volt coils. Answer the same questions as in problem 3.

With all low-voltage coils in parallel, 2.15 amp is read for 125 volts impressed on that side. What current is taken by each coil? What is the percentage exciting current? If the power input for the open-circuit test is 90 watts, what is the per cent iron loss? What is the equivalent parallel exciting circuit as viewed from the low-voltage side? From the high-voltage side? What exciting current would be taken if only one low-voltage coil were used in the open-circuit test? If 2 low-voltage coils were used in parallel? If 2 low-voltage coils were used in series?

5. A transformer for factory application has a rating of 25 kva, 2,200/1,100//440/220 volts, 60 cycles. The exciting current is 1.9%, and the core loss is 0.92%. What is the exciting current in amperes for a no-load measurement at 2,200 volts? At 440 volts? What is the core loss in watts? What is the equivalent parallel circuit of the transformer on the high-voltage side for the 2,200-volt connection? What would be the exciting current if only one of the high-voltage coils were used?

6. A low-power-factor wattmeter is rated 2.5/5 amp, 75/150 volts. Full scale is 75 watts direct reading for the 5-amp and 75-volt ranges. On an open-circuit test on a transformer, the scale reading is 15 watts for connections to the 5-amp current terminals and with a multiplier for 300 volts in the potential circuit. What is the actual core loss for this reading?

7. The components of the total core loss for the transformer of problem 4 are 22 watts eddy-current loss and 68 watts hysteresis loss. What will be the total iron loss of the transformer for rated voltage at 50 cycles?

8. A 55-kva, 2,400/480-volt, 50-cycle transformer has an eddy-current loss of 120 watts and a hysteresis loss of 350 watts at rated voltage. What will be the core loss for a rating of 50 kva, 2,200/440 volts, 60 cycles?

9. A 5-kva, 220/82-volt, 60-cycle synchronous-converter transformer has the following constants:

high-voltage winding: $R_1 = 0.0485$ ohm, $X_1 = 0.108$ ohm
low-voltage winding: $R_2 = 0.0078$ ohm, $X_2 = 0.0161$ ohm

Calculate the ohmic values of the equivalent resistance, reactance, and impedance of the transformer referred to the high-voltage winding; referred to the low-voltage winding. Determine the corresponding percentage values.

10. Calculated constants for the distribution transformer of problem 4 are:

H.V. winding: $R_1 = 9.24$ ohms, $X_1 = 11.82$ ohms
L.V. windings: $R_2 = 0.040$ ohm per coil, $X_2 = 0.048$ ohm per coil

Compute the percentage R, X, and Z of the transformer. What are the required ranges of instruments for a short-circuit test taken on the H.V. winding at twice the rated current? In the short-circuit test, does it make any difference how the L.V. coils are short-circuited upon themselves or through each other?

11. Prove that, at rated current, the per cent values of resistance, $\%R$, of resistance drop, $\%IR$, and of copper loss are the same numerically.

12. Redraw the vector diagram of Figure 5.5 for unity power factor and for a leading power factor. Is there some value of power factor for which the regulation would be zero? What value of power factor would give maximum regulation? Show that the % regulation is given approximately by the relation $n(\%R \cos\theta \pm \%X \sin\theta)$, where the plus sign is to be used for lagging pf and the minus sign for leading pf, and n is the ratio of load current to rated current.

13. The following test results were obtained on a 350-kva, 11,000/4,000-volt, 60-cycle substation transformer:

open-circuit test (L.V. wdg.): 4,000 volts, 1.38 amp, 2,260 watts
short-circuit test (H.V. wdg.): 352 volts, 31.8 amp, 3,040 watts

Calculate the ohmic values of the equivalent resistance, reactance, and impedance viewed from the H.V. side and from the L.V. side. Compute the percentage R, X, and Z of the transformer. Calculate R_1 and R_2 assuming copper losses equal in both windings. Compute the efficiency and voltage regulation of the transformer for a load of 300 kw, 0.8 pf lagging; for a load of 275 kva, 0.6 pf leading.

14. A 50-kva, 2,300/230-volt, 60-cycle transformer used in a manufacturing plant gave the following test results:

open-circuit test (L.V. wdg.): 230 volts, 4.1 amp, 344 watts
short-circuit test (H.V. wdg.): 128 volts, 32.6 amp, 1,450 watts

Calculate the ohmic values of the equivalent constants with respect to both windings and the percentage R, X, and Z. Calculate the efficiency and regulation for a load of 35 kva at 0.75 pf lagging. What is the maximum efficiency attainable at this pf?

15. A 10-kva, 2,200/440-volt, 60-cycle distribution transformer takes 104 watts at no load. If the maximum efficiency occurs at a load of 7 kw, 1.0 pf, what will be the efficiency at 5 kw, 0.8 pf?

16. A 20-kva, 2,300/230-volt, 60-cycle distribution transformer has a maximum efficiency of 98.2% for a load of 12 kva, 1.0 pf. If the impedance is 4%, compute the efficiency and primary voltage for a load of 12 kw, 230 volts, 0.8 pf leading.

17. A 50-kva, 11,000/2,200-volt, 60-cycle transformer has 1% R and 4% Z. Iron losses are 0.6%. What is the maximum efficiency at 0.8 pf?

18. Show mathematically that the maximum efficiency of a transformer is reached when the copper losses equal the iron losses.

19. A 25-kva, 2,300/230-volt, 60-cycle transformer has 1.5% R and 5% Z. Draw the vector diagram for the case of 5% voltage regulation at full kva loading. What is the pf for this case? What is the maximum possible positive voltage regulation for 120% rated kva? What is the maximum negative voltage regulation for rated kva, and at what pf is it obtained?

20. A 5-kva, 440/220-volt, 25-cycle distribution transformer was tested with the following results:

open-circuit test (L.V. side): 220 volts, 40 watts
short-circuit test (H.V. side): 14 volts, 11.37 amp, 50 watts

What is the kw output of this transformer when carrying rated kva with zero voltage regulation? When carrying load at 0.6 pf lagging with 2% regulation?

21. The following results were obtained on a 50-kva, 2,200/220-volt, 60-cycle transformer:

open-circuit test (L.V. side): 220 volts, 7.1 amp, 285 watts

The short-circuit test requires 90 volts on the high-voltage side for rated current. If maximum efficiency is attained at 30 kva, 1.0 pf, what will be the efficiency for a load of 40 kw, 0.8 pf lagging? What is the regulation for this load?

22. The transformer of problem 21 carries 50 kva at 0.8 pf lagging for 6 hours, 40 kva at 0.6 pf lagging for 2 hours, 10 kva at 1.0 pf for 4 hours, and no load the remainder of the day. Compute the all-day efficiency of the machine.

23. In preparing for an open-circuit test on a transformer having double windings on each side, a member of a test crew connects in parallel the 2 L.V. coils so that their mmf's would be in opposition. What would be the resulting effect of this connection? Sketch the proper connection of the coils. How should the coils be connected if placed in series for the test?

24. A transformer rated 7.5 kva, 460/230//230/115 volts, 60 cycles, is purchased with polarity marks to be placed on the terminals by the customer. All leads coming from the case are of the same wire size. State the manner in which you would ascertain which terminals are associated with each winding, and the manner of testing for the H.V. and the L.V. coils. Assume that once these tests are made, the H.V. terminals are labeled arbitrarily A and B, and C and D, respectively, for the 2 windings. Similarly, the terminals of the L.V. coils are labeled a and b, and c and d. From the following measurements, with A marked H_1 as a starting point, assign the correct polarity marks to all other terminals.

1. 100 volts between A and B gives a voltage reading of 100 volts between a and d, with b and c connected together.
2. 25 volts between c and d gives a voltage reading of 0 volts between A and D when B and C are connected together.
3. 50 volts between A and B gives a voltage reading between A and a of 75 volts when B and b are connected together.

25. Refer to problem 3. For the circuit arrangements shown by the connection diagrams, label all winding terminals properly with polarity marks. Could an open-circuit test be made on the L.V. side of the transformer with the H.V. coils connected in parallel? If so, what is the connection? What would be the output voltage if the L.V. coils were connected in series with X_2 tied to X_4, and 440 volts applied to the H.V. coils properly arranged? Show the several correct arrangements of the transformer coils for a short-circuit test taken on the H.V. side.

26. Two transformers, one of subtractive and the other of additive polarity, are to be connected in parallel. Show the proper connection diagram.

27. A small laboratory transformer has a voltage rating of 230/115//23/11.5 volts. A condenser and a variable resistor are placed in series across the low-voltage coils which in turn are connected in series. With respect to the secondary voltage, sketch the locus of the voltage between the midpoint of the secondary coils and the common junction of the load elements. Is the locus affected by interchanging the positions of the condenser and resistor?

28. A 25-kva, 2,300/230-volt transformer having 1.5% R and 3% Z is paralleled with a 40-kva, 2,300/230-volt transformer having 1% R and 4% Z. What is the load division between the transformers for a total of 50 kva at 0.8 pf lagging? Calculate this load division by using equivalent ohmic circuit constants, and by using percentage values referred to the same kva base. What maximum total load at 1.0 pf can they carry without overloading either transformer? Would the load division be changed if both transformers had voltage ratios of 6,600/440 volts? If the first transformer had a voltage ratio of 2,200/220 volts and the second 2,400/240 volts? Should these transformers be paralleled if the first has a voltage ratio 2,200/220 volts and the second 2,400/220 volts?

29. A 25-kva, 2,300/115-volt, 60-cycle transformer has a copper loss of 1.5% at rated kva, and a normal iron loss of 0.8%. Hysteresis loss is 4 times the eddy-current loss. For the same allowable flux density, what should be the new rating of the transformer on 25 cycles? Assume (a) that the copper loss is to remain the same, and (b) that the total full-load losses may be the same.

30. A single-phase transformer has a 4,000-volt primary winding, and two secondary windings of 115 volts each connected in series so as to give a single-phase, 3-wire supply. A load of 500 watts at 1.0 pf is connected between one line and neutral, a load of 300 watts at 0.9 pf leading between the other line and neutral, and a load of 2 kva at 0.8 pf lagging between outside lines. Calculate the currents in all primary and secondary leads. Neglect line and transformer impedance drops, and transformer losses.

31. Two 7.5-kva, 2,400/120-volt distribution transformers are connected for 3-wire service. Each transformer has 1.04% *R* and 3.65% *Z*. The impedance of each lead of the 3-wire circuit is $Z = 0.06 + j0.15$ ohm. The load impedances on each half of the system are 1.9 and 3.0 ohms, respectively, and are resistances only. For 2,400 volts impressed on the high-voltage side, what will be the voltages across each load? What is the input current to the transformers?

32. A 50-kva, 2,400//240/120-volt, 60-cycle distribution transformer has 1.2% *R*, 3.4% *Z*, and an open-circuit loss of 305 watts. The list price is $470. A similar transformer has 1.1% *R*, 3.7% *Z*, and 0.57% iron loss. Its list price is $490. Either transformer, to be purchased for the same duty, is expected to be on the line continuously throughout the year with an average daily load of full kva for 4 hours, 70% rated kva for 4 hours, and 35% rated kva for 16 hours. Compare the yearly cost of the losses for the 2 machines with respect to their list prices. Energy costs 8 mils per kwhr.

33. The several loads on a small plant are expected to be:

7 kva at 1.0 pf
8 kw at 0.8 pf lagging
6 kva at 0.9 pf leading
5 kva at 0.85 pf lagging
12 hp, 0.85 efficiency, at 0.8 pf lagging

If the loads are to be supplied at 440 and 220 volts, with those at the lower voltages to be evenly divided between the two halves of a 3-wire system, and the main supply voltage available is 4,000 volts, what should be the rating of a single-phase transformer to carry the plant load? What is the over-all pf indicated by these loads? What additional kva at 1.0 pf can be added without exceeding the transformer rating, and what would be required to achieve this result? At $10 per kva of capacitors, what would be the investment charge for this item alone per kilowatt of added load?

34. The single-phase section of the intercommunication switchboard on a ship receives power from a 450-volt supply and distributes it at 115 volts to a number of circuits for control and indicators. The total capacity of the feeders as shown by the fuses is 360 amp. Assuming the fuses to have a 100% overload rating, that only one-half of the connected load may be in full operation at any one time, and that the average pf is 70%, what should be the rating of each of 3 transformers required to supply the total load?

35. On full voltage, the starting current of a single-phase motor is 120 amp. If a compensator (autotransformer) having a 60% tap is used for starting, what will be the motor and line currents? Compare these two currents in terms of the turn ratio of the compensator.

36. To boost the voltage on a distribution feeder, a 15-kva, 1,100/110-volt transformer is connected as an autotransformer with 1,100 volts applied to the high-voltage winding. Normal current for the high-voltage winding is carried by the input circuit. What per cent of the input power is transferred by transformer action, and what per cent flows conductively from one circuit to the other? How does the total copper loss of the autotransformer compare with the normal copper loss as an ordinary transformer? Assume 1.1% *R*, and that as an ordinary transformer the copper loss divides equally between the two windings. How does the kva transformed by the autotransformer compare with that of a 1,100/1,210-volt transformer for the same service? How do the copper losses of the 2 machines compare, assuming the same per cent *R* and division of losses as given above?

37. An induction regulator supplies 40 kw, 0.8 pf lagging, at 440 volts to a load when boosting the primary voltage 10%. Calculate the currents in each winding of the regulator. Neglect exciting current.

38. A 5-kva induction regulator for general laboratory testing has four 220-volt coils, two for the exciting circuit and two for the load circuit. What connections of the coils will give 100% buck or boost on the load side with 220 volts applied to the regulator? What current can the regulator carry safely for these coil arrangements? What are the connections for full buck or boost when the supply voltage is 440 volts? What are the permissible currents in the coils? Are there other voltage connections that may be used? If so, what are the corresponding connections, input and output voltages, and currents?

39. In a single-phase supply circuit for an electric furnace, a customer proposes to insert, in series, the coils of 12.5-kva, 2,200/220-volt distribution transformers to produce a 5% line drop with 50 amp load current. He suggests using the low-voltage coils, leaving the high-voltage coils open. Comment on the feasibility of this idea. Could the transformers be used in such a way as to achieve the desired result?

40. The output tube of a certain amplifier can be considered as a simple generator source having an open-circuit voltage of 100 volts and a pure resistive impedance of 5,000 ohms. Calculate the power delivered to a pure resistance load of 8 ohms. Calculate the power delivered when a matching transformer, having an efficiency of 95% and a turn ratio of 25, is used.

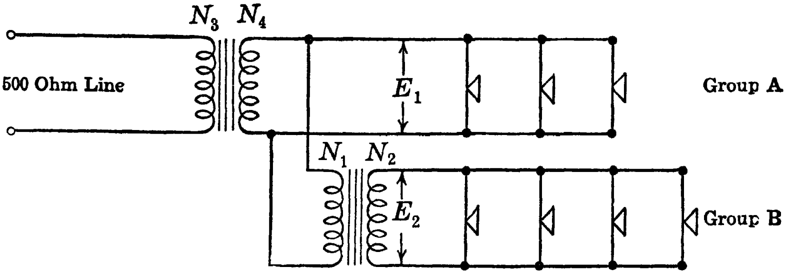
41. A transformer is used to couple a 500-ohm line to an amplifier having an input impedance of 200,000 ohms. At the lowest frequency, 40 cycles, the reactance of the primary winding should be at least twice the line impedance. Calculate the turn ratio and the minimum value of the primary inductance.

42. A step-down transformer is used to match an 8,000-ohm source to a 200-ohm load. The inductance of the primary winding is 30 henries. Determine the inductance of the secondary winding.

43. A 500-ohm line, having an open-circuit voltage of 30 volts, delivers power to a 15-ohm load through an ideal matching transformer having a turn ratio of 5. Calculate the power delivered to the load.

44. A 10,000-ohm generator is to be matched to a 200-ohm load. Determine the turn ratio of the ideal transformer. Using this transformer, the 200-ohm load was disconnected and the impedance was measured across the secondary terminals. What should be the value of the measured impedance?

45. Two groups of loud-speakers are supplied from a 500-ohm line as shown in the diagram. Each loud-speaker of group A is a 20-watt, 16-ohm speaker; those of group B are 10-watt, 8-ohm speakers. For rated input to each speaker, calculate, E_1, E_2, the turn ratio N_1 to N_2, the turn ratio N_3 to N_4, and the voltage of the 500-ohm line.



46. A matching transformer matches a 9,000-ohm amplifier to a 250-ohm speaker. A tap on the secondary matches the amplifier to a 90-ohm speaker if desired. What resistance for the other section of the secondary will be matched exactly to the amplifier?

CHAPTER VI

POLYPHASE SYSTEMS

6.1. Advantages of Polyphase Power. The a-c machine discussed in the chapter on the generation of electrical power is a single-phase generator. Commercial design is such that the terminal voltage is sinusoidal, in which case the current that flows when the generator is connected to a system of linear impedances is also sinusoidal. This type of generator, although frequently used, does not represent a very great part of the total generator kva met in practice. Power is almost always produced by the 3-phase generator, approximately 95 per cent of the total in this country. Electrical power is produced 3-phase, transmitted and distributed as such, and is used to a large extent in driving 3-phase motors. Some small 2-phase systems are still in existence, and there are many applica tions of six or more phases, but our primary interest here lies with the 3-phase circuit and machine.

There are several advantages for the 3-phase system. In the first place, the 3-phase machine allows much greater utilization of materials than does the single-phase machine. The power rating of a 3-phase machine is about 150 per cent that of the single-phase machine of the same physical size. Transmission of energy is less in cost for the 3-phase system because the copper required is less and the line losses are lower for the same amount of power transmitted. The power output of the single-phase generator is pulsating whereas that of the 3-phase generator is uniform and constant for balanced loading. The polyphase motor produces an armature reaction of constant magnitude and moves around the armature surface at synchronous speed whereas the armature reaction of the single-phase motor is pulsating. The polyphase motor therefore has a starting torque and a constant running torque for a given load. The single-phase induction motor has no inherent starting torque, and its running torque is pulsating. It is because of these many advantages that the 3-phase system is so widely used.

6.2. Notation for Voltage and Current. Polyphase circuits are but combinations of single-phase circuits. In a balanced system, whether generator or load, all single-phase components of the network are identical in character and magnitude of impedance, voltage, and current. The only difference is the time phase between the several voltages and currents of the individual elements, but it is this time phase which is of primary importance in the analysis of such circuits. The addition or subtraction of component voltages or currents in a polyphase network necessitates the adoption of a consistent system of notation so that all elements can be treated similarly.

The use of double subscripts has been found advantageous for designating voltages and currents in polyphase circuits. Either letters or numbers may be used, and corresponding terminals then are marked in cyclic order for convenience. Assumed positive direction of voltage, or current flow, is best taken in the same manner for all phase components, and is shown by the order of the subscripts. A reversal of the order of the subscripts is equivalent to a change in algebraic sign before the quantity, for example,

$$E_{AB} = -E_{BA} \tag{6.1}$$

Stated simply, if E_{AB} is a positive voltage sense of direction, then E_{BA} is negative.

In the following discussions on polyphase circuits, the conventions are those presented in the chapter on single-phase circuits. The direction of positive current flow in each of the three main lines has been chosen from the supply toward the load as shown in Figure 6.1. Positive direction of current flow in the neutral line has been taken in the opposite direction, i.e., from the load toward the supply. Since by now it is well understood that the arrow merely indicates the direction of positive current flow that has been adopted, the plus sign associated with it in the previous chapters need no longer be used. Positive voltage drop is taken in the direction of positive current flow and is given specifically at the outset by uniform subscript notation. Standard, or positive, sequence of voltages is in the order A–B–C–A.

Conventions are, of course, man-made and arbitrary and are purely for the convenience of analysis. Several, treating the same problem, may adopt their individual conventions; nevertheless, they will arrive at the same results if their work contains no numerical error. The basic requirement is that any adopted convention shall be logical, consistent for the component parts of the network, and that its use be continued without deviation throughout the analysis. It must be understood explicitly that no convention on the direction of current or voltage ever states that these quantities always are in a given direction. In an a-c circuit, they certainly are not. Conventions, either by directional arrows on a diagram or by subscript notation, are merely traffic signs which state that, for example, *if* we go with the arrow or in a given direction between two marked terminals, we shall say we are moving with positive current or with a positive driving voltage. If going in the opposite direction, we then meet negative current flow or a negative driving voltage. Conventions permit the proper time phase addition of voltages and currents, and, as we shall see, they hold equally for the unbalanced system as well as the balanced circuit.

6.3. Generation of Three-phase Power. In the analysis of a 3-phase system, the power source may be considered as consisting of 3 identical single-phase generators mechanically connected to a common drive, as is shown in Figure 6.1. Before the shaft of generator B is coupled to that of A, its rotor is rotated backwards so that its generated voltage lags that of *A* by 120 electrical degrees. Likewise, the rotor of generator *C* is keyed to the shaft so that its voltage lags

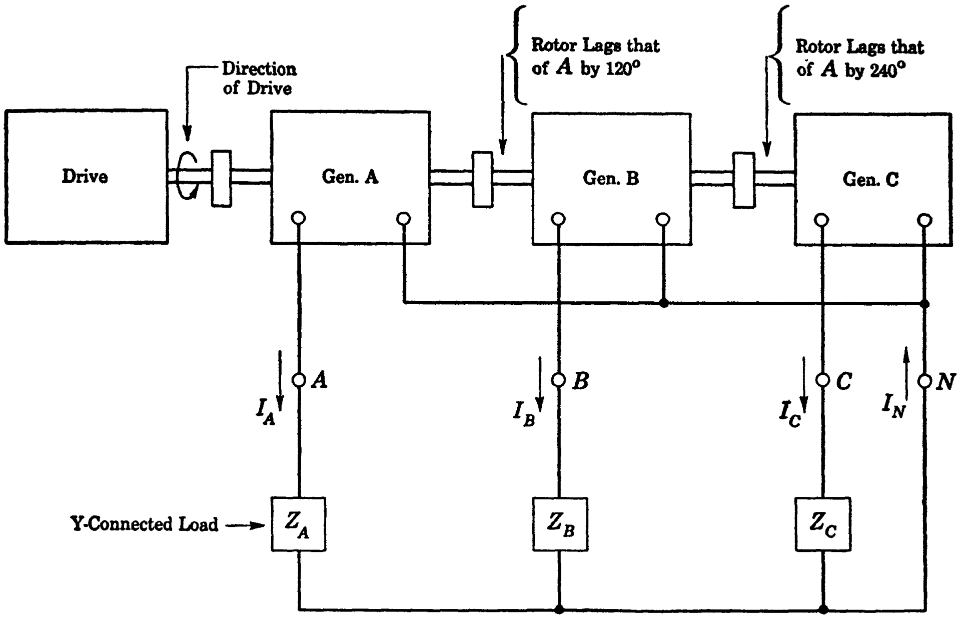


FIG. 6.1. A basic power source as represented by 3 single-phase generators connected in *Y*.

that of *A* by 240 electrical degrees. As a result of this arrangement, the instantaneous terminal voltages of the three machines reach their maximum values in the cyclic order *A–B–C*, as is shown in Figure 6.2a. This order of voltages, *A–B–C*, is termed *standard phase sequence* of the system. In Figure 6.2b is

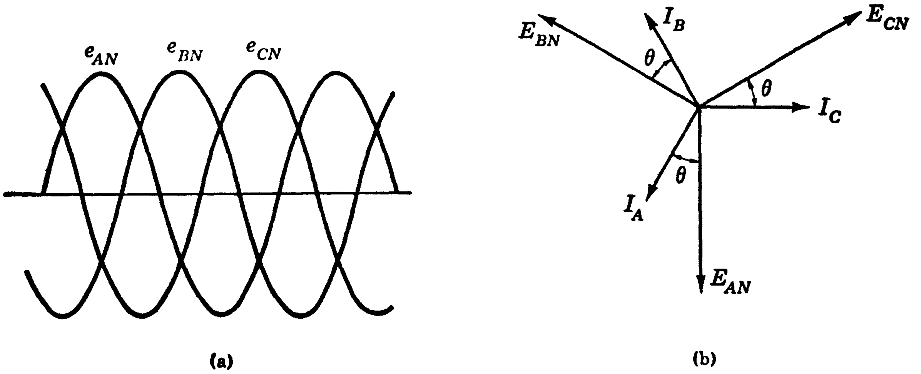


FIG. 6.2. Voltages and vector diagram of a 3-phase system, sequence *A-B-C*.

shown the vector diagram of machine voltages and line currents for identical loads of lagging power factor connected to each generator. These 3 identical impedances constitute a *balanced* 3-phase load and result in the line currents being equal in magnitude and appearing 120 electrical degrees apart in time as is indicated in the vector diagram.

A circuit connection in which one terminal of each phase or machine is connected to a common line, as is shown in Figure 6.1, is known as a wye (Y), or star, connection. Impedances arranged in a similar manner form a Y-connected load. The load connection of Figure 6.1 is of this type. As will be clear from later discussions, the common or *neutral* line may or may not be carried along in the system. The machines or the load impedances can also be connected in delta (Δ). The delta connection is discussed in later sections of this chapter and also in Chapter VII.

It should be clear, of course, that in an actual system 3-phase power would not be generated by three separate single-phase generators. The armature coils for the three phase voltages would be incorporated in one machine. The simplest 3-phase generator is a 2-pole machine with the entire armature winding grouped into three separate sets of coils. All phase groups contain the same number of coils and are so placed 120 electrical degrees apart in space position in the armature slots that the phase voltages produced by virtue of the rotation of the machine appear 120 electrical degrees in time from each other. The three sets of coils are connected either in wye or delta. We need not be concerned here with the details of armature windings or machine construction. A discussion of 3-phase voltage generation, and of voltage and current relations in such a system, is independent of these details. Furthermore, although some machines in a power network might be delta-connected, the general analysis of the system does not require that one type of connection be distinguished from the other. In the treatment of this chapter, therefore, we will assume that the windings of the machine are Y-connected and that the machine can be represented by the arrangement shown in Figure 6.1.

From Figure 6.1 it is evident that the system is but a combination of 3 single-phase circuits and that the total kvar and kw is simply the sum of these quantities supplied by the three generators. The power from each generator is the product of its terminal (line-to-neutral) voltage, its line current, and its power factor. The power factor angle of each generator is the angle of its leg impedance as is the case in any single-phase circuit. For a balanced system each leg impedance has the same magnitude and phase angle, and the total power is three times the product of line-to-neutral voltage, line current, and power factor. Furthermore, the *instantaneous* power flow in a balanced 3-phase circuit is constant, in contrast to the pulsating single-phase power, and hence is equal to the average power. As will be shown in later sections, these general relationships still apply when the load is delta connected.

6.4. Line-to-Line Voltages. It is obvious that each line-to-line voltage is composed of two leg[1] or line-to-neutral voltages. As a result of the connection between the two generators, or windings in the 3-phase machine, the instantaneous voltage of A with respect to B, e_{AB}, is equal to the voltage of A with respect to N, e_{AN}, minus the voltage of B with respect to N, e_{BN}, as is illustrated in Figure 6.3a. The line voltage e_{AB}, therefore, has a maximum value $\sqrt{3}$ times the maximum value of either e_{AN} or e_{BN}, and it appears 30 electrical degrees ahead of e_{AN} in time. These several voltages would be shown on an oscillograph screen exactly as pictured, in relative magnitude and time position, if leads from A, B, and N were carried to the oscillograph in the manner consistent with the conventions adopted here.

---

[1] The term *leg* refers to quantities associated with one of the three impedances. *Leg* and *line* voltages and currents are used throughout this book purposely to avoid confusion. The term *phase* too often refers to either leg or line quantities and frequently without definition.

The results can be obtained more readily by treating the voltages as vector quantities in the usual manner, i.e., E_{AB} is the vector difference of E_{AN} and E_{BN}.

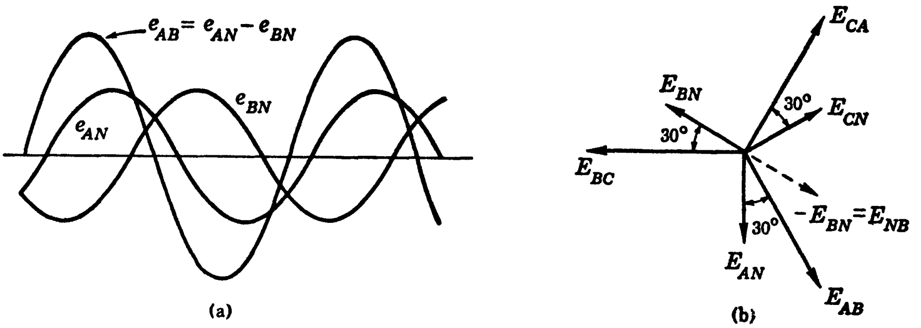


FIG. 6.3. (a) Combination of Y-circuit leg voltages to obtain line voltage. (b) Vector diagram of line and leg voltages.

In an exactly similar way, the line voltages E_{BC} and E_{CA} are obtained. The three line voltages are all equal in magnitude and appear 120 electrical degrees apart in time as is shown in Figure 6.3b. In themselves they form a balanced 3-phase system of voltages. The line-to-line voltages have a magnitude equal to $\sqrt{3}$ times the magnitude of the line-to-neutral voltages and on the vector diagram are rotated with respect to them by 30 degrees.

6.5. Analysis of the Balanced Y Load. Figure 6.4 shows a set of balanced impedances connected in Y. The component elements of the impedances will be the same in character and in magnitude. With balanced 3-phase line voltages impressed between the load terminals, the voltages across the three impedances will be equal and will be the effective values of the line voltages divided by $\sqrt{3}$, in magnitude. From Figure 6.4, it is observed that the voltages impressed between load terminals are, respectively, the line-to-line voltages at the supply terminals. Consequently, the line-to-neutral voltages of the supply are equal to the corresponding leg voltages of the load, that is, $E_{AN} = E_{AO}$, etc., in magnitude and time phase. The complete voltage diagram for all line and leg voltages is shown in Figure 6.5a. The diagram also can be shown with all vectors drawn from a common point, Figure 6.5b. Whether the line voltages are drawn between extremities of the leg voltages or from a common point is entirely immaterial. Each form has certain advantages, the choice depending upon the particular problem to be solved. Either graph is solely for the purpose of showing proper effective magnitudes and time phase angles, and Figures 6.5a and 6.5b are exactly equivalent. It is with the very definite purpose of assisting in determining the phase angles between quantities that the arrowheads are displayed on the vectors.

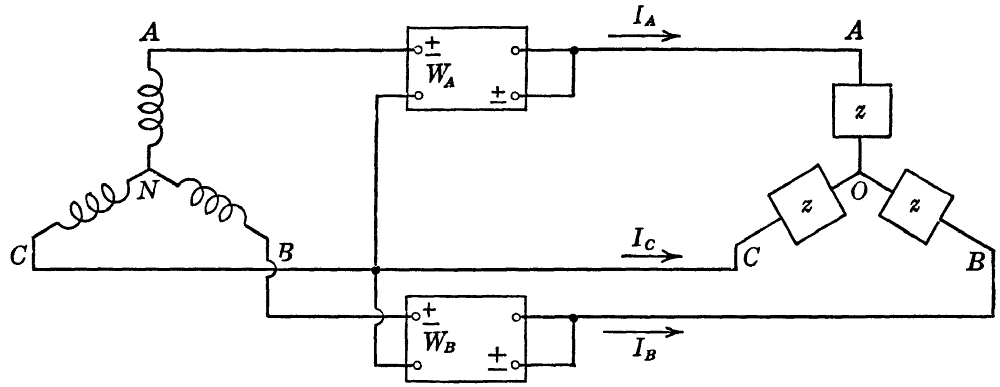


FIG. 6.4. Balanced Y-connected impedances connected to a Y-connected generator.

It might be remarked that, although there may be geometric similarity between the connection diagram, which is a space diagram, and the corresponding voltage diagram, which is a time diagram, the two should not be confused. It is the magnitudes and time angle positions of the leg and line voltages and currents with which we are concerned here by way of the sine-wave and vector diagram

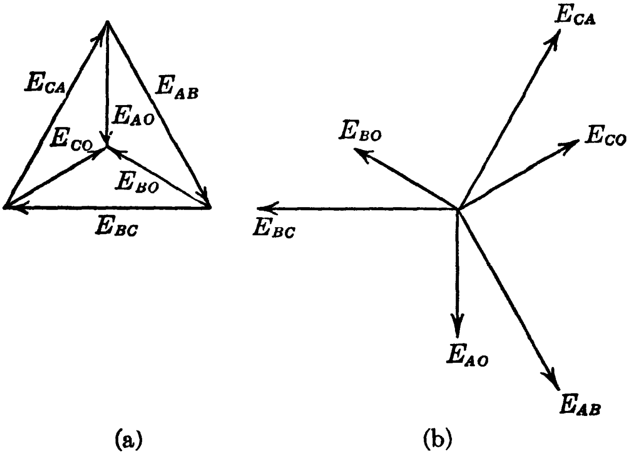


FIG. 6.5. Vector diagram of leg and line voltages for Y-connected system.

discussions. In practice, diagrams and blueprints very often show the Y, or star, connection in a manner different from the arrangement displayed here. The circuit arrangement is a connection diagram only, not a time diagram. The only reason for ever showing the Y circuit in the form of a Y is that it may assist in picturing the corresponding star array of leg voltages which, for the balanced circuit, requires 120 degrees between the adjacent arms. This same comment holds true for the delta circuit.

The direction of flow of current is as indicated in Figure 6.4, and each load current can be designated by a single subscript, for example, $I_{AO} = I_A$. In effective magnitude, they will be

$$I_A = \frac{E_{AO}}{Z} = \frac{E_{AN}}{Z} = \frac{E_{AB}}{\sqrt{3}Z} \text{ amperes}$$

and

$$I_A = I_B = I_C \tag{6.2}$$

The three currents form a balanced 3-phase system in themselves, and, since their sum is zero, there would be no current in a line connecting the neutrals N and O. Because the load currents also are the respective line and leg currents of the supply, the phase angles between load voltages and currents will be the same for the corresponding voltages and currents of the supply. The complete

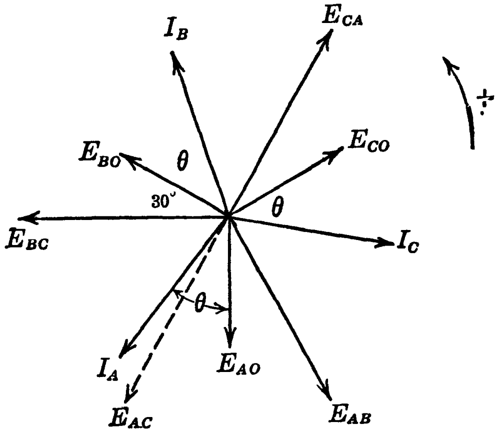


FIG. 6.6. Vector diagram for the Y-circuit of Fig. 6.4, lagging power factor, sequence A-B-C.

vector diagrams for load and supply, therefore, are the same diagram Figure 6.6, in which the leg currents are assumed to lag their corresponding voltages by the angle θ.

6.6. Power in the Balanced Y System. The total power in the balanced Y system is

$$\begin{aligned} \text{power} &= 3I_{\text{leg}}^2 R_{\text{leg}} \\ &= 3E_{\text{leg}} I_{\text{leg}} \cos\theta \qquad (6.3) \\ &= \sqrt{3} E_{\text{line}} I_{\text{line}} \cos\theta \text{ watts} \end{aligned}$$

since $\sqrt{3}E_{\text{leg}} = E_{\text{line}}$, and $I_{\text{leg}} = I_{\text{line}}$. In practice, this power is usually measured by using two wattmeters, as shown in Figure 6.4. The wattmeter readings will be

$$\begin{aligned} W_A &= E_{AC} I_A \cos(E_{AC}, I_A) \\ W_B &= E_{BC} I_B \cos(E_{BC}, I_B) \text{ watts} \qquad (6.4) \end{aligned}$$

Referring to the vector diagram, Figure 6.6, it will be seen that the angle between E_{AC} (E_{CA} reversed) and I_A is $(\theta - 30°)$, and that the angle between E_{BC} and I_B is $(\theta + 30°)$. Hence,

$$\begin{aligned} W_A &= E_{AC} I_A \cos(\theta - 30°) = E_{\text{line}} I_{\text{line}} \cos(\theta - 30°) \\ W_B &= E_{BC} I_B \cos(\theta + 30°) = E_{\text{line}} I_{\text{line}} \cos(\theta + 30°) \text{ watts} \end{aligned} \qquad (6.5)$$

and

$$W_A + W_B = \sqrt{3} E_{\text{line}} I_{\text{line}} \cos\theta \text{ watts} \qquad (6.6)$$

when the indicated mathematical steps are performed. Note that the subscripts for the wattmeter readings refer only to the lines in which the current coils are placed. The common line for the potential circuits is always the remaining third lead to the load. The same total power is obtained no matter in which two lines wattmeters are placed, i.e., $W_A + W_B = W_B + W_C = W_C + W_A$. In general, however, W_A will not have the same numerical value when associated with W_C as when associated with W_B. This holds true for W_B and W_C also. To repeat, the sum of each pair of wattmeter readings gives the total 3-phase power and, furthermore, this is true whether the load is balanced or unbalanced. The manner in which a wattmeter should be connected, and the method of calculating the reading it is to give, are discussed under wattmeters in Chapter XVIII on electrical instruments.

Observe from the above discussion that the total 3-phase volt-amperes for the system is $\sqrt{3}E_{\text{line}} I_{\text{line}}$, and that the power factor, which is the ratio of the 3-phase power in watts to the 3-phase volt-amperes, is simply the cosine of the phase angle between leg current and leg voltage and can be computed from the relation

$$\text{Power factor} = \frac{\text{total watts}}{\sqrt{3} E_{\text{line}} I_{\text{line}}} \qquad (6.7)$$

Specifically, it is the cosine of the impedance angle of the leg. The power factor for the 3-phase circuit is the power factor of the leg, all legs having the same value for the balanced system. It should be understood that power factor has its usual interpretation only for balanced loads. The total reactive volt-amperes for the circuit are $\sqrt{3}E_{\text{line}}I_{\text{line}}\sin\theta$.

If the power-factor angle is greater than 60 degrees, one instrument reads negatively with respect to the other, and the total power then is the difference between the two readings. This condition is often met in practice, for example, when measuring power taken by a very lightly loaded induction motor. With both instruments connected to the lines in an identical manner, the pointer on one will move off scale to the left, and that instrument will have to be reconnected (reverse leads to current coil) to give an on-scale reading. If readings of the first instrument are assumed positive, those for the reconnected wattmeter will be negative.

Expansion of equations 6.5 will show that

$$\tan\theta = \sqrt{3}\frac{W_A - W_B}{W_A + W_B} = \sqrt{3}\frac{W_B - W_C}{W_B + W_C} = \sqrt{3}\frac{W_C - W_A}{W_C + W_A} \tag{6.8}$$

from which the power factor can be obtained. The tangent is related directly to the impedance angle of the leg in that it is the ratio of the net leg reactance to the leg resistance for the equivalent series circuit of the leg. Tan θ therefore is positive for reactance which is predominantly inductive, and is negative for net reactance which is capacitive. The reverse of these statements holds if the voltage sequence is A–C–B–A.

6.7. Example on the Y System. Figure 6.7 shows a star-connected set of impedances with line impedance $1 + j3$ between the supply terminals and the load. At the load, the voltage is 220 volts, with sequence A–B–C. $W_B = 866$ watts, and $W_C = 1{,}732$ watts. Let the problem be to calculate the voltage, power, and power factor at the supply terminals.

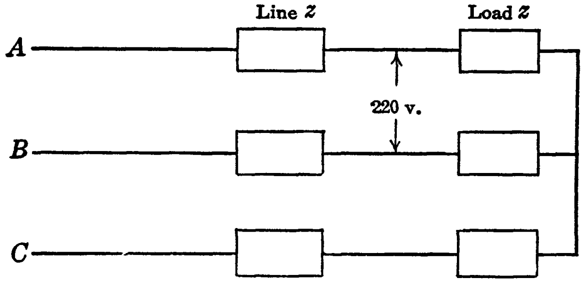


FIG. 6.7. Star-connected load with line impedances between supply and load.

Solution:

From equation 6.8, $\tan\theta = -0.577$, whence $\theta = 30°$, leading power factor

$$\text{The current is } I = \frac{866 + 1{,}732}{\sqrt{3} \times 220 \times 0.866}$$

$$= 7.87 \text{ amperes}$$

$$\text{Leg impedance is } Z = \frac{220/\sqrt{3}}{7.87}$$

$$= 16.14 \text{ ohms mag.} = 14.0 - j8.1$$

$$\text{Over-all leg impedance, including one line and one leg} = 15.0 - j5.1$$

$$= 15.9 \text{ ohms mag.}$$

Line voltage at input terminals $= \sqrt{3} \times 7.87 \times 15.9 = 216$ volts, which also can be obtained by adding the impedance drop of one line to the leg voltage of 127 volts and multiplying the result by $\sqrt{3}$.

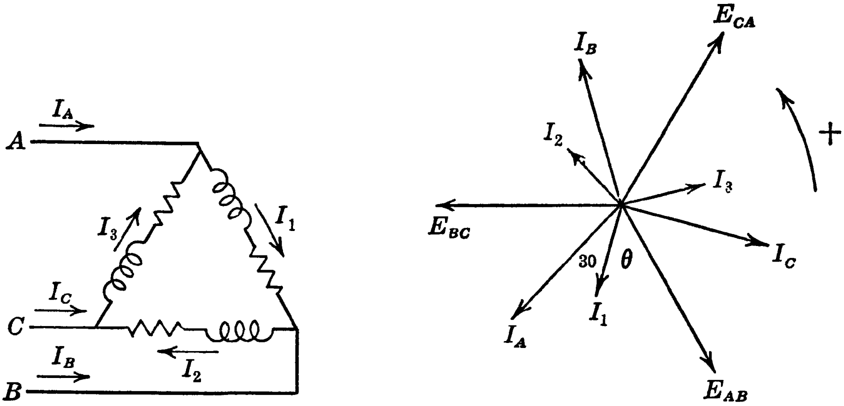
$$\text{Power factor at input terminals} = \frac{15}{15.9} = 0.943 \text{ leading}$$

$$\text{Input power} = \sqrt{3} \times 216 \times 7.87 \times 0.943 = 2{,}785 \text{ watts}$$

$$= 3 \times (7.87)^2 \times 15$$

$= (866 + 1{,}732) + 3 \times (7.87)^2 \times 1$, the sum of the power taken by the load and the line losses

6.8. The Balanced Delta System. Refer to Figure 6.8a. While the line voltages for the delta circuit now are equal to the leg voltages, any line current is composed of the currents in the two legs connected to the line in question.



(a) (b)

FIG. 6.8. Vector diagram for the delta system, lagging power factor, sequence A–B–C.

Figure 6.8b shows a balanced set of leg currents, I_1, I_2, and I_3, lagging their respective leg voltages by the angle θ. The current at A will be the vector difference of the currents caused to flow by the voltages E_{AB} and E_{CA}. Without resorting to a subtraction of these two currents by sine waves to find their difference, we may go directly to the vector treatment. Thus, in magnitude

$$I_A = I_{AB} + I_{AC} = I_1 - I_3 = \sqrt{3}I_1 \text{ in magnitude} \tag{6.9}$$

The line currents for the balanced delta system are therefore $\sqrt{3}$ times the leg currents.

The complete voltage and current diagram for the delta circuit is given by Figure 6.8b. The line voltages could have been drawn in the form of a delta as was done in Figure 6.5a.

Similar conditions hold for a delta-connected generator. The leg voltages, of course, are the corresponding line voltages. Any leg current is equal to the leg voltage divided by the leg impedance, and the line currents are $\sqrt{3}$ times the leg currents, in magnitude.

6.9. Power in the Balanced Delta System. As for the Y system, the total power in the balanced delta system is

$$\begin{aligned} \text{power} &= 3I_{\text{leg}}{}^2R_{\text{leg}} \\ &= 3E_{\text{leg}}I_{\text{leg}} \cos\theta \\ &= \sqrt{3}E_{\text{line}}I_{\text{line}} \cos\theta \text{ watts} \end{aligned} \tag{6.10}$$

since $E_{\text{line}} = E_{\text{leg}}$, and $I_{\text{line}} = \sqrt{3}I_{\text{leg}}$. Again, the total power usually is measured by two wattmeters, and what has been said for the Y circuit with respect to conventions holds also for the delta circuit. It is necessary to determine the currents in the two lines in which the wattmeters are placed, and beyond this point the treatment is identical to that already discussed. The student is asked to carry out the analysis, by aid of the vector diagram, to show that two wattmeters will measure the total power in the delta system, and that it is immaterial in which two lines wattmeters are placed.

6.10. Example on the Balanced Delta System. Three impedances of $15 + j9$ ohms each are in delta on a 440-volt, 3-phase power supply. What are line current and load power?

Solution:

Leg impedance is 17.49 ohms mag., whence leg current $= 440/17.49 = 25.2$ amp, and line current $= \sqrt{3} \times 25.2 = 43.6$ amp. Power factor $= 15/17.49 = 0.86$ approx. The phase angle for the leg is 31° approx.

$$\begin{aligned} \text{3-phase power} &= 3 \times (25.2)^2 \times 15 = 28{,}500 \text{ watts} \\ &= \sqrt{3} \times 440 \times 43.6 \times \frac{15}{17.49} \end{aligned}$$

Leg voltage for an equivalent Y system is 254 volts, leg current is 43.6 amp, and leg impedance then is $254/43.6 = 5.83$ ohms mag. $= 5 + j3$. The equivalent Y system has the same leg power factor and takes the same total power. Note that for the equivalent systems the impedance ratio between the legs of the delta and the Y is 3. Once the equivalent Y circuit is obtained, line impedances can be taken into account easily, as shown in Section 6.7.

With wattmeters in lines A and C (this step is left to the student),

$$W_A = 43.6 \times 440 \times \cos 61^\circ = 9{,}320 \text{ watts}$$

$$W_C = 43.6 \times 440 \times \cos 1^\circ = 19{,}180 \text{ watts}$$

6.11. Combinations of 3-Phase Balanced Loads. In general, a 3-phase power source supplies many parallel loads of varying character. The total power measured at the supply terminals will be the algebraic sum of the values of power taken by the individual loads. Similarly, with due respect to lagging and leading power factor, the input reactive kva will be the sum of the reactive kva of the separate loads. The over-all kva for the system then is but the resultant of the total kw and the total kvar. With total kva known, the input line current and the over-all power factor are readily computed. There is no distinction in this form of calculation for the 3-phase system over that for the previous single-phase system. For combinations of parallel loads it is simplest to tabulate in column form the kw and kvar of the several loads. This type of problem seldom requires a graph or more than the brief tabulation of power components.

6.12. Equivalent Y and Δ Balanced Loads. Readings of voltage, current, and power in a 3-phase circuit will not tell whether the load is Y- or delta-connected, or a combination of these arrangements. From the terminal measurements one can assume an infinite number of load configurations, the simplest, however, being but a single Y or a single delta. By definition, all assumed circuit arrangements are equivalent if they show the same measured terminal quantities of voltage, current, power, and character of power factor, whether lagging or leading.

For purposes of calculation, then, any configuration of circuits can be replaced by some other equivalent system, specifically, a single Y or delta. A delta load can be replaced by a Y load to give the same over-all data, and either a single Y or delta load can be substituted for a combination of loads. It is generally simplest to substitute a single Y for any or all that may be given, whether the manner of connection is or is not stated. The objective in replacing a single load or a combination of loads by an equivalent Y circuit is to reduce the problem to a single-phase circuit involving but one leg of the equivalent Y.

One great advantage of the scheme lies in that type of problem in which line impedance must be taken into account. With load conditions known and the equivalent Y substitution made, the impedance of one line now can be added directly to the impedance of one leg of the load. The problem hence is reduced to the single-phase circuit already fully developed in Chapter 4. Ahead of the line impedance, at the input terminals of the complete system, the line-to-line voltage is merely $\sqrt{3}$ times the calculated line-to-neutral voltage at that point for the single-phase circuit, and the total 3-phase power input is simply 3 times the leg power represented by the single-phase circuit.

In treating 3-phase machines and power systems, it usually is much simpler to work with one leg of an equivalent Y circuit.

6.13. Example on Combined Balanced Loads. Figure 6.9 shows a delta load of leg impedance 18 + $j6$ ohms in parallel with a Y load of leg impedance 5 − $j8$ ohms, taking power from a 230-volt supply. Let the problem be to find the total power to the loads, the over-all power factor, the resultant line current, and the circuit constants of the equivalent Y system for the combination.

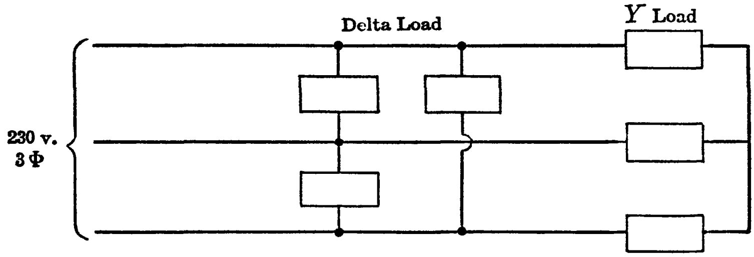


FIG. 6.9. Delta and Y loads in parallel.

Solution:

For the delta load:

leg voltage = 230 volts

leg impedance = 18.97 ohms mag.

leg current = 12.12 amp mag.

leg power = $I^2R - jI^2X_L$ = 2,650 watts − $j883$ reactive volt-amperes

For the Y load:

$$\text{leg voltage} = \frac{230}{\sqrt{3}} = 132.7 \text{ volts}$$

leg impedance = 9.43 ohms mag.

leg current = 14.08 amp mag.

leg power = 990 watts + $j1{,}585$ reactive volt-amperes

For the entire system:

power = 3(3,640 watts + $j702$ reactive volt-amperes)
= 10,920 watts + $j2{,}106$ reactive volt-amperes
= 11,140 volt-amperes mag.

$$\tan\theta = \frac{702}{3{,}640} = 0.193, \text{ and pf} = 0.98 \text{ leading}$$

$$\text{line current} = \frac{11{,}140}{\sqrt{3} \times 230} = 27.9 \text{ amp mag.}$$

For the equivalent Y circuit:

$$\text{leg voltage} = 132.7 \text{ volts}$$

$$\text{leg current} = 27.9 \text{ amp mag.}$$

$$\text{leg impedance} = 4.75 \text{ ohms mag.}, = 4.66 - j0.90 \text{ ohm}$$

$$= \frac{(6 + j2)(5 - j8)}{11 - j6} = 4.66 - j0.90$$

Any combination of loads would be treated in the above manner by adding the respective leg watts and reactive volt-amperes. Line impedance ahead of the loads is then taken into account through analysis using the equivalent Y for the parallel loads.

6.14. Reversed Phase Sequence. In all of the discussion thus far, it has been assumed that the time sequence of the line voltages was E_{AB}, E_{BC}, E_{CA}, called standard sequence, and usually indicated by the order A–B–C. Should the voltages rise to their maximum values in the order E_{AB}, E_{CA}, E_{BC}, the phase sequence is said to be reversed. It then is indicated by the notation A–C–B. Reversing the direction of rotation of a generator supplying its own load will reverse the sequence of voltages, as will the interchange of any 2 supply leads to the load. Obviously, generators are installed to run in a given direction, and consequently phase sequence is changed by switching line leads.

Phase sequence for a 3-phase power supply can generally be determined only by test. There are several ways of doing this. One form of sequence indicator is essentially a small induction motor having 3 fixed coils connected either Y or delta to produce a revolving magnetic field when clipped to the supply lines. The clips are marked, and the legend on the face of the device states the voltage sequence corresponding to the direction in which the disk rotor turns. Interchanging the clips to the supply lines reverses the motion of the revolving field within the case and with it the direction of motion of the rotor.

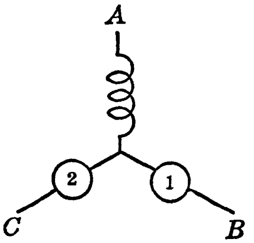


FIG. 6.10. Phase rotation indicator.

An indicator often used in the laboratory is composed of an inductance and 2 lamps connected in Y, Figure 6.10. This is purposely an unbalanced circuit. Analysis of the circuit (the method of analysis appears later in this chapter) supposes the coil to be pure reactance and the lamps to be pure resistance elements of equal ohmic values. Calculating the currents through the 2 lamps for assumed standard sequence of terminal voltages, it is observed that lamp 1 will burn dimly and lamp 2 brightly. Hence if such conditions are realized with the indicator connected to a 3-phase circuit, the sequence of voltages then is A–B–C. Reversed voltage sequence causes lamp 1 to burn brightly, and lamp 2 to burn dimly. These 3 elements, enclosed in a case, with terminals marked as shown and with the case carrying the legend, become a sequence indicator. Commercial indicators may employ neon tubes rather than a lamp, and the inductance may be replaced by a capacitor. In such case, the indication on sequence is opposite to that we have discussed for the inductance.

If the character of the load is known for a 3-phase circuit, that is, whether the power factor is lagging or leading, voltage sequence can be ascertained from the readings of 2 wattmeters connected in correct manner to measure 3-phase power. Again, by connecting a known artificial load to a 3-phase supply of unknown sequence, and observing the indications of 2 wattmeters, the voltage sequence of the supply can be found. These methods are essentially equivalent.

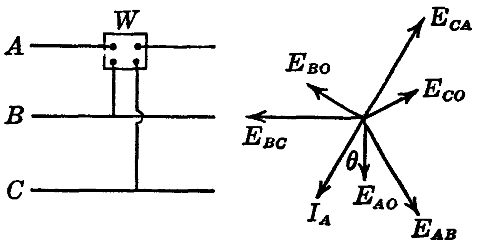
With balanced line voltages, opposite voltage sequence for a balanced 3-phase load cannot affect the magnitudes of the line currents or the total power. It will, however, interchange the readings of 2 wattmeters connected to indicate that total power. Observe that this is the manner in which sequence is ascertained in the preceding paragraph.

For the balanced system, Y or delta, there is but one solution for the magnitudes of all quantities to be calculated, except the individual readings of the wattmeters. The vector diagram for sequence *A–C–B* may be shown simply by interchanging any two leg voltage vectors and then proceeding from there in the usual way. A second manner of displaying the vector diagram is to draw the voltages as for sequence *A–B–C*, but to treat positive angles in clockwise sense rather than counterclockwise, as is the usual convention. These methods give the same results, for example, the reading of a particular wattmeter.

6.15. Measurement of Reactive Power. In a balanced 3-phase system the reactive power is given by the expression

$$\text{reactive power} = \sqrt{3}E_{\text{line}}I_{\text{line}}\sin\theta \tag{6.11}$$

and can be measured with one wattmeter as is shown in Figure 6.11a. The vector diagram is shown in Figure 6.11b. The current coil of the wattmeter carries the line current I_A and the voltage coil is connected across the lines *B* and *C*. The wattmeter reading will be



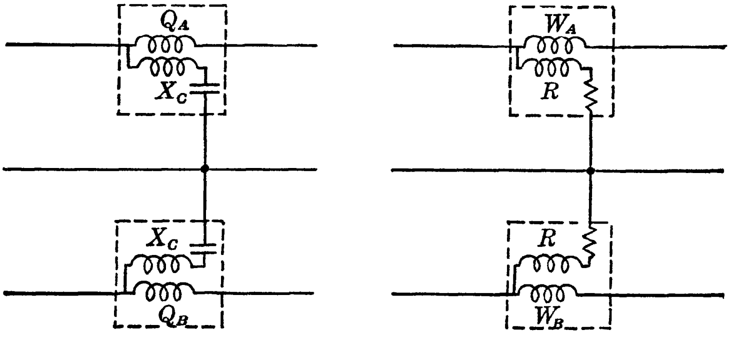
(a) (b)

FIG. 6.11. Measurement of reactive power in a balanced 3-phase system.

$$\begin{aligned} W &= E_{BC}I_A\sin\theta \\ &= E_{\text{line}}I_{\text{line}}\sin\theta \\ &= \frac{\text{reactive power}}{\sqrt{3}} \end{aligned}$$

Therefore, the reactive power is equal to $\sqrt{3}$ times the wattmeter reading.

The reactive power in a 3-wire, 3-phase system can be measured, whether the connected load is balanced or unbalanced, by using 2 reactive-power meters, as shown in Figure 6.12a. For comparison, the connection for measuring true power is shown in Figure 6.12b. The reactive-power meters are essentially wattmeters so modified that the current in the voltage coil is shifted in phase 90 degrees. The 90-degree phase shift is obtained by substituting a high reactance for the resistor used in the usual wattmeter.



(a) (b)

FIG. 6.12. Measurement of (a) reactive power and of (b) active power in a 3-phase system.

The reactive-power meter reads the product of voltage and current and the sine of the angle between them. Referring to equations 6.5, it is evident that the individual readings of the reactive-power meters for a balanced load will be

$$Q_A = E_{AC}I_A \sin(\theta - 30)$$

$$Q_B = E_{BC}I_B \sin(\theta + 30)$$

and

$$Q_A + Q_B = \sqrt{3}E_{\text{line}}I_{\text{line}} \sin\theta \text{ reactive volt-amperes}$$

Often wattmeters are used in connection with "phasing transformers" to measure reactive power.

Although power factor has its usual physical significance for only balanced loads, it can be defined for unbalanced loads if the total active power and the net reactive power are known. This representative power factor is given by the expression

$$\text{power factor} = \frac{\text{active power}}{\sqrt{(\text{active power})^2 + (\text{reactive power})^2}} \tag{6.12}$$

Often power companies meter both active and reactive power and determine the power factor of the load from these two sets of readings.

6.16. Unbalanced 3-Phase Systems. In the balanced 3-phase system, it is understood that all quantities of the same character are equal, and that like voltages and currents are 120 degrees apart in time. All line and leg voltages and currents, in themselves, form balanced 3-phase systems. Actually, however, the balanced circuit is a condition which may be closely approached in practice but one which usually never is reached exactly. The general case is the unbalanced system, the balanced circuit being but a special case. Although not strictly balanced, in perhaps most instances the system may be sufficiently near it to allow of the treatments presented up to this point.

The analysis of the unbalanced circuit is relatively simple, both in method and calculation. The solutions following for the unbalanced Y and delta circuits are carried through with numerical values. The desire is to show the methods employed in obtaining results, the same schemes carrying over to other problems having different numbers. The methods are general, regardless of magnitude or kind of unbalance, but specific cases are more to the point in understanding. In general, the solutions can be obtained graphically or by means of complex numbers. A good graphical analysis is fully as satisfactory as any other method, and time requirements are practically the same for all.

6.17. The 4-Wire System. We have seen that a lead between the common point of the 3 legs of a completely balanced Y circuit and the neutral point of the power source carries no current because the sum of the currents at the junction of the Y is zero. With a set of unequal impedances in Y, however, the sum of the 3 leg currents will in general not be zero, and the fourth, or neutral, line will carry a current which is the vector sum of the 3 leg currents; that is,

$$I_{ON} = I_{AO} + I_{BO} + I_{CO} \tag{6.13}$$

The presence of a neutral lead of zero impedance maintains at the load the same leg voltages as at the supply, and each leg current then can be found immediately in magnitude and in phase position with respect to its corresponding leg voltage. From the standpoint of the vector diagram, the fourth line fixes the neutral at the geometric center of the triangle of line voltages, regardless of voltage unbalance of the lines or unbalance of the load impedances. The solution of such a circuit thus is obtained by solving 3 simple single-phase circuits.

Power in the 4-wire system cannot be measured by 2 wattmeters only, because current flowing in the third main line returns to the power source through the neutral lead. The 4-wire circuit practically is a system of 3 separate single-phase circuits having a common return and 3 wattmeters are required, their voltage coils being placed across the respective legs of the Y. The total power will be, of course, ΣI^2R for the 3 legs, and the total reactive volt-amperes will be ΣI^2X. Each leg has its own power factor, and there is no single value of power factor, such as an average. The instantaneous power factor of any unbalanced system never has been defined.

6.18. The Unbalanced 3-Wire Y System. Figure 6.13 shows a group of 3 unequal impedances placed across a 3-phase, 4-wire supply. With the switch in the fourth line closed, the neutral current, I_{ON}, is the vector sum of the 3 leg currents, that is,

$$I_{ON} = I_{AO} + I_{BO} + I_{CO} = \frac{E_{AN}}{Z_1} + \frac{E_{BN}}{Z_2} + \frac{E_{CN}}{Z_3} \tag{6.14}$$

All currents can be found graphically or by means of complex numbers.

If the switch is opened, the current in the neutral becomes zero, and a voltage, E_{NO}, appears across the switch. This voltage may be conceived to be that of an equivalent single-phase generator in the neutral line which sends a current $I_{NO} = -I_{ON}$ through that lead, thereby giving zero resultant neutral current. The voltage E_{NO} acts on the 3 leg impedances in parallel, as shown in Figure 6.14, and consequently

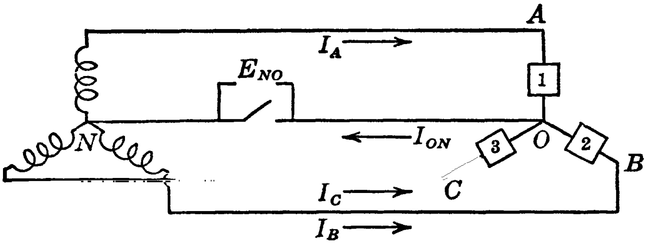


FIG. 6.13. Unbalanced Y-connected load.

$$I_{NO} = -I_{ON} = E_{NO}\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] \tag{6.15}$$

The voltage across any leg, E_{AO}, for example, is no longer the line-to-neutral voltage maintained with the switch closed, but is given by the equation

$$E_{AO} = E_{AN} + E_{NO}$$

Likewise,

$$E_{BO} = E_{BN} + E_{NO} \tag{6.16}$$

and

$$E_{CO} = E_{CN} + E_{NO}$$

These quantities are shown in the vector diagram of Figure 6.15, a general case.

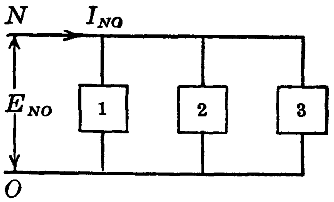


FIG. 6.14. Determination of the voltage E_{ON} of Fig. 6.13.

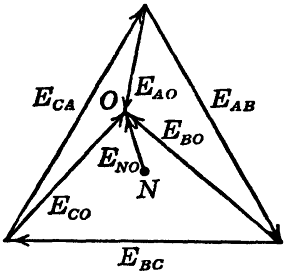


FIG. 6.15. Location of neutral in unbalanced Y-circuit.

The determination of E_{NO} locates the point O on the diagram, since N is fixed at the geometric center of the triangle of line voltages, and O is displaced from N by the vector E_{NO}. All leg voltages of the unbalanced Y system then become known immediately, and all leg currents can be readily calculated, together with the corresponding values of power and reactive volt-amperes. The point O, often called the floating neutral because its position is altered with a change in the impedance of any leg, may be inside or outside the triangle of line voltages.

The unbalanced Y circuit also can be solved in a different way using complex quantities only. From Figure 6.13 we may write

$$\begin{aligned} E_{AB} &= E_{AO} - E_{BO} \\ &= I_A Z_1 - I_B Z_2 \\ E_{BC} &= I_B Z_2 - I_C Z_3 \\ E_{CA} &= I_C Z_3 - I_A Z_1 \\ I_A + I_B + I_C &= 0 \end{aligned} \tag{6.17}$$

All voltages, currents, and impedances are expressed by complex numbers. These equations constitute a set of three (any two of the voltage equations give the third, because $E_{AB} + E_{BC} + E_{CA} = 0$) from which the leg currents can be computed. The leg voltages and total power then follow.

6.19. Power in the Unbalanced 3-Wire Y System. Regardless of the character or magnitude of unbalance in the circuit, the total power can be measured by using 2 wattmeters, and this will be equal to the sum of the values of I^2R for the 3 legs. As before, for wattmeters in lines A and B,

$$\begin{aligned} W_A &= E_{AC} I_A \cos (E_{AC}, I_A) \\ &= E_{AC} \times \text{projection of } I_A \text{ on } E_{AC} \\ W_B &= E_{BC} I_B \cos (E_{BC}, I_B) \\ &= E_{BC} \times \text{projection of } I_B \text{ on } E_{BC} \end{aligned} \tag{6.18}$$

$$\begin{aligned} W_A + W_B &= \text{total power in watts} \\ &= I_A{}^2 R_1 + I_B{}^2 R_2 + I_C{}^2 R_3 \text{ watts} \end{aligned} \tag{6.19}$$

where R_1, R_2, and R_3 are the respective resistances in the three legs.

The general proof for the above statements, in terms of the voltages and currents expressed symbolically, can be shown from the complete vector diagram for the circuit. Purely from the geometry of the vector diagram it can be demonstrated that

$$\begin{aligned} W_A + W_B &= (E_{AO} \times \text{projection of } I_A \text{ on } E_{AO}) + (E_{BO} \times \text{projection of } I_B \text{ on } \\ &\qquad E_{BO}) + (E_{CO} \times \text{projection of } I_C \text{ on } E_{CO}) \\ &= W_A + W_C \\ &= W_B + W_C \end{aligned} \tag{6.20}$$

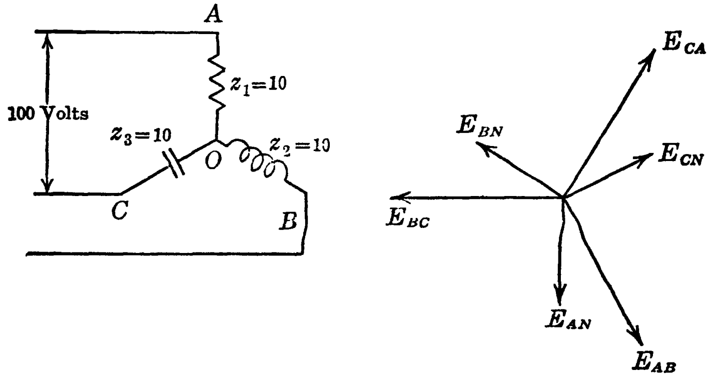
The proof is suggested as a problem assignment for the student.

In measuring power with 2 wattmeters, there will be 3 pairs of readings, depending upon the choice of the 2 lines in which the instruments are to be placed. In general, no 2 wattmeter readings of the 6 possible will be the same for an unbalanced system, although each of the proper pair of readings will show the same total power. The power taken by the system is fixed by the leg impedances and the line voltages and is given by any two wattmeters connected correctly.

In general, with any unbalanced system, reversed voltage sequence will result in three entirely different readings for the line currents. Also, in general, all 6 new possible wattmeter readings will be different from each other and, in turn, different from the 6 possible wattmeter readings for standard phase sequence. Taken in proper pairs, however, the 6 possible combinations of wattmeter readings for the 2 sequences of voltage will give the same total power. This is equal to ΣI^2R for the legs. The total circuit power is, of course, independent of voltage sequence if the same voltages are applied to the same impedances.

Any unbalanced circuit, Y or delta, therefore, may require either one of two solutions. In an actual case, for instance on the laboratory floor, if voltage sequence is not ascertained, there is an even chance that a single solution carried out beforehand may agree with the instrument readings later found.

6.20. Example on the Unbalanced Y System. Figure 6.16a shows an unbalanced set of impedances connected in Y to which are applied balanced line voltages. The impedances are given by their ohmic values and all the figures have been chosen as numbers easily handled in order to show the method of solution with a minimum of numerical computation. The same analysis will hold for other impedances and for unbalanced voltages applied to them.



(a) (b)

FIG. 6.16. Example of an unbalanced Y-circuit.

Solving the problem first with a neutral line, *ON*, assumed present,

$$
\begin{aligned}
I_{ON} &= \frac{E_{AN}}{Z_1} + \frac{E_{BN}}{Z_2} + \frac{E_{CN}}{Z_3} \\
&= \frac{-j57.7}{10} + \frac{-50 + j28.85}{j10} + \frac{50 + j28.85}{-j10} \\
&= j4\ 23 \text{ amperes}
\end{aligned}
$$

where the complex magnitudes of the leg voltages have been determined from the voltage diagram of Figure 6.16b.

Now

$$
\begin{aligned}
-I_{ON} &= E_{NO}\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] \\
-j4.23 &= E_{NO}\left[\frac{1}{10} + \frac{1}{j10} + \frac{1}{-j10}\right] \\
&= 0.1E_{NO}
\end{aligned}
$$

whence

$$E_{NO} = -j42.3 \text{ volts}$$

The leg voltages thus become, with the neutral line open,

$$
\begin{aligned}
E_{AO} &= E_{AN} + E_{NO} \\
&= -j57.7 - j42.3 \\
&= -j100 \\
E_{BO} &= -50 - j13.45 \\
E_{CO} &= 50 - j13.45 \text{ volts}
\end{aligned}
$$

The leg (and line) currents are

$$
\begin{aligned}
I_A &= \frac{E_{AO}}{Z_1} = \frac{-j100}{10} = -j10 \\
I_B &= \frac{E_{BO}}{Z_2} = -1.34 + j5 \\
I_C &= \frac{E_{CO}}{Z_3} = 1.34 + j5 \text{ amperes}
\end{aligned}
$$

The total power will be measured by wattmeters placed in any 2 of the 3 lines, and it can be calculated either by the graphical method, by means of complex numbers, or as the sum of I^2R for the 3 legs. The latter usually is the most direct and for this problem shows 1,000 watts immediately, all the power being consumed in the 10-ohm resistance of the one leg. The student can show that the other methods give the same result.

It should be stressed that, although the solution to this problem is shown by way of complex numbers, each step and the entire analysis can be carried out graphically. The second form of solution, following directly, in which impedance drops of adjacent legs are added to give line voltages, permits of analysis only in terms of complex numbers.

The second solution is carried through as follows: Referring to Figure 6.16a and equations 6.17

$$
\begin{aligned}
E_{BC} &= -100 + j0 = I_B(j10) - I_C(-j10) \\
E_{AB} &= 50 - j86.6 = I_A(10) - I_B(j10) \\
E_{CA} &= 50 + j86.6 = I_C(-j10) - I_A(10) \\
& I_A + I_B + I_C = 0
\end{aligned}
$$

Solving these equations for the leg currents, we have,

$$
\begin{aligned}
I_A &= -j10 \\
I_B &= -1.34 + j5 \\
I_C &= 1.34 + j5 \text{ amperes}
\end{aligned}
$$

from which the leg voltages are found to be

$$
\begin{aligned}
E_{AO} &= I_{AO}Z_{AO} = -j100 \\
E_{BO} &= -50 - j13.4 \\
E_{CO} &= 50 - j13.4 \text{ volts}
\end{aligned}
$$

The voltage diagram for the circuit is shown in Figure 6.17. The floating neutral O now is outside the triangle of line voltages.

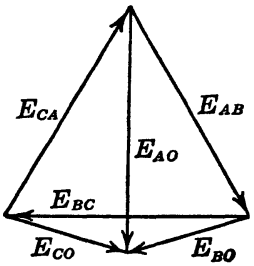


FIG. 6.17. Vector diagram for the unbalanced Y-circuit of Fig. 6.16.

6.21. Example on the Unbalanced Delta System. A delta system, unbalanced in every respect, is shown in Figure 6.18. The numbers opposite the circuit elements are ohmic values, and the graphical solution is given. It might be said that measurement of circuit constants, voltages, currents, etc., in practice may not be sufficiently accurate, nor such accurate results desired in general, to require a solution better than good graphics afford. Further, the graphical method, when it can be used, gives a better visualization of the problem and the steps in solution.

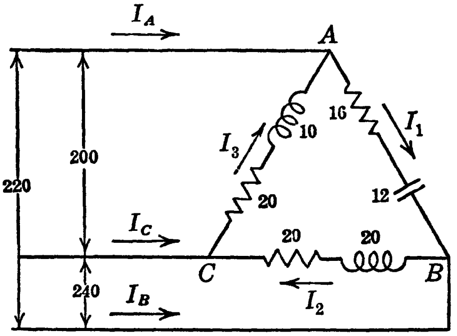
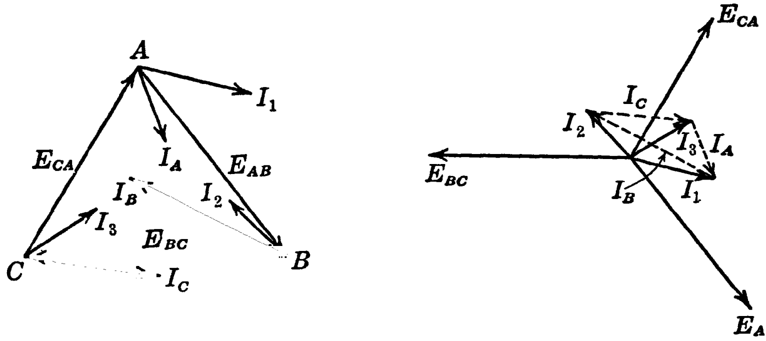


FIG. 6.18. A completely unbalanced delta circuit.

Using E_{BC} as the reference and base of the triangle of vector voltages, the point A may be found as the intersection of the arcs of 220 volts and 200 volts swung about the points B and C, respectively, Figure 6.19a. This is merely a statement of the fact that the three line voltages form a closed system. Any convenient voltage scale may be chosen. Figure 6.19b shows the corresponding star array of vectors. The leg currents are



(a) Delta form of diagram (b) Star form of diagram

FIG. 6.19. Vector diagram for the unbalanced delta circuit of Fig. 6.18.

$$I_1 = \frac{220}{\sqrt{(16)^2 + (12)^2}} = 11 \text{ amperes, leading } E_{AB} \text{ by the angle } \tan^{-1}\frac{12}{16}$$

$$I_2 = \frac{240}{\sqrt{(20)^2 + (20)^2}} = 8.49 \text{ amperes, lagging } E_{BC} \text{ by the angle } \tan^{-1}\frac{20}{20}$$

$$I_3 = \frac{200}{\sqrt{(20)^2 + (10)^2}} = 8.95 \text{ amperes, lagging } E_{CA} \text{ by the angle } \tan^{-1}\frac{10}{20}$$

The actual phase angles in degrees between any quantities need not be known, only their trigonometric values. These functions locate the positions of the currents with respect to their corresponding voltages, and in the end the projections of line currents on line voltages determine the circuit power. For example, the position of I_1 is found by going along the voltage E_{AB} 16 units from A and out from the delta 12 units. Any magnitude of unit may be chosen so long as the two measurements are in the ratio 16/12. To a convenient current scale, I_1 is located in magnitude by measuring 11 units along this construction line and, therefore, is completely established. The same scheme determines the other two leg currents on the diagram.

The line currents next are found by combining the leg currents in pairs. For example, I_A is obtained by shifting I_3 parallel to itself and adding its negative, $-I_3$, to the tip of I_1, Figure 6.19. Scaling the 3 line currents thus to be found in this manner, we have

$$I_A = 8.2 \text{ amperes},\quad I_B = 18.8 \text{ amperes},\quad \text{and}\quad I_C = 13.6 \text{ amperes}$$

Wattmeters placed in lines A and B will give the following readings:

$$\begin{aligned} W_A &= E_{AC} \times \text{projection of } I_A \text{ on } E_{AC} \\ &= 200 \times 4.9 = 980 \text{ watts} \\ W_B &= E_{BC} \times \text{projection of } I_B \text{ on } E_{BC} \\ &= 240 \times 16.7 = 4{,}010 \text{ watts} \\ W_A + W_B &= 4{,}990 \text{ watts} \end{aligned}$$

The total circuit power calculated as the sum of the power taken by the three legs is

$$\Sigma I^2R = (11)^2 \times 16 + (8.49)^2 \times 20 + (8.95)^2 \times 20 = 4{,}980 \text{ watts}$$

This latter calculation always gives the total power most readily and is the best check on the results of other methods. The student may carry out the solutions for the individual wattmeter readings and the total power for wattmeters placed in lines A and C, as well as in lines B and C.

It will be seen that the discrepancy between the numerical results for power calculated by the two different methods is about 0.2 per cent. This is well within usual required accuracy since, as stated before, measurement of circuit voltages and impedances may not be so exact as to demand refinements in subsequent computations. Again, calculations always are limited to some extent by the method of analysis used, and both graphical solutions and calculations by use of the slide-rule are subject to slight errors. Care exercised in these methods, however, should readily permit accuracy within an acceptable range.

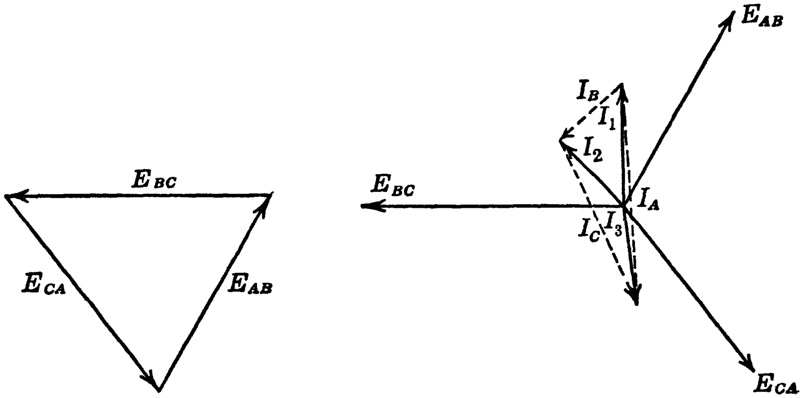
Although the entire discussion on the unbalanced delta circuit has been carried through for a specific example, nevertheless the methods are general and apply exactly to any other unbalanced delta circuit.

6.22. Examples on Reversed Voltage Sequence. Referring to Section 6.20, for reversed voltage sequence we may treat the problem by assuming any two leg voltages to be interchanged. For example, let E_{BN} and E_{CN} of Figure 6.16 exchange positions in the vector diagram. Two of the line voltages also will have different positions. Note that the respective leg impedances are not to change, and that the lettering on the connection diagram of Figure 6.16a is not to be altered. A leading current is still ahead of its driving voltage in the counterclockwise direction. Consequently,

$$
\begin{aligned}
I_{ON} &= \frac{E_{AN}}{Z_1} + \frac{E_{BN}}{Z_2} + \frac{E_{CN}}{Z_3} \\
&= \frac{-j57.7}{10} + \frac{50 + j28.85}{+j10} + \frac{-50 + j28.85}{-j10} \\
&= -j15.77 \text{ amperes}
\end{aligned}
$$

The student is asked to carry through the solution from this point to obtain the new leg voltages and currents. The equivalent impedance of the parallel legs is the same as before.

In a similar manner, for reversed phase sequence on the unbalanced delta system of Figure 6.18, the new diagram for the voltages is shown in Figure 6.20.



(a) Triangle of line voltages (b) Star form of diagram

Fig. 6.20. Solution of the circuit of Fig. 6.18 with reversed voltage sequence.

E_{AB} and E_{CA} have been interchanged arbitrarily in this case. The magnitudes of the leg currents do not change, and they have the same angular positions with respect to their corresponding voltages. Figure 6.20b shows the complete vector diagram for the leg and line currents. Thus, in magnitude,

$$
\begin{aligned}
I_A &= I_1 - I_3 = 19.8 \text{ amperes} \\
I_B &= I_2 - I_1 = 8.1 \text{ amperes} \\
I_C &= I_3 - I_2 = 16.4 \text{ amperes}
\end{aligned}
$$

Wattmeters placed in lines A and B give

$$\begin{aligned} W_A &= E_{AC} \times \text{projection of } I_A \text{ on } E_{AC} \\ &= 200 \times 17.30 = 3{,}460 \text{ watts} \\ W_B &= E_{BC} \times \text{projection of } I_B \text{ on } E_{BC} \\ &= 240 \times 6.35 = 1{,}524 \text{ watts} \\ W_A + W_B &= 4{,}984 \text{ watts} \end{aligned}$$

The student should continue the problem from this point to obtain the other pairs of wattmeter readings. As pointed out earlier, the problem can be treated also under reversed voltage sequence by changing the convention for leading and lagging power factor and without an interchange of vectors in the voltage diagram. Note, however, that with unbalanced line voltages, as in the present example, the leg currents and total power will not remain the same if reversed sequence is achieved by interchanging any two line leads. In that case the delta impedances have different voltages impressed on them.

6.23. Combined 3-Phase and Single-phase Loads. Single-phase loads often are taken from two of the lines supplying power to 3-phase circuits. If the system is 4-wire Y, single-phase loads then may also be connected line-to-neutral. Viewed from the supply terminals, the over-all circuit is generally unbalanced, at least to some extent, and the problem may be that of any such system, that is, the calculation of line currents, total power, the readings on individual wattmeters, etc.

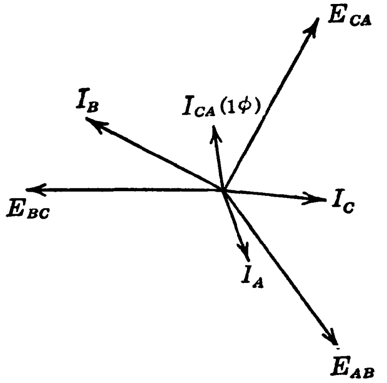


FIG. 6.21. Combined 3-phase load of Fig. 6.18 and a single-phase load.

By way of a single outlined example, let a single-phase impedance of $20 - j15$ be connected between lines A and C of the circuit shown in Figure 6.18, sequence A–B–C to remain. To the currents already computed for lines A and C, there now must be added the single-phase current $I_{CA}(1\phi) = 8$ amp, at leading power factor with respect to the voltage E_{CA}. The new line currents, Figure 6.21, are

$$\begin{aligned} I_A &= I_1 - I_3 - I_{CA}(1\phi) = I_A(3\phi) - I_{CA}(1\phi) \\ I_B &= I_2 - I_1, \text{ as before} \\ I_C &= I_3 - I_2 + I_{CA}(1\phi) = I_C(3\phi) + I_{CA}(1\phi) \end{aligned}$$

Figure 6.21 is basically Figure 6.19, but with the vectors now drawn from a common point. The entire problem can be handled easily by graphics to obtain the magnitudes of the resultant line currents and the readings of individual wattmeters. The total power in watts is that already calculated for the unbalanced delta circuit plus $8^2 \times 20 = 1{,}280$ watts for the added single-phase load. The total reactive volt-amperes are simply ΣI^2X for the 4 branches of the circuit.

In like manner, combined single-phase loads of any number and 3-phase loads of any number and character can be handled according to the general scheme just outlined. The analysis of any polyphase system is simply the calculation on single-phase circuits in combination, and any others added are treated by the same principles. Note also that in the above example the 2 impedances in parallel between lines A and C could have been combined to give a single impedance, thus leading to a new unbalanced delta circuit.

6.24. General Equivalence of Y and Delta Systems. A balanced or unbalanced set of impedances in Y may be represented by an equivalent set of impedances in delta, and vice versa. Such equivalent systems yield the same line currents, the same phase relations between line quantities, and the same power. It is understood that the line voltages between respective terminals are to remain the same. The equivalent system is useful in breaking down a complicated network, single-phase or polyphase, into a much simpler one, and in treating Y and delta loads in parallel. The equivalent Y circuit may be of advantage when voltage drops across line impedances between load and supply must be considered, and the equivalent Y circuit for a 3-phase machine usually leads to a simpler analysis.

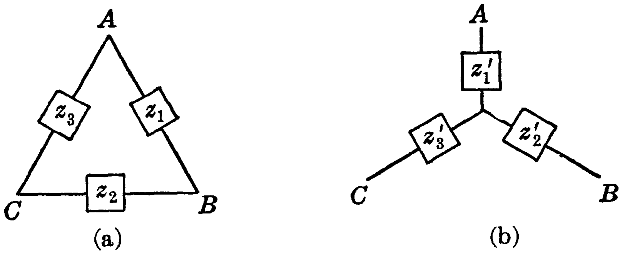


FIG. 6.22. Equivalent delta and Y systems.

Referring to Figures 6.22a and 6.22b, which are to represent equivalent systems,

$$Z_{AB} = \frac{Z_1(Z_2 + Z_3)}{Z_1 + Z_2 + Z_3} = Z_1' + Z_2'$$

$$Z_{BC} = \frac{Z_2(Z_1 + Z_3)}{Z_1 + Z_2 + Z_3} = Z_2' + Z_3' \tag{6.21}$$

$$Z_{CA} = \frac{Z_3(Z_1 + Z_2)}{Z_1 + Z_2 + Z_3} = Z_3' + Z_1'$$

All impedances are to be given by complex numbers. These three equations can be solved for Z_1, Z_2, and Z_3 in terms of Z_1', Z_2', and Z_3', or can be solved for the latter three impedances in terms of the first. The results are

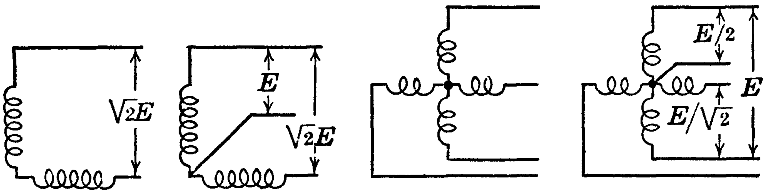
$$\begin{gathered} Z_1 = \frac{Z_1'Z_2' + Z_2'Z_3' + Z_3'Z_1'}{Z_3'} = \frac{Z'}{Z_3'} \\ Z_2 = \frac{Z'}{Z_1'}, \quad Z_3 = \frac{Z'}{Z_2'} \end{gathered} \tag{6.22}$$

and

$$\begin{gathered} Z_1' = \frac{Z_1Z_3}{Z_1 + Z_2 + Z_3} = \frac{Z_1Z_3}{Z} \\ Z_2' = \frac{Z_1Z_2}{Z}, \quad Z_3' = \frac{Z_2Z_3}{Z} \end{gathered} \tag{6.23}$$

For balanced systems of impedances, a delta circuit and a Y circuit are equivalent when the impedance of the delta leg is three times that of the Y leg.

6.25. The 2-Phase System. Although the first concern is for the 3-phase system, a few words may be said with respect to the 2-phase (and quarter-phase) system. If the total coils on a generator are grouped into two parts of equal number, and these winding groups placed on the armature at 90 electrical degrees from each other, they will produce voltages which are 90 electrical degrees apart in time. It is possible to bring the four separate leads for the two phase groups from the machine and have two distinct 1-phase systems. The usual practice, however, is to tie the phases together and as a result have a 2-phase system of 2, 3, 4, or 5 wires as shown in Figure 6.23.



(a) 2-phase 2-wire (b) 2-phase 3-wire (c) 2-phase 4-wire (d) 2-phase 5-wire

FIG. 6.23. Two-phase systems.

If the phase voltage for one set of coils is E, as shown in Figure 6.23a, the line-to-line voltage is $\sqrt{2}E$. In Figure 6.23b the line-to-neutral voltage is E, while the line-to-line voltage is the same as before. If the midpoints of the phases are connected as shown in Figure 6.23c, the line-to-line voltages now become $\sqrt{2}(E/2)$ and E. The voltages shown in Figure 6.23d are apparent from this discussion. Figures 6.23c and 6.23d show quarter-phase systems in that the line-to-line voltages are all equal and appear in time phase 90 electrical degrees from each other, taking the coil terminals in cyclic order. These relations on 2-phase systems are left to the student to verify.

PROBLEMS

1. Draw 3 vectors, representing effective magnitudes of voltages, of approximately the same lengths but at various angles with each other. Label each terminal of each vector and indicate an assumed positive sense of direction of voltage for each. By diagrams, and by corresponding equations, show the addition of the 3 vectors, all taken in the positive sense. Show the addition of the first and last and the subtraction of the second from this sum. Show the negative addition (subtraction) of the first two to the third. By a vector, and by subscript notation, show the negative of the resultant voltage obtained in each case.

2. Refer to Figures 6.1 and 6.2. When e_{AN} is maximum and is driving a current from A to N in a load circuit, what are the magnitudes and directions of the other two leg voltages? If current in line A is increasing and is 70.7% of its maximum positive value, what are the magnitudes and directions of the currents in the other two lines? Assume the three leg impedances to be identical.

3. Refer to Figure 6.1. Three oscillograph elements are connected to measure the three line voltages but the connections to the element between lines A and C are inadvertently reversed with respect to the cyclic order shown by the voltage waves. Sketch the voltage waves as they would appear on the oscillograph screen, or on a film, labeling each wave.

4. A Y-connected generator delivers 7.5 kva at 220 volts, 0.8 pf lagging. Calculate the in-phase and out-of-phase components of the line currents. Draw the complete vector diagram showing all line and leg voltages and currents.

5. A balanced Y load of 30 ohms, 0.866 pf lagging, per leg is connected to a 220-volt, 3-phase supply. Determine the power taken by the load.

6. At full load, a 40-hp motor takes 34.5 kva at 1.0 pf on 440 volts. Calculate the line and leg voltages and currents if the machine is Y-connected; if it is delta-connected. Draw the complete vector diagram for each case.

7. A balanced delta-connected load draws 15 amp per line at 0.8 pf leading from a 220-volt, 3-phase source. Determine the impedance components in each leg of the circuit.

8. Three impedance coils are connected in Y to a 110-volt, 3-phase supply. Each coil has 6 ohms resistance and 3 ohms reactance. Three other coils connected in delta to the same supply draw the same line current and total power. Calculate the resistance and reactance of the second set of coils. Generalize from your results to give the ratio of delta leg impedance to Y leg impedance for equivalent balanced 3-phase systems. Check your conclusion by a second or third example of your own choosing.

9. Three equal impedances of 4 ohms resistance and 3 ohms capacitive reactance are connected in delta to a 220-volt supply. Draw the complete vector diagram for the system. Calculate the power taken if the impedances are connected in Y to the same lines. What power is taken, in either case, if one line is opened?

10. A set of balanced impedances in Y takes 2,650 watts at 0.85 pf leading from a 115-volt, 3-phase supply. Calculate the impedance components of each leg.

11. On a 220-volt, 3-phase supply, the wattmeter readings for a balanced Y load are $W_A = +2{,}775$ watts and $W_B = +1{,}095$ watts. What are the impedance components in each leg of the load? Phase sequence is A–B–C.

12. $W_C = +1{,}160$ watts and $W_B = +311$ watts for a balanced Y load supplied from a 120-volt, 3-phase, 60-cycle power source. For phase sequence A–B–C calculate the circuit elements in each leg.

13. A balanced delta load takes 12 amp line current from a 230-volt, 3-phase power supply. If the power factor is 0.8 leading, what are the representative circuit constants for each leg of the delta? What is the equivalent Y circuit?

14. Three capacitive reactances of 25 ohms each are connected in delta to a 220-volt 3-phase supply. For voltage sequence A–B–C, what should be the readings on wattmeters connected in lines A and C?

15. What are the equivalent delta circuits for the Y loads of problems 10, 11, and 12?

16. In measuring the power taken by a balanced delta load connected to a 230-volt, 3-phase supply of voltage sequence A–B–C, $W_A = +1{,}446$ watts and $W_C = +824$ watts. What are the equivalent circuit constants of each leg? Repeat for the load Y-connected? Answer the same questions should $W_B = +1{,}763$ watts and $W_C = -400$ watts.

17. A 150-hp, 3-phase motor takes its power from a 4,000-volt supply. The motor has an efficiency of 92%, and the power factor is 0.9 leading. What is the line current? What ohmic values of impedance connected in delta could replace the motor so as to give the same terminal measurements? Repeat for impedances connected in Y. For voltage sequence A–B–C, what will be the readings of W_A and W_C?

18. A balanced delta load takes 20 amp per terminal from a 220-volt supply. The leg pf is 0.8 lagging. A Y-connected load on the same lines takes 15 amp per terminal at 0.707 pf leading. What is the line current, what is the power factor, and what is the power for these combined parallel loads? What single set of impedances in Y could replace the combination so as to give the same over-all instrument readings?

19. A balanced set of impedances in Y, $Z = 12 + j8$ ohms per leg, is in parallel with a balanced delta set of impedances having $Z = 20 - j10$ ohms per leg. On a 220-volt supply, what line current is drawn by the combination, and what is the total power?

20. Should all 4 loads given in problems 18 and 19 be in parallel, what is the simplest set of balanced impedances in Y that can represent the over-all terminal measurements?

21. A factory takes 100 kw at 1.0 pf, 200 kva at 0.8 pf lagging, 150 kw at 0.9 pf leading, and 25 kva at zero pf leading from a 440-volt supply. What is the power factor at the input to the factory, and what is the total power?

22. A plant has the following connected loads:

(a) 80 kw at 1.0 pf.
(b) 100 kva in motors, 0.8 pf leading.
(c) 300 hp in motors, 0.85 pf lagging and 86% efficiency.
(d) 250 kva at 0.75 pf lagging.

Determine the total plant load and power factor. It is desired to add a load of 50 kw at 0.8 pf leading. What kva in static condensers also must be added so that the supply transformers will not be overloaded if they are carrying full kva initially?

23. A 3-phase, 350-hp, 2,300-volt motor having an efficiency of 92% and operating at 0.6 pf lagging is in parallel with a motor taking 100 amp per terminal at 0.8 pf leading. Determine the line current to the combination and the resultant power factor.

24. An industrial load on a 2,300-volt, 3-phase supply takes 60 kw at 0.8 pf lagging, 20 kva at 1.0 pf, 30 hp at 85% efficiency and 0.9 pf lagging, and 20 hp at 90% efficiency and 0.75 pf leading. Calculate the total power, the kva, and the over-all power factor. What equivalent resistance and reactance per leg, delta-connected, could represent the combined loads?

25. The present load of a factory is 266 kva and 200 kw, lagging power factor. A 50-hp motor, 90% efficient, 1.0 pf, and a 40-hp motor, 88% efficient, 0.8 pf leading, are to be added. With these motors installed, what is the over-all power factor of the plant? At $10 per kva of capacitors, what is the cost of raising the power factor of the plant to 95%?

26. A balanced Y load draws 40 kw at 0.8 pf lagging at a terminal voltage of 440 volts. What are the equivalent ohmic values of the leg impedances? If the power is supplied over lines each having an impedance of $0.15 + j0.25$ ohms, what are the voltage, power, and power factor at the input terminals of the lines? What is the efficiency of transmission for the system?

27. A balanced delta load has 220 volts applied to it and draws 7 amp at 0.707 pf lagging. If the supply lines have 3 ohms resistance and 6 ohms inductive reactance each, what is the voltage at their input terminals? What power is taken by the load, and what is lost in the lines? Calculate the pf at the input terminals of the lines. Recalculate the problem for a line impedance of $3 - j6$ ohms.

28. A 200-hp, 3-phase motor, 93% efficient, takes 47 amp per terminal at 2,300 volts when carrying full load. If the power is supplied over lines each having an impedance of $1 + j3$ ohms, what are the voltage, power, and power factor at the input terminals of the lines?

29. A Y load takes 15 kw, 0.8 pf lagging at 230 volts. A parallel delta load draws 20 kva, 0.707 pf leading. Each line supplying the total power has an impedance of $0.3 + j0.6$ ohms. What are the voltage, power, and power factor at the input terminals of the supply lines?

30. A balanced delta load has a leg impedance of $54 + j36$ ohms. A parallel motor load takes 11.4 kva at leading power factor for 9.5 hp output at 88% efficiency. If the load voltage is 440 volts, and each supply line has an impedance of $0.7 + j2.4$ ohms, what are the voltage, power, and power factor at the input terminals of the supply lines?

31. At the input terminals of a supply system the voltage is 220 volts, the current is 12.7 amp, and the power is 3.87 kw, leading power factor. Each line to the load has an impedance of $2 + j5$ ohms. What are the impedance elements of each leg of the delta-connected load?

32. A 3-phase motor takes 2.46 kw at 0.4 pf lagging from a 220-volt supply. What should be the wattmeter readings W_A and W_B if the voltage sequence is A–B–C? If the voltage sequence is A–C–B?

33. The power taken by a balanced inductive load is shown by wattmeter readings of +930 and +1,525 watts for the 2 instruments. If the voltage sequence is A–C–B, associate the position of the instruments with the corresponding indications.

34. In making measurements on a 3-phase, 230-volt load containing resistance and capacitance, $W_A = 4.62$ kw and $W_B = 2.95$ kw. What is the load current, power factor, and voltage sequence?

35. From the results of preceding problems, generalize your conclusions with respect to wattmeter readings, voltage sequence, and character of power factor (leading or lagging). Is it possible for 2 wattmeters to give information on both voltage sequence and character of power factor? Will a polyphase wattmeter lend itself to the determination of either voltage sequence or character of power factor?

36. The power factor of a 3-phase motor is to be adjusted to 0.8 leading for an input of 20 kw. What will be the wattmeter readings if two are used to determine the power and the power factor?

37. Draw the connection diagrams for the following arrangements in measuring power taken by a 3-phase load, showing all polarity marks:

(a) 2 wattmeters, potential coils connected directly to the lines but current coils connected to the secondaries of current transformers.
(b) 2 wattmeters, potential coils with multipliers, current coils carrying line currents.
(c) 2 wattmeters, both potential transformers and current transformers used in making power measurements.
(d) A polyphase wattmeter with both potential and current transformers.

38. Two wattmeters are used to measure the power taken by a 3-phase load. Each instrument is rated 30/60 amp, 100/200 volts with 50% overload capacity indefinitely for both potential and current coils. Full scale for each wattmeter is 30 hektowatts for the 30-amp and 100-volt ranges. On a 449-volt supply, the instrument readings from the scales are +29.3 and +11.7 hektowatts, respectively. What power is being meas-

ured if the 60-amp current terminals and 600-volt multipliers are used? What is the line current?

39. Two wattmeters having ratings given in problem 38 are employed to measure the power taken by a 3-phase synchronous motor drawing 19.3 amp at 0.8 pf lagging from a 545-volt supply. How would you connect these instruments, and what should be their scale readings?

40. A polyphase wattmeter is used in measuring the power taken by a 3-phase, 4,000-volt motor. The instrument has a maximum rating of 5/10 amp, 150/300 volts, and full scale is 10 hektowatts for the lower ranges of current and voltage. If the instrument indicates 4.15 hektowatts for connections to the 5-amp current coils through 40:1 current transformers, and with 20:1 potential transformers ahead of the 300-volt potential coils, what current is being taken by the motor? Assume 86% pf.

41. Ahead of a balanced delta load of leg impedance $8 - j3$ ohms on a 220-volt supply is a single-phase wattmeter with current coil in line C and potential circuit between lines A and B. The current terminal having the polarity mark is on the supply side of the instrument, and the marked potential terminal is connected to line B. What reading is obtained from the wattmeter, taking into account any multiplying factors for its scale indication? The voltage sequence is A–B–C. If the voltage sequence is reversed, what reconnection must be made for the instrument, and what reading results?

42. Using 2 current transformers and 3 ammeters, sketch the connection diagram for the measurement of the line currents in a 3-phase, 3-wire circuit.

43. Show that for a balanced 3-phase load

$$\cos\theta = \frac{1}{\sqrt{1 + 3(W_A - W_B)^2/(W_A + W_B)^2}}.$$

44. A 3-phase, 4-wire system supplies 120 amp per terminal to a balanced delta-connected load having a power factor of 0.8 leading. Three 1.0-pf loads between the main lines and the neutral take 60, 80, and 75 amp, respectively. Calculate the current in each of the 4 lines. Obtain solutions for both voltage sequence A–B–C and A–C–B.

45. Three impedances connected in star on a 230-volt, 4-wire, 3-phase supply have the respective magnitudes $15 + j7$, $20 - j12$, and $14 + j5$ ohms. What is the current in the neutral line for voltage sequence A–B–C? For sequence A–C–B? Calculate the total power.

46. On a 100-volt, 3-phase, 4-wire system, 3 impedances in Y have the magnitudes: $Z_1 = 5.77 + j0$, $Z_2 = 5 + j2.885$, and $Z_3 = 2.885 - j5$. What current is carried by the neutral wire under voltage sequence A–B–C? With sequence A–C–B? Calculate the total power.

47. On a 100-volt, 3-phase, 4-wire supply $Z_2 = 8.66 - j5$, and $Z_3 = 8.66 + j5$ ohms. What must be Z_1 if the neutral current is zero for sequence A—B—C?

48. Three impedances, $Z_1 = 4.33 - j2.5$, $Z_2 = 5.77 + j0$, and $Z_3 = 8.66 - j5$ ohms, are connected in star on a 220-volt, 3-phase supply. What voltage is read across an open switch in the neutral wire if the voltage sequence is A–B–C? Draw the complete vector diagram for this circuit. What power should be indicated by wattmeters in lines A and C?

49. Repeat problem 48 for voltage sequence A–C–B. The neutral wire is to be open in obtaining wattmeter readings.

50. Solve the example shown in Figure 6.16 of the book but with $Z_1 = 17.32 + j0$. What conclusions do you draw from your solution? (This is a practical circuit. It is used on the University of California campus to supply several hundred kw d-c excitation, through transformers and rectifiers, to the field coils of a large cyclotron.)

51. A phase sequence indicator, Y-connected, has a pure inductive reactance of 1,000 ohms between terminal A and the common junction O, and two 1,000-ohm lamps of resistance only in the other two legs. For voltage sequence *A–B–C*, which lamp burns brighter? For sequence *A–C–B*? Repeat the problem for a 1,000-ohm capacitor replacing the inductor. Assume a 220-volt, 3-phase supply.

52. Three impedances in Y (see Figure 6.13) have the respective values $Z_1 = 10 + j0$, $Z_2 = 17.32 + j10$, and $Z_3 = 10 - j10$ ohms. If a neutral line is present, calculate all line currents and the total power for $100\sqrt{3}$ volts between lines, sequence *A–B–C*. Calculate the leg voltages and currents, and the total power, if the neutral line is opened.

53. On a 440-volt, 4-wire supply, 3 loads in Y take, respectively, 9.12 kw at 0.707 pf leading, 12.9 kw at 1.0 pf, and 10.32 kw at 0.8 pf lagging. Voltage sequence is *A–B–C*. What is the neutral line current? What power is taken by each leg if the neutral line is opened?

54. Refer to Figure 6.18. The currents I_1, I_2, and I_3 flow through impedances of $20 + j10$, $10 + j10$, and $10 + j20$ ohms, respectively. On 110 volts, sequence *A–B–C*, what will be the readings of wattmeters placed in lines *A* and *B*? Repeat for sequence *A–C–B*.

55. Show that for any delta circuit, balanced or unbalanced, the line currents can be shown by the delta set of closing vectors of the leg currents if these are drawn in star array.

56. Refer to Figure 6.18. Let the three currents meet, respectively, the impedances: $8.66 + j5$, $10 + j0$, and $8.66 - j5$ ohms. On a 200-volt supply, sequence *A–B–C*, determine the readings of wattmeters placed in lines *A* and *B*, in lines *A* and *C*, in lines *B* and *C*. Repeat for sequence *A–C–B*.

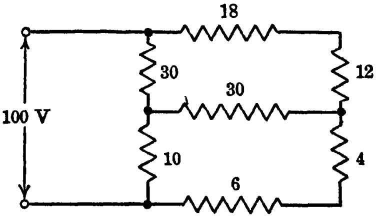
57. Refer to Figure 6.18. $E_{AB} = 200$ volts applied to $12 + j5$ ohms; $E_{BC} = 220$ volts applied to $10 - j10$ ohms; and $E_{CA} = 250$ volts applied to $10 + j0$ ohms. What will be the readings of wattmeters placed in lines *A* and *B* if the sequence is *A–B–C*? If the sequence is *A–C–B*?

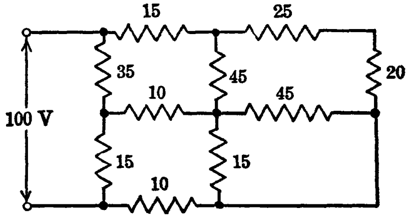
58. Three impedances, $Z_1 = 10 - j10$, $Z_2 = 20 + j0$, and $Z_3 = 12 + j5$ are connected in delta on a 3-phase supply. Calculate the corresponding values of the leg impedances for the equivalent Y system.

59. Calculate the leg impedances of the equivalent delta circuits using the Y values for the sequence indicators of problem 51.

60. Four resistors of 15 ohms each form a square, and 15-ohm resistors also are connected between the diagonal points. By means of the Y-delta transformation, compute the power taken by the network if 150 volts is applied between 2 adjacent terminals.

61. Calculate the line currents for the resistance networks shown.





62. The voltage between each main line and neutral of a 3-wire, 2-phase supply is 220 volts. One phase carries 20 kw at 1.0 pf, the other phase carries 16 kw at 0.8 pf lagging, and between the main lines there is a 30-kw, 0.8-pf lagging load. Compute the currents in the 3 lines. The voltage across the 20-kw load leads the voltage of the second phase by 90°.

63. On a 250-volt, 3-phase supply with voltage sequence *A–B–C*, a delta circuit has leg impedances $Z_{AB} = 15 - j10$, $Z_{BC} = 12.5 + j21.65$, and $Z_{CA} = 25 + j0$ ohms. Between lines *B* and *C* is a single-phase load having an impedance of $25 + j0$ ohms. What is the supply current of line *C*? Of lines *A* and *B*?

64. Between lines A and B, and B and C, of a 3-phase supply feeding an unbalanced Y load are single-phase impedances. Using double-subscript notation, write the general equations for the three line currents in terms of their branch currents. Repeat for the same connections of the single-phase loads, but with an unbalanced delta load replacing the Y system.

65. On a 200-volt, 3-phase supply of voltage sequence A–B–C, a delta load has impedances $Z_{AB} = 8 - j6$, $Z_{BC} = 25 + j0$, and $Z_{CA} = 17.32 + j10$ ohms. Between lines B and C is a single-phase load having an impedance of $10 - j10$ ohms. The total power to the loads is measured by a polyphase wattmeter having a rating of 20/40 amp, 150/300 volts, 4 kw full scale. What should be the scale reading of the wattmeter?

CHAPTER VII

TRANSFORMER CONNECTIONS

7.1. General. Our interest in the single-phase transformer centered around the fact that it was a very important link in the single-phase system and was more than just a machine. For that reason it was discussed before taking up the polyphase system. In exactly the same way, the polyphase transformer or groups of single-phase transformers are major elements in the transmission of power over a polyphase network.

Of first concern is the function of the 3-phase transformer or of the 3-phase bank of single-phase machines in raising or lowering the system voltage. While many ways of connecting units may be devised, only a few, because of their practicability, are of commercial importance. Determining factors are simplicity, cost, reliability, efficiency, and safety.

Secondly, banks of transformers are used to convert from one number of phases to another number of phases. Three- and 2-phase systems are mutually convertible, for example, and from 3-phase circuits are obtained 6-phase, 12-phase, etc., circuits for the operation of synchronous converters and mercury-arc rectifiers. However, although polyphase power of one number of phases can be converted to polyphase power of another number of phases or can supply single-phase power, it is not feasible to draw polyphase power from a single-phase supply by means of transformer connections.

The usual manner of connecting the primary and secondary windings of single-phase transformers comprising a 3-phase bank follows the Y- and delta-circuit arrangements discussed in the preceding chapter. Of importance are the bank requirements for balanced loads, the effect on load capacity of the open-delta connection, the effects of single-phase loads on a 3-phase bank, the effect of the neutral line for a Y-connected set of windings, the presence of third-harmonic voltages and currents in 3-phase banks, the transformation of polyphase power, and the parallel operation of polyphase banks.

7.2. Three-phase Transformers. Although our attention is directed mainly toward banks of single-phase transformers connected to transform 3-phase power, brief mention might be made of the 3-phase transformer itself. The magnetic circuit comprises 3 vertical arms which are connected at top and bottom for the returning flux, the flux of any one leg returning through the other two. Each arm carries the primary and secondary windings of one phase, each set of high-voltage and of low-voltage coils being, in general, connected in one manner or another within the tank. Only 6 leads then are brought out of the case.

The 3-phase transformer thus becomes a single machine. As such, it contains less iron, weighs less, requires less space, and is a little more efficient than a bank of 3 single-phase transformers. In the larger sizes it is cheaper than 3 single-phase transformers, but the cost of spare units and of upkeep is, of course, higher. With the increase in transformer reliability that has resulted from design improvements and better protection practices, the trend is toward the use of three-phase units in the larger sizes.

As a consequence of the mutual flux paths for the 3 sets of windings on the polyphase transformer, the analysis of this machine is not so simple as that of the single-phase transformer or the polyphase bank. A discussion on the reactions within the polyphase unit, though, is beyond the scope of this book.

7.3. Third-harmonic Voltage and Current. A sinusoidal voltage impressed on any coil requires a sinusoidal flux for the equal and opposite induced voltage. Consequently, in any iron-core circuit giving rise to magnetic saturation, the exciting current cannot be sinusoidal. The current wave will have an unsymmetrical shape and will be peaked increasingly as the degree of saturation is increased. Analysis of the wave shape by Fourier series shows the third-harmonic component to be by far the strongest of the harmonics present in the wave.

In any transformer bank having saturation, either the third-harmonic voltage or the third-harmonic current will be present. The third-harmonic current will appear if a closed path is provided for it, such as in the delta connection. The third-harmonic voltage will be observed if there is no path through which it can force a current, for example, the Y-Y connection without a neutral wire. In this case a third-harmonic component exists in the line-to-neutral voltage of the bank. It should be understood that when third-harmonic current flows it only appears as part of the exciting current, I_0, and in comparison with the load component it represents but a few per cent of the total current.

Harmonics represent distortion in the circuit, and hence certain disadvantages are, in general, associated with them. They do not contribute to the power flow as does the fundamental; harmonic currents lead to local heating and hence to a decrease in kva capacity of the apparatus; harmonic voltages give rise to an oscillating neutral in a Y-Y bank; resonant conditions may yield very large third-harmonic voltages with attendant insulation stress; and, of major practical concern, third-harmonic currents produce telephone interference where power and telephone lines run parallel to each other at relatively close separation. It is usual to suppress harmonics as much as possible, although there are some instances in which a given harmonic is employed purposely.

7.4. Delta-delta Connection. In connecting 3 single-phase transformers for delta-delta operation, Figure 7.1, the primaries are closed upon themselves, and, likewise, the secondaries. The secondaries must be connected with due regard for the relative polarities of the windings; otherwise the windings might be short-circuited upon themselves. Properly connected, the instantaneous sum of the three voltages, 120 degrees apart in time, around the closed loop is zero.

As a precaution, before closing the delta secondary of a bank, a voltage reading can be taken between the 2 open ends so as to make certain that one secondary is not reversed, under which condition twice rated secondary voltage would be measured. The same precaution applies to any connection in which the transformer secondaries close a path through themselves.

The delta-delta connection is of advantage for relatively low voltages and high current. It allows third-harmonic components of the exciting current to flow in the closed paths and, hence, eliminates third-harmonic voltages between terminals. These voltages do appear between line and junction of a Y-Y bank. Transformers of different kva ratings can operate in delta-delta, and a bank still is operative if one transformer is taken out of service. On the other hand, no neutral is available for loading or grounding, high circulating currents are present if the ratios of transformation are not the same, and the division of a balanced 3-phase load is imperfect if each transformer does not have the same impedance.

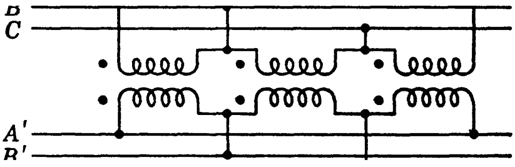


FIG. 7 1 Delta-delta transformer connections.

The resultant current through any unit of a delta-delta bank can be computed as the superposed vector sum of the individual currents due to the 3 line-to-line loads. Balanced 3-phase loading on a bank of 3 identical transformers will, of course, load the transformers equally in current, kva, and kw, the power factor of each transformer being that of the load. As is the case in the delta-load, the transformer currents are less than the line currents by the factor $\sqrt{3}$.

Much low-voltage distribution, factory, and general commercial wiring is of the delta-delta type because of the advantages stated above. Primary voltages for these circuits are in the general range of 2.4 to 12 kv, and typical secondary voltages are 480/240 and 240/120 volts. Code practice usually requires the grounding of the midpoint of one transformer for the protection of life and apparatus. Single-phase lighting loads of an appreciable number of kw are normally carried by a separate transformer and are not superposed on the nominally balanced 3-phase load of the bank.

7.5. Open-delta Connection. If one transformer of a delta-delta bank is removed, the two remaining can continue to supply 3-phase power to the secondary lines. Under load, the voltages of this open-delta bank will be slightly unbalanced because of unequal voltage regulation for the 2 transformers. The bank is economical for systems which are later to expand and where the investment for the complete bank is not desired at the present time.

The leg currents for the open-delta bank now are the line currents as well and are $\sqrt{3}$ times their former values. For safety to the individual units, then, the load on the open-delta bank should not exceed $1/\sqrt{3}$, or 57.7 per cent, of full load for the closed-delta bank. If one unit of a closed-delta bank becomes defective, the open-delta serves as a temporary means of carrying power.

Open-delta banks are used frequently. In the large central-valley agricultural region of California, for example, there are many such connections, the greater part of the output being for pump motors used for irrigation. These banks may carry a large overload over part of the daily cycle if there is the opportunity for the machines to cool at night. Much of the pumping load is at night; hence, the transformers can carry high overloads if the daytime load has not exceeded their capacity. Implied in these statements is the fact that in the initial rating of any electrical apparatus there is the consideration of the ambient temperature under which it must perform.

7.6. Y-Y Connection. Because of insulation requirements, the Y-Y bank is better suited to systems having high line voltages and relatively low currents. The arrangement provides a neutral for grounding or for line-to-neutral loads. Such loads, however, cannot be carried on the secondary unless the primary also has its neutral connected back to the power source. Without the primary neutral, a line-to-neutral load on one secondary requires load components in all primary windings, and current would have to flow through the extremely high open-circuit impedances of the other 2 transformers. The primary currents are therefore limited, and there is a large shift of the secondary neutral toward the loaded line.

The neutral also is unstable because of third-harmonic components in the exciting currents, and perhaps because of unequal magnetizing currents for the 3 units. Third-harmonic voltages exist between each line and the junction point. Circulating currents cannot flow, however, because of either the third-harmonic voltages or the unequal ratios of transformation. Unequal impedances of the transformers have little effect on load division. The Y-Y bank in practice is not operated in open delta if one transformer is removed, because the voltage stress on the other two usually is greater than that permissible.

Frequently the Y-Y bank has a delta tertiary which may serve either one or both of two purposes. It stabilizes the neutral, and it can carry a load by itself. Synchronous condensers for voltage regulation are connected in many instances to high-voltage transmission lines by means of the delta tertiary. This type of connection, with both Y-neutrals grounded, is used to greater extent than any of the many forms of the Y-Y bank because of the advantage of the delta tertiary. Its application is almost solely to high-voltage transmission systems.

7.7. Y-delta and Delta-Y Connections. This form of bank, Figure 7.2, with the neutral of the Y practically always grounded, has a stable neutral because of the delta, and good current balance is obtained even with transformers having widely different impedances. The general combination of a Y and a delta is probably the most satisfactory connection for a 3-phase bank, and with a 4-wire Y it has a wide application in low-voltage distribution systems. Its disadvantages lie in the fact that the bank is inoperative if one transformer fails or must be removed, and that the bank cannot be paralleled with a delta-delta or a Y-Y bank because of the phase difference of 30 degrees between line voltages of the banks on the secondary side.

The Y-delta bank, with the primaries connected 4-wire Y, is used to a fairly large extent for supplying factory power. Line voltages on the primary side range from 4.16 to about 14.4 kv, with low-side voltages from 120 to 600 volts. Delta-Y banks are in common use for commercial combined power and lighting service and are popular for office buildings because the over-all loading can be closely balanced. High-side voltages are in the general range from 2.4 to 14.4 kv, with secondary voltages of 208/120 volts being most common for the 4-wire Y.

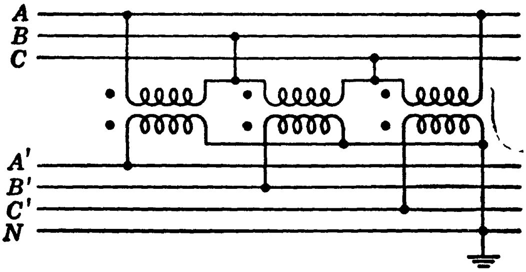


FIG. 7.2. Delta-Y or Y-delta transformer connections.

Lighting loads are connected line-to-neutral, and motor loads, elevator motors for example, are connected across the main wires. While 220- and 230-volt motors will not show the same performance on reduced voltage that they will under nameplate rating, general-purpose machines usually are built to operate satisfactorily within a voltage deviation of ±10%. Manufacturers now supply standard three-phase motors rated at 208 volts. One of the more recent approaches to supplying a combined light and power service to industrial plants and large office buildings is the 480/270 volts 4-wire Y. The main part of the lighting load is carried by 277-volt fluorescent lamps. Economic considerations favor the higher voltage system where both the motor and lighting loads are relatively high. The 120 volts required for incandescent lamps and small appliances can be supplied from small 480 to 120/240-volt single-phase transformers.

The delta-Y connection is practically always used in stepping up the voltage at the generator end of a high-voltage transmission line, while the Y-delta connection is used at the receiving end. One important reason for this choice is that the high-voltage winding need be insulated only for the line-to-neutral rather than the line-to-line voltage. In addition, since one end of the winding is either grounded directly or through a current-limiting impedance, only one high-voltage bushing need be used.

7.8. Single-phase Loads. A considerable amount of power is delivered single phase from low-voltage, 3-phase distribution systems. The single-phase power may be in addition to 3-phase loads or may form a fairly balanced 3-phase load if an approximately even distribution in loading occurs between lines. In some instances only one single-phase load is carried. A single-phase load is an unbalanced condition for the polyphase system and, consequently, leads to unequal currents in the supply lines, unbalanced voltages, a loss in efficiency, and a decrease in economy of apparatus.

It is evident at this point, from what has been said thus far, that a great many connections are possible when taking single-phase power from a 3-phase system. Figure 7.3 shows some typical connections, the transformers being indicated by heavy lines. Although the diagrams show but a single load, it is standard practice to distribute the various single-phase loads of the system among the phases so that the combination approximates a balanced 3-phase load.

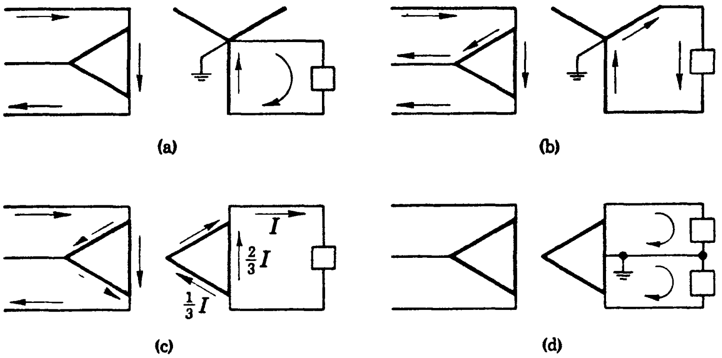


FIG. 7.3. Typical connections for taking single-phase power from a 3-phase system.

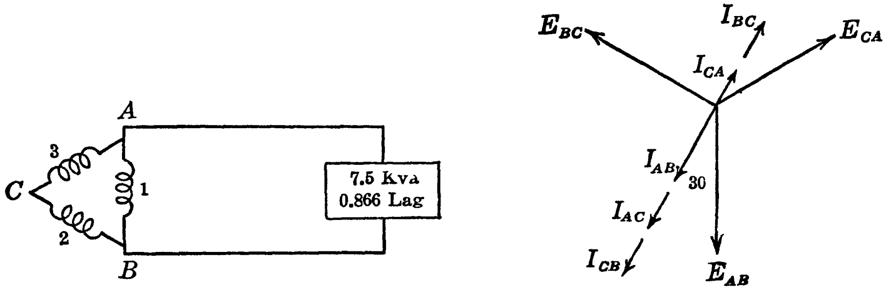
In (a) and (b), the secondary currents can flow in only one series path. With current in the secondary of any transformer, there must be a corresponding current in the primary winding. In (c), the secondaries are delta connected and the single-phase current will divide between the 2 parallel paths of the bank. For identical transformers or for 3 having equal ohmic impedances, one will carry two-thirds of the total load current and the other 2 will carry the remainder. The loading in (d) is the same as in (c) if the 2 single-phase loads are equal.

As has been pointed out, a line-to-neutral load on a secondary of a Y-Y bank cannot be carried unless both neutrals are present, or unless there is a delta tertiary to maintain the secondary neutral.

7.9. Combined 3-Phase and Single-phase Loads on Delta-delta Banks. To some extent, primarily for light commercial loads, a limited single-phase power load may be supplied by 2 lines from the secondary side of a delta-delta transformer bank. The single-phase load is in addition to the nominally balanced 3-phase load. In these instances, the transformer between the lines carrying the single-phase load is larger than the other two. Typical ratings are 2 transformers of 5 kva and a third of 7.5 or 10 kva, or 2 of 7.5 kva and a third of 10 or 15 kva. The largest transformer carries its share of the 3-phase load and, in addition, the greater part of the single-phase load. The single-phase load is usually less than 50 per cent of the 3-phase load.

Since the bank is a generator with respect to any load on it, the current, kva, and kw of each transformer can be calculated by way of the corresponding quantities supplied the load system. The vector diagram for the bank and the notation used with it will then have the same appearance as for the load itself. By this treatment there is eliminated the necessity of introducing a new set of conventions, with its ramifications, for the supply over that already adopted for the load.

As an example, Figure 7.4a shows a single-phase load on a 15-kva, 3-phase bank, primary omitted, and Figure 7.4b the corresponding current distribution with voltage sequence A–B–C. Transformer 1 carries two-thirds of the single-phase load current, and transformers 2 and 3 carry the remaining third. The magnitude of the system voltage is not necessary because any current and its associated kva are directly proportional. Thus,



(a) (b)

FIG. 7.4. One-phase load on a 3-phase transformer bank.

$$
\begin{aligned}
\text{kva}_1 &= E_{AB}I_{AB} = 5 & \text{kw}_1 &= 5\cos 30^\circ = 4.33\\
\text{kva}_2 &= E_{CB}I_{CB} = E_{BC}I_{BC} = 2.5 & \text{kw}_2 &= 2.5\cos 90^\circ = 0\\
\text{kva}_3 &= E_{AC}I_{AC} = E_{CA}I_{CA} = 2.5 & \text{kw}_3 &= 2.5\cos 30^\circ = 2.17
\end{aligned}
$$

The total power delivered by the 3 transformers = 6.5 kw = 7.5 × 0.866 and must agree with the sum of the kw's for the individual units. Note that transformer 1 has reached its full kva, and only a limited load at a power factor highly leading could be added without exceeding the rating of this unit.

Suppose, for a new condition, that transformer 1 has a rating of 10 kva, and transformers 2 and 3 each are rated 7.5 kva. Assume, also, that all 3 have the same ohmic impedance quantities. Let the single-phase load be as before, and, in addition, let the bank carry 18 kva, 3-phase, at 0.866 pf leading. Figure 7.5a shows the circuit with the bank primary omitted, and Figure 7.5b shows the associated distribution of currents.

The kva of the single-phase loads on each transformer are as before, but each now also carries 6 kva of the 3-phase load. Combining currents or kva com-

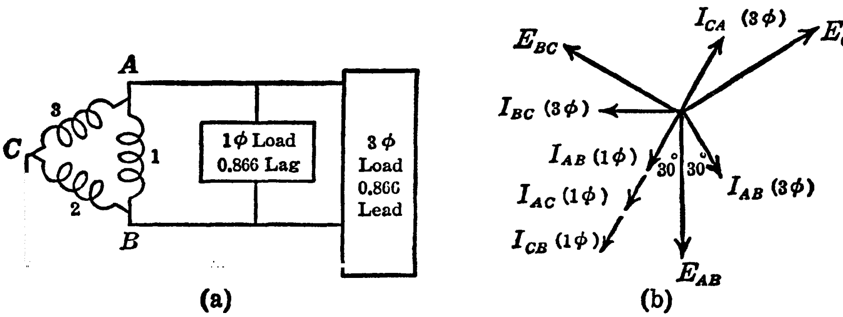


FIG. 7.5. Combined 3-phase and 1-phase loads on a delta-delta transformer bank.

ponents vectorially to obtain the resultant loading on each transformer, we have

$$\begin{aligned} \text{kva}_1 &= E_{AB}[I_{AB}(3\phi) + I_{AB}(1\phi)] \\ &= \text{kva}_1(3\phi) + \text{kva}_1(1\phi) = 9.55 \qquad \text{kw}_1 = 9.52 \\ \text{kva}_2 &= E_{BC}[I_{BC}(3\phi) + I_{BC}(1\phi)] \\ &= \text{kva}_2(3\phi) + \text{kva}_2(1\phi) = 5.22 \qquad \text{kw}_2 = 5.20 \\ \text{kva}_3 &= E_{CA}[I_{CA}(3\phi) + I_{CA}(1\phi)] \\ &= \text{kva}_3(3\phi) + \text{kva}_3(1\phi) = 8.50 \qquad \text{kw}_3 = 7.36 \end{aligned}$$

Note that $\Sigma\text{kw} = 22.08 = 7.5 \times 0.866 + 18 \times 0.866$. Observe, also, that transformer 3 is overloaded. It is rather easy to overload one of the transformers, and possibly two of them, when a single-phase load is placed on a bank in addition to a balanced 3-phase load.

The kva carried by a given transformer of a delta-delta bank can be calculated, regardless of the number of single-phase loads or their distribution between lines, by superposing the kva requirements of the several loads on that transformer. Any 3-phase load is merely a combination of single-phase loads and can be treated in this manner if desired. The same method is applicable to banks in which the percentage impedances are unequal. For delta-delta banks, the calculation of the division for each single-phase load becomes a problem on the parallel single-phase circuit discussed in Chapter IV.

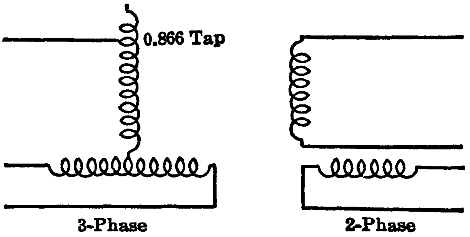


FIG. 7.6. Scott connection of transformers.

7.10. Conversion of 3-Phase to 2-Phase Power. A few applications of 2-phase power still exist in this country, and for a long period of time the Scott connection, Figure 7.6, has remained the predominant method for transforming 3-phase to 2-phase power. Only 2 transformers are employed, the primary of one being connected to a midtap of the other. The transformer having the midtap is called the main unit, and the other is called the teaser transformer. The teaser primary winding must have an 86.6 per cent tap to act as one of the supply terminals, and the full secondary winding used in order to obtain equal voltages across the 2 secondaries.

With balanced 3-phase voltages applied to the primary terminals, the voltages across the 2 transformers are in quadrature with each other, thus giving a 90-degree displacement between the secondary voltages. The secondaries may be distinct and may supply separate single-phase loads or, by diametrical connections to the armatures of motors or synchronous converters, may drive them as 2-phase machines. For distribution systems, the end of one secondary often is joined to an end of the other so as to form a neutral lead, and power is then supplied by 3 wires. The voltage between the outer lines is $\sqrt{2}$ times the voltage of either winding. The several arrangements have been shown essentially, in discussing the 2-phase system, in the chapter on polyphase circuits.

If the 2-phase load is balanced, the supply circuit also is balanced, except for very slight differences due to unequal voltage regulation of the 2 transformers. It can be said that, in general, with any polyphase-to-polyphase transformation of power, balanced conditions of the supply system exist with balanced loading of the bank.

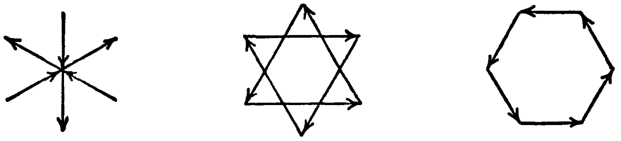
7.11. Conversion of 3-Phase to 6-Phase Power. Six or more phases are desired primarily for the operation of synchronous converters or mercury-arc rectifiers. In converting a 3-phase supply to a 6-phase supply, several types of connection are possible: diametrical, double Y, double delta, double T, and ring (mesh). For rotating machines, the diametrical form is by far the simplest and is almost always used.

In the diametrical arrangement for the operation of synchronous converters, consider, for example, a 2-pole machine. The ends of one transformer secondary are connected to taps on the armature winding 180 degrees apart, or diametrically opposite each other. The second and third secondaries are connected, similarly, to taps 120 degrees and 240 degrees, respectively, farther around the armature winding. For a multipolar machine, all points at the same potential on the armature are connected together. A 6-pole converter would have 18 taps in all, the end of each transformer secondary being joined to 3 of them 360 degrees apart.

Only by the connections to the closed paths through the armature winding does the output of the transformer bank become a 6-phase power supply for the machine, since the secondaries are independent of each other. This statement holds for all the methods of transformation given above. If, however, the midpoints of the secondaries are joined together, a true 6-phase star system of voltages is obtained without regard to connections to any load circuit.

In all but the diametrical arrangement, double secondaries are required. Two Y's, or deltas, or T's are formed by the secondaries with, in each case, the voltages of one 3-phase grouping rotated 180 degrees with respect to the other set of coils. The double Y becomes the 6-phase star transformation if the 2 neutrals are joined together. Figure 7.7 shows the vector diagrams for the double-Y, double-delta, and ring connections. Because the required neutral is available, the 6-phase star and double-Y circuits are adapted to rectifiers.

Since a 2-phase supply can be transformed to a 3-phase system, it also can be transformed to a 6-phase source of power by means of double secondaries. Parallel operation of 3-phase banks, with relative phase displacements between their output voltages, are employed to obtain a greater number than six phases.



(a) Double Y (b) Double delta (c) Ring or mesh

FIG. 7.7. Vector diagrams of secondaries for transformation from 3-phase to 6-phase.

7.12. Autotransformers and Autostarters. Three-phase autotransformers or groups of single-phase autotransformers are employed to a great extent to transform power 3-phase to 3-phase. The connections are the usual ones presented for the ordinary transformer. Single-phase units in combination also are used to transform power from one number of phases to another. For economic reasons, the application of autotransformers is usually limited to service in which the secondary voltage is greater than 50 per cent of the primary voltage.

Autostarters, or starting compensators, are autotransformers used to reduce the voltage to a motor during its starting period. They are used where the line current at start must be reduced or where the starting torque must be reduced to meet the load requirement. Autostarters may be 3-phase units or combinations of single-phase units, Y or open-delta being the standard manner of connection. Usually 50, 65, and 80 per cent taps are provided. The starting current taken by the motor varies directly with the percentage tap or starting voltage at the motor terminals, and the line current ahead of the compensator varies with the square of the percentage tap. For example, if the motor is started on the 65 per cent tap, the starting current of the motor is reduced to 65 per cent and the line current, ahead of the compensator, to 42.3 per cent.

7.13. Parallel Operation of 3-Phase Banks. Parallel operation of 3-phase banks of transformers requires the same general conditions to be met as for paralleling 1-phase transformers. In addition, the polyphase banks must have the same voltage sequence of secondary terminal voltages. Further, there should be no phase difference between corresponding line voltages of the banks to be paralleled. Y-Y and delta-delta banks can be paralleled with each other but not with a Y-delta combination because of a 30-degree phase shift between the output voltages. Again, if 2 Y-Y banks, for example, are not connected similarly with respect to polarity markings, both can have the same sequence on their secondaries, but a 180-degree phase displacement may exist between corresponding terminal voltages.

PROBLEMS

1. To a fair size, draw the hysteresis loop (flux vs. exciting current) for an air-core coil. For one cycle of the sinusoidal flux wave having a peak value equal to the maximum of the hysteresis loop, draw the associated wave of exciting current. Repeat for a coil having an iron core with the magnitude of flux being the same as before and the core giving rise to appreciable saturation. Observe the phase displacement between the flux and current waves. If the flux wave and hysteresis loop are both increased 20% in magnitude, what is the relative effect on the fundamental and harmonic content of the exciting current?

2. Three inductive coils having negligible resistance are connected in Y across 220-volt, 3-phase lines. What is the character of the leg currents? Of the leg voltages? Assume (a) no saturation, and (b) saturation.

3. A voltage wave is given by the equation

$$e = 160 \sin \omega t + 60 \sin 3(\omega t - 30°)$$

What is the effective value of the voltage as read on a voltmeter? What is the peak value of the voltage wave?

4. A high-resistance voltmeter indicates 131 volts line-to-neutral for the circuit of problem 2. What is the magnitude of the third-harmonic component in the voltage wave if it is the only harmonic of appreciable magnitude present?

5. Sketch the wave shapes representing the fundamental components of the exciting currents in the 3 legs of the delta primaries of a transformer bank. On the same diagram draw the wave shapes of the third-harmonic components of the exciting currents assuming magnitudes of 30% of the fundamental currents. For the purpose of this problem, assume that each third-harmonic crosses the axis in a positive direction along with its fundamental. From this sketch, state your conclusions with respect to the phase angles between the third-harmonic currents.

6. A fully loaded, delta-delta bank of transformers, stepping down the voltage from 12,000 to 480 volts, is carrying 360 kw at 60% pf lagging. Calculate all leg and line currents for the bank. Can this bank carry an additional load of 120 kw at 1.0 pf? If so, under what conditions?

7. A Y-connected, 11,000-volt generator supplies power to a delta-delta bank of transformers from which power is transmitted at 132 kv. At the receiving end, the voltage is reduced to 4,000 by means of a Y-Y bank. If the load is 240 hp with an efficiency of 80% and 0.75 pf, calculate all line and leg currents and voltages for the entire system. Neglect losses in the transformers and transmission lines.

8. A Y-connected, 14,000-volt generator feeds a delta-Y transformer bank where the voltage is increased to 140 kv. At the load end, voltage is reduced to 26.4 kv through a Y-delta bank. Calculate all line and leg currents and voltages of the system if 10,000 kw at 0.8 pf are taken by the load. Neglect losses in the transformers and transmission lines.

9. Three 150 kva single-phase transformers form a Y-delta bank between a generator and a load. Each transformer has 1.3% *R* and 3.8% *Z*. At no load the generator voltage is 11 kv and the load voltage is 4 kv. What should be the generator voltage for a load of 350 hp, 85% efficient, at 4 kv, 0.8 pf?

10. Three balanced 3-phase loads of 10 kw at 1.0 pf, 15 kva at 0.8 pf lagging, and 5 kva at 0.6 pf leading are supplied by a Y-delta bank which reduces the voltage from 4,000 to 220 volts. Specify the ratings of the transformers.

11. A 3-phase, 800-hp induction motor load, 84% efficient, 0.7 pf lagging, and a 3-phase synchronous motor load of 500 hp, 92% efficient, 0.8 pf leading, are supplied by a Y-delta bank of transformers. If the voltage is stepped down from 11,000 to 440 volts, specify the ratings of the transformers.

12. An industrial plant has the following balanced 3-phase loads taking power at 440 volts: 225 kw at 1.0 pf, 450 kva at 0.8 pf leading, 600 kw at 0.75 pf lagging, and 150 kw at 0.8 pf lagging. What are the line currents in the high- and low-voltage mains if the plant is supplied at 2,300 volts through a bank of transformers connected Y-delta to step down the voltage? Specify the individual ratings of the transformers.

13. The voltage between primary lines for a Y-connected bank is 132 kv. A Y-connected secondary supplies 20,000 kw, 0.7 pf lagging at 13.2 kv, and a delta-connected secondary supplies 4,500 kva, 0.9 pf leading, at 2.3 kv. What current flows in the high-voltage lines?

14. Balanced 3-phase loads on a plant are 500 kva, 0.8 pf lagging, 440 volts; 270 kw, 0.9 pf leading, 440 volts; and 150 kw, 1.0 pf, 220 volts. The supply voltage of 2,200 volts is reduced to 440/220 volts by a transformer bank having midtaps on the secondaries. What will be the current in each section of the secondary windings if they are connected in Y?

15. The 460-volt secondaries of a delta-delta transformer bank are tapped at the midpoints to give 230 volts 3-phase.

(a) What are the currents in each section of any secondary winding if a 3-phase load at 460 volts is 60 kw at 0.8 pf lagging, and a 230-volt, 3-phase load is 20 kva at 0.9 pf leading?

(b) Repeat (a) for 60 kw, 460 volts, 0.866 pf lagging, and 20 kw, 230 volts, 0.866 pf leading.

(c) If the bank had a rating of 100 kva, what 3-phase load, in kva, at only 230 volts could be placed on the bank without overloading it?

16. A delta-delta bank of transformers steps down the voltage from 2,300 to 230 volts and supplies a load of 200 kw, 0.8 pf lagging. Calculate the initial current in each transformer. If one transformer is taken out of service, what will be the currents in the two that remain, assuming the same load?

17. Draw the vector diagram for an open-delta transformer bank carrying a balanced 3-phase, 1.0-pf load and show that the power factor of each transformer is 0.866. Draw the vector diagram for the bank when carrying a balanced 3-phase load at 0.8 pf lagging.

18. Two identical transformers in open-delta supply a balanced 3-phase load of 75 kw at 0.8 pf lagging. If the voltage ratio is 2,300/460 volts, what current is carried by each transformer? What is the required rating of each transformer? What kva at full load could be carried by a delta-delta bank, each transformer to have the rating just calculated?

19. For a given 3-phase load, what is the ratio of the copper loss in a delta-delta transformer bank to that when carrying the same load in open-delta?

20. An open-delta transformer bank comprises two 15-kva machines. If the load pf is 0.8 lagging, and the transformers can carry 15% overload for a short interval, what permissible load can be taken from them?

21. The present 3-phase balanced load on a plant is 75 kw at 0.8 pf lagging and is supplied by an open-delta transformer bank. What should be the rating of each transformer if the load eventually is to be 150 kw at 0.85 pf and is to be supplied by the full delta-delta bank?

22. The transformers in an open-delta bank are each rated 75 kva, 2,300/230 volts. If the bank carries 125 kva at 0.8 pf lagging at present, what additional load at 1.0 pf can be added if the third transformer is installed to give a full delta bank?

23. A delta-delta transformer bank carries 105 kw at 0.75 pf when fully loaded. On open-delta it must carry 75 kw at 0.8 pf. What is the per cent load on each transformer when in open-delta?

24. The pumping load for an irrigation district requires a transformer bank of 150 kva. This rating includes 30% overload capacity. Since the maximum pump load is carried during the cooler hours of the night, normal kva loading during the day is considerably less. What kva are required of each transformer if the bank is connected full delta? If connected open-delta? What is the economy of apparatus of the full delta bank over the open-delta connection?

25. What kw load is carried by each transformer of an open-delta transformer bank which steps down the voltage from 2,300 to 230 volts for a 20-kva load at 0.5 pf? What effect does phase sequence have on the transformer currents? What should be the minimum rating of each transformer? If each transformer has a rating of 15 kva, what additional unity-pf load can be carried by the bank should a third 15-kva transformer be added?

26. The electrical specifications for a ship call for a delta-delta transformer bank having a voltage ratio 450/120 volts to supply single-phase loads. If the transformer ratings are based on an open-delta connection with balanced single-phase loads of 8 kw total, 0.9 pf, what should be the normal kva of each transformer, and what kva can the full delta-delta bank carry so as not to exceed the temperature rise when on open-delta?

27. An office building is supplied by a 500-kva transformer bank delta-connected on the input side for 4,160 volts. The secondaries are connected in Y for 208/120 volts.

(a) What are normal leg and line currents for the bank?

(b) If the line-to-neutral loads (for lighting) at 120 volts are balanced and are 350 kw total at 1.0 pf, what 3-phase motor load at 208 volts, 0.8 pf lagging, can be carried without overloading the bank?

(c) What effect does a single-phase load have upon the bank?

28. An office building is supplied by a 3-phase transformer bank connected for 6,900 volts delta on the high-voltage side, and for 208/120 volts on the low-voltage side. If the connected 3-phase load is 75 hp, 0.8 eff., 0.707 pf lagging, and the single-phase load is 300 kw, 1.0 pf, balanced among the 3 phases, what currents are carried in the windings of each transformer?

29. In preparing a statement on the energy cost for a feeder circuit in a shipyard, an ammeter test with a clip-on instrument showed 80, 100, 75, and 25 amp, respectively, in the 3 main lines and the neutral wire. The system voltage was 208/120 volts, with loads of resistance character only. At 7 mils per kwhr, what is the energy cost for a 30-day month, assuming these loads substantially constant over a 24-hour day?

30. The primaries of a transformer bank are Y-connected. Each secondary has double windings, one set connected in Y and the other in delta. What is the character of the wave of exciting current when the delta is open, and when it is closed? What is the character of the line and leg voltages of the Y-connected secondary coils when the delta is open, and when it is closed? What is the character of the voltage wave between 2 terminals of the delta when it is open? With the delta open, what effects would be observed in placing a load on one leg of the Y-connected secondary coils? How does closing the delta change these effects? There is no neutral wire on either primary or secondary of the Y circuits.

31. Three similar transformers have 1,000-volt primary windings, and 500-volt secondary windings with 3 equidistant taps so as to give 125 volts per section. Show by a sketch some of the voltage ratios obtainable for delta-extended-delta arrangement of the coils.

32. Three identical 10-kva transformers compose a delta-delta bank. What maximum single-phase load in kva can be carried by the bank? If the single-phase load is 12 kw at 1.0 pf, what kva and kw are carried by each transformer? Repeat for a single-phase load of 6 kva at 0.866 pf lagging; for a 10-kw, single-phase load at 0.8 pf leading. What effect does reversed sequence have on the loading of each transformer?

33. Three 10-kva transformers are connected in delta-delta. Across terminals *A* and *B* is placed a single-phase load of 7.5 kva at 0.8 pf lagging, and between terminals *B* and *C* is a 1-phase load of 8 kw at 0.9 pf leading. Calculate the kva and kw of each transformer for sequence *A–B–C*.

34. Show that for 3 equal single-phase loads, forming a symmetrical loading on a delta-delta bank, *all transformers carry the same kva and kw. Note that this method of* superposition reduces to the simpler treatment for balanced 3-phase loading on the bank.

35. Two transformers of 15 kva and one of 20 kva compose a delta-connected bank. If they carry a balanced 3-phase load of 15 kva at 0.866 pf lagging, what maximum single-phase load can be carried by the 20-kva transformer at 1.0 pf without overloading any unit of the bank? Assume the transformers have the same ohmic values of resistance and reactance. Repeat, assuming all transformers have the same $\%R$ and $\%X$.

36. A balanced 3-phase load of 30 kva at 0.866 pf lagging and a single-phase load of 10 kva at 1.0 pf are placed on a delta-delta transformer bank. What kva and kw are carried by each transformer? Assume equal impedances and sequence *A–B–C*.

37. Occasionally, in residential districts, an open-delta bank may be composed of a large transformer and a small one for supplying a very little 3-phase power in addition to the much greater load of lighting and single-phase appliances. What is the advantage of the open-delta bank for this application rather than the delta-delta bank? For a single-phase load of 40 kw, 1.0 pf, across the large transformer, and a 3-phase load of 3 kw at 0.866 pf lagging, what kva and kw are carried by each transformer?

38. A single-phase current of 20 amp is drawn from a bank of 3 transformers having turn ratios of 4:1. What is the current distribution throughout the bank for each of the following connections?

(a) Delta primary—Y secondary (line-to-neutral load).
(b) Delta primary—Y secondary (line-to-line load).
(c) Delta primary—delta secondary.
(d) Delta primary—one secondary reversed, all in series.
(e) Y primary—delta secondary.

39. Two transformers, both having voltage ratings of 1,200/240 volts, are connected T-T. What are the voltages across each winding of the transformers with 1,200 volts, 3-phase, impressed on the high-voltage coils? What are the line voltages on the secondary side?

40. Two 10-kva, 1,100/2,200//110/220-volt transformers are connected to convert 3-phase power to 2-phase power. What are the output voltages between lines if the low-voltage windings are arranged for 3-wire service? What should be the ratios of transformation for the 2 transformers if the leg voltages of the secondaries are to be equal? In both cases, the low-voltage coils of each machine are placed in series.

41. A 2-phase, 200-hp, 0.8-pf motor having an efficiency of 93% takes its power from a 3-phase, 2,300-volt supply through a Scott-connected transformer bank. If each secondary voltage is 440 volts, what is the kva loading of each winding of the transformer?

42. Six-phase power is obtained from a 3-phase, 220-volt source through a delta-star connected transformer bank. If 42.5 volts are desired for the 6-phase load, what ratio of transformation is required of each transformer? What is the kva rating of the bank if each secondary carries 60 amp?

43. With variable-tap autotransformers, what are limiting ratios of secondary to primary voltage for Y and for delta connection?

44. One campus of the University of California obtains its power from the Pacific Gas and Electric Company through two 3-phase autotransformers, each rated 2,500 kva, 12,000/4,160 volts, 60 cycles, Y, 4.3% *Z*. For full loading on one of these transformers, what are the currents in the series and common sections of the winding on each leg, and what are the voltages across these sections?

45. A 440-volt motor draws 150 amp when starting on full voltage. What are the currents to the motor and to the compensator when started on the 65% taps of an autostarter?

46. A motor draws 560% rated current on full-voltage starting. What percentage taps on a starting compensator will limit the current to the compensator to 200% of rated motor current?

47. Each set of primary windings of 2 Y-Y banks is connected in an identical manner, both in forming the Y's and in connecting them to the same supply lines. By connection diagrams, and related voltage vector diagrams, show that both banks can have the same sequence on the secondary side, but that corresponding line voltages may be 180° out of phase. Show polarity marks on all terminals of each winding in the connection diagram. Might this phase displacement also be present in paralleling delta-delta banks or delta-Y banks?

48. A generator is connected at its terminals to one transmission line through a delta-Y bank, and to a second transmission line by a delta-delta bank. It is proposed that the lines be paralleled at a distant point by means of a third transformer bank. Specify the type of connection.

49. A steel pipe and fabricating plant has an average load of 500 kw at 75% pf supplied by three 200-kva transformers in a delta-delta bank. What corrective kva of condensers brings the kva load on the plant just within the rating of the bank?

50. A local industrial company now has installed three 275-kva transformers. The average power factor for the plant is 83% lagging. An additional load of 100 hp, 88% efficient, 0.8 pf lagging, is to be installed. If the bank is fully loaded at present, what is the cost at $12 per kva of capacitors to bring the new over-all plant load within the rating of the transformer bank?

CHAPTER VIII

ELECTRICAL MACHINES

8.1. Introduction. Up to this point little has been said on machines with the exception of the transformer which was introduced primarily as a circuit element. The presence of voltages of various magnitudes and frequencies has been assumed, our analyses starting from these given supplies. Now, however, we must extend our treatment on circuits to include rotating electrical machinery. This chapter will consider only the fundamental principles involved and the general points of similarity among all types of machines. The details of mechanical construction and of operation of particular machines are presented in later chapters.

Although there are many variations among the different types of rotating machines, they all have certain common features. In terms of physical principles of operation, all electrical machines have decidedly similar properties, these basic similarities far outweighing those that are dissimilar. The similar characteristics are related to the electric and magnetic circuits of the machines, whereas the dissimilar characteristics are dependent primarily upon mechanical considerations. Except for the transformer, the dissimilarities arise from the speed and torque requirements of the prime mover or the driven machine, and they result in modification of mechanical design to yield certain electrical changes, or from desired variations in electrical qualities that necessitate alterations in mechanical design. The endeavor in this chapter is to present all machines from the same viewpoint with respect to excitation, induced voltage, torque, frequency, speed, phases, energy conversion, losses, etc.

From the standpoint of use, a rotating machine may be classified as either a motor or a generator. However, as was pointed out in Chapter II, the conductors of a motor cut lines of flux and the motor is, therefore, at the same time generating a voltage. Likewise, a generator under load has current-carrying conductors in a magnetic field, and the resultant force action is fundamental to every motor. The difference between a motor and a generator is the use to which the machine is put and not its construction. To stress this further, consider a motor-generator set acting as a tie between 2 electrical systems, Figure 8.1. If the power flow is from system A to system B, machine No. 1 is acting as a motor driving machine No. 2 as a generator. A slight mechanical adjustment on one of the machines or a change in electrical terminal conditions may interchange their functions, in which case the power flow is from system B to system A. This change in direction of power flow would not be indicated by external observation of the machines, and, if it were not for the electrical instruments connected in the lines, it would be difficult to identify the function performed by each machine.

In Figure 8.1, either system may be a-c or d-c. If power is transformed from a-c to d-c, or vice versa, the set then is essentially a rotary converter or inverter, respectively. Actually, however, in the rotary converter the 2 machines are combined into one having a single armature winding connected to a commutato at one end of the shaft and to slip rings at the other end. If the frequencies of systems A and B are unlike, the set then is a frequency-changer, also true if one system is a-c and the other d-c. Figure 8.1 may be generalized further by assuming either system to be mechanical rather than electrical, whereupon power is transformed from one kind to another.

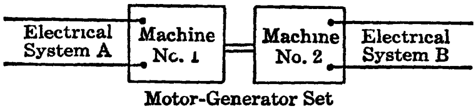


FIG. 8.1. Motor-generator set connecting two electrical systems.

The relationship of the various rotating machines to each other can be indicated by a diagram such as Figure 8.2. Starting with the alternator, for example, which also can be used as a synchronous motor, the substitution of a commutator for the slip rings yields the d-c generator. The d-c generator, without any change in construction, also can be used as a d-c motor. The combination of the a-c and d-c machines gives the rotary converter. Substitution of a squirrel-cage winding for the d-c field coils of the synchronous machine yields the induction machine, either motor or generator. Single-phase machines employ the principles of the synchronous machine, the induction machine, the d-c machine, or a combination of these principles.

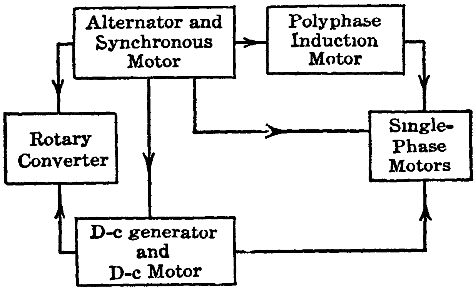

Alternator and Synchronous Motor
Polyphase Induction Motor
Rotary Converter
Single-Phase Motors
D-c generator and D-c Motor

FIG. 8.2. Diagram showing the interrelationship of the various rotating machines.

8.2. Basic Circuits in Machines. All machines have 2 fundamental electric circuits which are linked by a common magnetic circuit. The first of the electric circuits is called the field circuit because it provides the excitation, or flux, of the magnetic field. The exciting current may be d-c or a-c. Although, in genral, the excitation of the machine is provided by this one electric circuit only, t times it also may be supplied in part by the second electric circuit. The coils n which the field current flows may or may not move, and the volt-amperes equired for excitation are normally but a few per cent of the nameplate kva of he machine.

The second electric circuit is on the armature of the machine. It is the source of the generated voltage and of the developed torque, and hence is the power winding in which electrical power is converted to mechanical power, or vice versa. This winding may or may not move, but one of these 2 electric circuits must move with respect to the other, or else, as with the transformer, there must be an equivalent relative motion by way of the variation of the mutual magnetic field.

Both field and armature windings must be built for the same number of magnetic poles, obviously always of even number and with alternate poles of opposite polarity. Further, there must be an air gap between them to permit of relative motion if energy is to be converted from mechanical to electrical, or if frequency is to be changed. Since both electric circuits work on the same mutual magnetic paths, they always will react against each other, and a change in the mmf of one will require a corresponding change in the mmf of the other. This effect of the mmf of the armature current is called armature reaction.

It should be recognized that in any motor or generator the conversion of mechanical energy to electrical energy is made through the intermediate link of the magnetic circuit. This also is true in the transformer, wherein electrical energy is really transferred and not transformed in kind. The generality of the magnetic link between the electric circuits, thereby also relating the electrical power to the mechanical power of a machine, can be readily understood by considering briefly the fundamentals discussed in Chapter II. Mechanical power is the product of force and velocity, and electrical power is the product of voltage and current. The mechanical force results from the current and the flux, equation 2.3, whereas the voltage results from the flux and the velocity, equation 2.1. Thus, the flux of the magnetic circuit ties together the power expressions of the 2 systems.

8.3. The Revolving Magnetic Field. In all electrical machines, the mutual flux of the 2 windings varies with respect to the armature circuit. In the transformer and the induction machine it varies with respect to the exciting winding as well. Each coil of any machine is a part of a single-phase circuit within that device, and hence the enclosed flux pulsates at normal frequency through the coil. In rotating machines, because of the mechanical spacing of groups of coils in consecutive slots, the maximum value of the pulsating flux is transferred uniformly from one group of coils to the next in a given direction, thus giving rise to a revolving magnetic field. In any generator it is the forced motion of the rotor that produces ultimately the revolving magnetic field of the armature when it is carrying current, and, in any motor, it is the revolving magnetic field existing under the influence of the impressed voltage that causes the rotor to turn.

Although our attention is directed mainly toward the 3-phase machine, the basic ideas presented are applicable to machines of other numbers of phases, including the d-c machine, which is but a special case of the general a-c machine.

In several previous instances, it has been pointed out that relative motion between a magnetic field and armature conductors leads to generated voltages in those coils, and that it is immaterial whether the exciting winding or the armature winding has the motion. In the study on 3-phase circuits, it was noted specifically that uniform motion of a generator gives rise to sinusoidally induced voltages in the three separate phase windings, and that these voltages appear 120 degrees in time from each other because of the symmetrical manner in which the coils are placed on the armature. As a consequence of the rotating field of flux of the rotor, there exists at the stator terminals a symmetrical 3-phase system of supply voltages.

If, now, this 3-phase voltage is impressed on the armature terminals of a second machine of like construction, it will force sinusoidal currents, 120 electrical degrees in time from each other, through the three phase windings. As a result of the position of the windings, the maximum current and its attendant flux will appear first in phase A, then in phase B, etc. This flux field moves progressively and at synchronous speed around the armature periphery from one phase winding to the next and is called a revolving field. It is reversed in direction by interchanging any two of the supply leads.

Figure 8.3 shows the flux wave for phase winding A, with β_1, the maximum flux density at a particular instant, in the axis of that winding. The maximum flux densities for that same instant of time, and for phase windings B and C, are located 120 and 240 electrical degrees, respectively, along the armature surface. Using phase A as the reference, the resultant flux density β_λ at angle λ measured from the zero of the first flux wave is

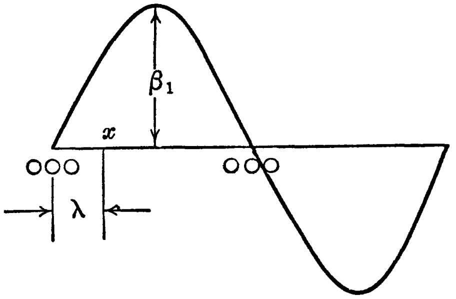


FIG. 8.3. Flux distribution of phase A at any particular instant.

$$\beta_\lambda = \beta_1 \sin \lambda + \beta_2 \sin (\lambda - 120) + \beta_3 \sin (\lambda - 240)$$

which is the sum of the flux components at point x due to the position of the three phase windings. It is taken at any chosen instant of time, and, in general, $\beta_1 \neq \beta_2 \neq \beta_3$.

Now β_1, β_2, and β_3 are flux densities, all having the same maximum value β_m but varying sinusoidally in time 120 electrical degrees from each other. Thus, with each component modified by this time factor, for example, $\beta_1 = \beta_m \sin \omega t$,

$$\begin{aligned}\beta_\lambda &= \beta_m[\sin \lambda \sin \omega t + \sin (\lambda - 120) \sin (\omega t - 120) + \\ &\qquad \sin (\lambda - 240) \sin (\omega t - 240)] \\ &= \tfrac{3}{2}\beta_m \sin\left(\omega t + \frac{\pi}{2} - \lambda\right) \\ &= \tfrac{3}{2}\beta_m \cos (\lambda - \omega t) \qquad (8.1)\end{aligned}$$

Equation 8.1 is a special form of the *wave equation,* one of the small group of fundamental relations underlying mathematical physics. It is a function of both distance and time, and its interpretation shows motion. Observe, for instance, that if the position x is fixed, the resultant flux wave at that point varies sinusoidally in time, whereas at a given instant of time the space variation of the wave along the armature surface is sinusoidal, with one cycle per pair of poles. If the space angle λ is made to increase at the uniform rate ω, that is, λ and ωt increase at the same rate, the equation shows uniformly progressive motion of the flux wave around the armature periphery, with maximum value $\frac{3}{2}\beta_m$. The synchronous speed in rpm is $120f/P$.

8.4. The Transformer as a Coupled Circuit. From what has been said thus far, it is evident that any machine is merely a particular application of the coupled circuit. In view of this, consider Figure 8.4 showing 2 electric circuits linked by a flux Φ. Either coil may be the exciting winding, the exciting current may be of any form, and the coils may have any position with respect to each other.

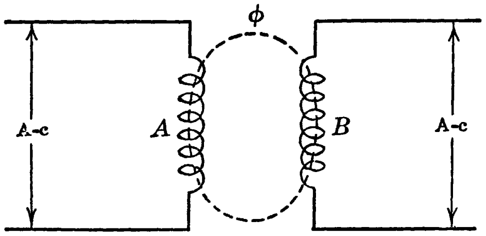


Fig. 8.4. Application of the coupled circuit to the transformer.

The simplest application of the coupled circuit is that of the transformer in which the 2 windings are fixed in position on the same iron core. With an a-c voltage applied to A, the field or exciting winding, an a-c voltage is induced in B. Current hence flows in both coils when the circuit is completed between the terminals of B. The frequency of both primary and secondary is the same, but in practice there usually is a change in voltage and current between them, this effect obtained by way of the turn ratio.

Since neither winding moves, and therefore the input and output energies are not changed in form, except for the small amount of heat loss in the copper and iron, the electrical rating of the exciting winding is equal approximately to that of the secondary power winding. The volt-amperes for magnetization are but a very few per cent of the nameplate rating of the device. The transformer is merely a stationary 2-pole machine with no consequent torque but with a strong mechanical force of repulsion between the coils.

The power factor at the terminals of the output winding always is that of the connected load, whereas at the terminals of the exciting winding the power factor is altered slightly from this value because of the losses and the volt-amperes for magnetization. As previously discussed, extensions of the basic idea include the single-phase autotransformer and the induction regulator. Further, *A* and *B* may be said to represent groups of coils arranged for polyphase transference of power, either group to be a single polyphase device or a connected set of single-phase units to serve the same purpose.

8.5. Application of the Coupled Circuit to the Synchronous Machine. Figure 8.5 shows the exciting winding *A* carrying direct current and mounted on a shaft. For an actual machine, *A* represents groups of coils, each of which is placed on a rotor pole. The power winding *B* also represents many coils, these coils being placed in slots in a cylindrical iron core surrounding the rotor. They are connected in groups for single-phase or polyphase operation and for the same number of magnetic poles as are on the rotor. The power winding will have either an a-c voltage induced between its terminals, if the machine is to be driven mechanically for generator action, or will have an a-c voltage applied to it, if the machine is to be a motor. In practically every instance, it is the power winding that is stationary, although occasionally, in small sizes, that winding is shaft-mounted. Except for a few single-phase generators, and the great number of tiny single-phase clock motors, *B* will be a 3-phase winding. Few 2-phase machines exist, and practically none of any other number of phases. Six-phase synchronous converters are adaptations for a 3-phase power supply.

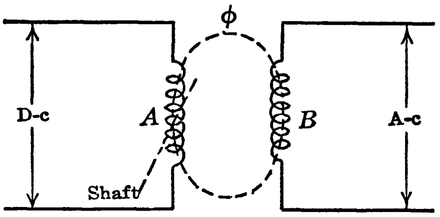


FIG. 8.5. Application of the coupled circuit to the synchronous machine.

This application of the coupled circuit is the synchronous machine. As an a-c generator, or alternator, the frequency in *B* is fixed by the number of magnetic poles and by the speed of the rotor. With polyphase power supplied *B*, a revolving magnetic field will be set up in the air gap by this winding. Once the rotor is brought near the synchronous speed of the stator field, the rotor magnetic poles will be forced to line up with the moving stator magnetic poles, the rotor thus being carried along also at synchronous speed. To say that machines are running in synchronism merely means that they are in step electrically with each other. This applies, of course, only to the present type of machine and to those connected to the same power system, the controlling tie among them being the system frequency. All synchronous generators on the same lines must run at such speeds as to produce the same frequency, and all synchronous motors must run at those speeds that are dependent upon the frequency and upon their respective numbers of poles. Any synchronous machine, therefore, must run at one and only one speed if connected with several others, else it cannot carry load. Briefly, the synchronous machine is an inherently invariable-speed machine.

The revolving field produced by the 3-phase stator currents in the motor can be considered as actual physical poles mechanically moved at synchronous speed about the axis of the machine, as shown in Figure 8.6. Although the diagram is for a 2-pole motor, it must be remembered that what is said about the electric and magnetic characteristics of one pair of poles on a multipolar machine is true also for the other pairs of poles. It is easier, therefore, to confine any discussion to the simplest construction. The greater the magnetic strength of either stator or rotor poles, the greater is the attraction between them. The greater then will be the load torque that the shaft can carry and still allow the machine to run at synchronous speed. The driving torque in the air gap is, of course, tangential to the armature surface and directed along the periphery. The torque is zero if the stator and rotor poles are directly opposite each other, and it increases, as they separate, up to a maximum angle that depends upon several factors, as we shall see, but which for no synchronous machine can be greater than 90 degrees.

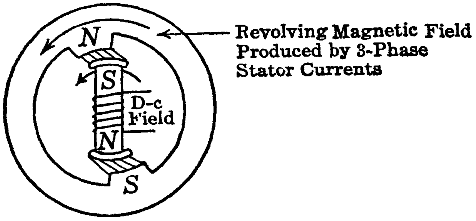

N
S
N
S
D-c Field
Revolving Magnetic Field Produced by 3-Phase Stator Currents

FIG. 8.6. Elements of the synchronous motor.

As load is applied to the motor, the rotor poles lag in position behind the revolving stator magnetic poles. This angle of lag is called the torque angle, power angle, or load angle of the machine. The rotor shifts in position with change in load, but it runs at synchronous speed thereafter. If load is too great, the rotor is forced past that angle of maximum torque and thus out of synchronism, which it can regain only by dropping some load.

In the case of the alternator, the rotor is driven. It now leads, by the power angle, the revolving magnetic poles of the stator. In fact, whether or not a synchronous machine is functioning as a motor or as a generator can be determined easily by observing the backward or forward motion, respectively, of a stroboscopic disk on the shaft. The displacement from no load is the power angle.

If the rotor is of the type indicated in Figure 8.6 (salient-pole rotor), the motor can carry some load with no direct current in the field windings. Under this condition, the stator mmf induces magnetic poles in the rotor, and the rotor tends to line up so that the stator mmf acts on the path of minimum reluctance. For this type of operation, the machine is said to run on reluctance torque. Electric clock motors, for example, are reluctance-torque salient-pole synchronous motors with shading coils simply to start them. Many large salient-pole motors can carry appreciable load on reluctance torque alone.

When the rotor is at standstill, the magnetic poles of the stator field are rapidly passing those of the rotor. As a stator pole passes consecutive rotor poles, the direction of the torque is reversed, with the result that the average torque is zero. There is not sufficient time for the stator mmf to magnetize the rotor projecting iron poles by induction with fixed polarities, nor, in general, would there be sufficient torque to accelerate the inertia of the rotor and perhaps with it any shaft load. The synchronous motor, therefore, has no inherent starting torque and the rotor must be brought nearly to synchronous speed by some other means before it will "lock in" with the revolving stator magnetic field. In practice, the rotor is supplied with an induction-motor squirrel-cage winding so that the machine can start and be brought near synchronism. In starting, the d-c field circuit is opened, or closed upon itself, so that it cannot prevent the stator flux wave from reaching across the gap to the squirrel-cage.

In the synchronous machine, the excitation is supplied by either or both windings, although the field winding generally carries more than that required for normal open-circuit voltage. From fundamental physical principles on induced voltage, it is recognized that for a given voltage at a given frequency for the machine the air-gap flux then is fixed in magnitude. It hence can be reasoned at this point that there can be a particular value of current in the field winding such that it will supply all the required flux. Further, if winding *A* supplies more, or less, than this amount, the current in *B* will change in both magnitude and phase position so as to maintain the required flux in the air gap. This phenomenon is of considerable consequence in the application of synchronous motors in that the power factor of the power winding can be controlled by the amount of excitation of the field winding. Further discussion on this point is given in the chapter on synchronous machines.

Since the synchronous machine transforms energy from mechanical to electrical, or vice versa, the electrical rating of the power winding, as given on the nameplate, is approximately equal to the equivalent mechanical shaft power. For the generator, the prime mover must supply the windage, friction, and iron losses, and the copper losses in the armature winding. Likewise, the input to the motor must be greater than the output by the amount of these losses. In both cases, the copper loss of the exciting circuit may or may not be supplied by the main source of energy; it may be taken from a separate excitation system. The excitation voltage and current have no relation in magnitudes to the voltage and current of the power winding, but the power for excitation is only a few per cent of the kw or kva rating of the machine.

8.6. The Induction Motor as a Coupled Circuit. In Figure 8.7, winding *A* is stationary and carries a-c excitation, while *B* is mounted on a shaft so that it is completely free to rotate within the influence of the magnetic field of *A*. In practice, *B* is short-circuited upon itself, or is closed through some small value of resistance. No power can be transferred, or transformed, of course, if current cannot flow in the secondary of any coupled circuit.

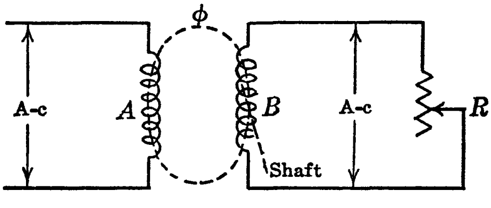


FIG. 8.7. Application of the coupled circuit to the induction motor.

The mutual force action between the 2 windings of the coupled circuit again becomes one of torque, and hence *B* will rotate by virtue of the magnetic field f *A*. Consequently, electrical power supplied *B* from *A*, through the magnetic ield, will be converted to mechanical power at the shaft. The machine is the nduction motor in this particular application of circuit theory. If *A* is a single-phase winding, the motor must be started in some manner, but if *A* is a polyphase winding, a revolving magnetic field is produced and the motor will be self-starting with a large starting torque.

In moving past the rotor conductors, the stator field induces voltages in them, and hence currents will flow through the closed rotor paths. In turn, a revolving magnetic field also is produced by the rotor currents, this field being exactly in time phase with the moving stator field but in space position at a large angle. The interaction of the 2 fields gives rise to the starting and running torque of the machine. If the rotor had resistance only, the torque angle would be 90 degrees, but, because of reactance and the subsequent time lag of the rotor current behind the induced voltage, the rotor produces its magnetic field at an angle greater than 90 degrees with respect to the axis of the stator mmf wave. The interaction of the revolving stator field on the rotor can be understood from Figure 8.8 in which the stator field again is represented as an equivalent set of fixed magnetic poles on a frame that rotates about its central axis.

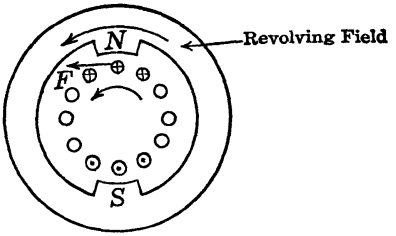


Fig. 8.8. Elements of the induction motor.

The rotor always moves at some speed less than the synchronous speed of the revolving field. If it were to rotate at synchronous speed, there would be no flux cut by the rotor conductors, no induced voltage or current in them, and hence no torque. The difference between the synchronous speed of the field and the actual speed of the rotor is called the "slip." When load is applied to the motor, the rotor speed decreases until the increased rotor current at the lower speed produces sufficient torque to carry the added load. The drop in speed from no load to full load is of the general order of 5 per cent for the usual induction motor, and this type of machine is classed as a constant-speed motor. The rotor will continue to lose speed as load is increased up to a maximum torque called the pull-out, or breakdown, torque. Beyond that point the machine will come to rest.

Except for details in design and construction that may appear, the stator of the induction motor is the same as that for the synchronous machine. In fact, the same stator often is used for some small machines having interchangeable rotors. The simplest type of rotor for the induction machine consists of copper, brass, aluminum, etc., bars placed uninsulated in longitudinal slots in a laminated cylindrical iron core. The ends of the bars are electrically connected by end rings at each extremity of the armature, thus giving rise to the name "squirrel-cage" for this form of rotor construction. In many machines, the rotor carries a winding in its slots similar to that of the stator, connected Y or delta, and with the terminals brought out through slip rings so that the rotor-circuit resistance can be varied externally. This is the "wound-rotor," or "slip-ring," type of induction machine. It is a squirrel-cage winding on the synchronous motor that gives that machine its starting torque.

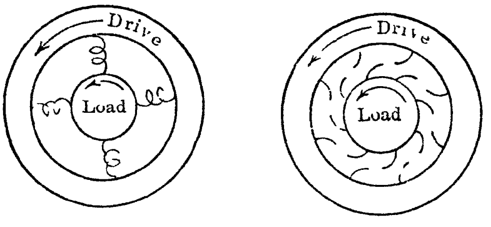
Note that an increase in resistance of the power-winding circuit decreases the current and hence the torque at a particular speed. For a given torque, the slip then increases as rotor-circuit resistance is increased so that a greater induced voltage can maintain the necessary current for the torque. A given torque therefore can be obtained at any speed between a maximum speed, determined by the internal impedance of the windings, and standstill, by adjusting the resistance. This is the reason for the slip-ring induction motor. Starting torque can be made very high—equal to the pull-out torque if desired. As a corollary, speed control of this type of motor can be obtained by changing the external resistance, although the efficiency suffers because a good part of the energy crossing the gap to the rotor now is converted into heat rather than to mechanical energy at the shaft.

Referring to Figure 8.7, the magnitudes of the induced voltage and the current in *B* depend upon the turn ratio of the windings and also upon the actual speed of *B*. At standstill, the frequency in *B* is the same as that in *A*, and it decreases toward zero as *B* increases in speed. Observe that, if *B* is of the wound type and is driven, the machine can be used as a variable-frequency supply with the frequency carried above rated value by driving *B* in the direction opposite that of the stator field. This application, though, is limited to special laboratory testing. The frequency is zero at forward synchronous rotor speed (slip = 0), is rated value at standstill (slip = 1), and is twice rated value at reversed synchronous speed (slip = 2). Note also that the rotor reactance varies with frequency over the speed range of the machine.

The phase angle between the induced voltage in the rotor and the rotor current is fixed always by the relative values of net resistance and reactance of that circuit. At the terminals of the exciting winding, the power factor depends upon the amount of shaft power, the amount of rotor power converted to heat in any external resistance, the copper, iron, friction, and windage losses within the machine, and the magnetizing volt-amperes. For the motor, the current always lags the voltage at the input terminals of the machine, whereas for the induction generator it always leads. The volt-ampere rating of the field winding *A* of the motor is somewhat higher than that of the armature winding *B* because of these losses and the excitation.

The induction machine is the most general type of all, as exemplified by the many ways in which it can be used. If the power winding *B* is similar to that of *A*, the machine at standstill is a transformer and can be so used. It also has been connected as an induction regulator, both single-phase and 3-phase, for laboratory work. If d-c excitation is supplied *B*, that winding changes in function from an armature circuit to a field circuit, and the machine becomes the alternator or the synchronous motor. Again, it has been so used, and is the only true cylindrical-rotor type of synchronous machine.

The synchronous motor and the induction motor can be compared with the 2 mechanical power transmission systems shown in Figure 8.9. Figure 8.9a shows power being transmitted through spring connecting links. The springs are analogous to the magnetic lines of force in the air gap of the synchronous motor. The load shaft must follow at the same speed as the driving shaft. As load is applied, the torque angle increases until such a point is reached that the connecting links break and the load is suddenly dropped. The same general relations apply to the synchronous motor. Once the motor is stopped by overload, the load must be decreased and the motor restarted.



(a) Spring-drive (b) Fluid-drive

Fig. 8.9. Mechanical power transmission systems.

A fluid-drive system is shown in Figure 8.9b. This system exhibits certain characteristics common to the induction motor in that the slip between the drive and load shafts increases with the load. Also, there is no definite "break" after a certain load is exceeded, and the motor will automatically pick up in speed if the excess in load is dropped.

8.7. The D-c Machine as a Coupled Circuit. Figure 8.10 shows winding *A* stationary and carrying direct current for excitation, while the power winding *B* is shaft-mounted. At the immediate terminals of *B* the machine is a single-phase alternator or synchronous motor, but by connecting *B* to the external lines through an automatic switching circuit, called a commutator, the entire system becomes the d-c machine. Electrical power hence is converted from a finite frequency to zero frequency, or vice versa, depending upon generator or motor action.

As any particular coil moves in position from the influence of one field pole to the next, the commutator at the same time reverses the coil connections with respect to the external circuit. Outside the commutator, the current is therefore unidirectional, but within the armature winding it is alternating at a frequency dependent upon the speed of the rotor and the number of poles, just as with the synchronous machine. By using a large number of coils and an equal number of automatic switching elements (commutator bars), the output voltage and current of the generator can be made practically free of any pulsations

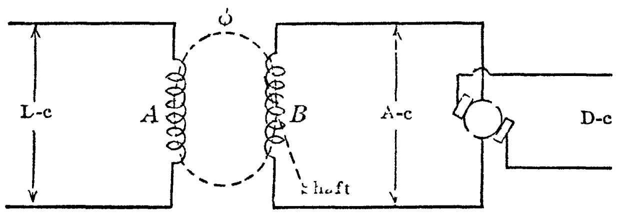


Fig. 8.10. Application of the coupled circuit to the direct-current machine.

Conversely, the torque under motor action also is then more uniform. The commutator is intended to be automatic in its switching performance regardless of the speed of the machine. Although conceivably the field coils might be rotated past a stationary armature, electrical simplicity and corresponding mechanical requirements dictate mounting the armature winding and associated commutator on the rotor.

As with all generators, the speed and internal frequency in the d-c generator are determined by the prime mover. The induced voltage varies also directly with the speed and with the magnitude of the exciting current, saturation neglected. The d-c machine has no synchronous speed, and the motor can be adjusted in speed over a wider range than that for any other type of motor. Speed control is achieved by varying the magnitude of the voltage applied to the armature, by changing the exciting current, or by both methods. The motor will run at that speed at which its generated voltage (counter-emf, or back-emf) allows just that power current to flow to produce the required torque. The motor therefore will react by increasing in speed if either the applied voltage is raised or the field current decreased.

Since the external circuit is d-c, no reactive volt-amperes are either taken from or supplied winding B. Further, the voltage and current in B must reverse together as the individual coils pass under the brushes on the commutator. The power factor within B, therefore, is always unity. The brushes must be set for current reversal when the axis of each coil, in turn, centers on consecutive field poles so that no voltage will be induced in the coil as it is short-circuited. Only a few degrees departure is permitted from this brush position without sparking at the commutator. Consequently, the axis of the armature mmf is held essentially at 90 degrees, the power angle of the d-c machine, with respect to the axis of the field mmf. The losses of the d-c machine do not differ from those of any electrical rotating machine.

Adaptations of the d-c machine to give it shunt, series, or compound characteristics lie only in the manner of arranging winding A for excitation. Since the external circuit is d-c, winding A may be connected across the lines for shunt excitation, in series with the lines for series excitation, or split with part of the winding across the lines and part in series for compound action. In the generator, these methods lead to different voltage characteristics for the external circuit. In the motor, they control the torque and speed characteristics. These schemes of excitation and resulting effects are discussed in detail in the chapter on d-c machines. The power required for excitation seldom exceeds 5 per cent of the nameplate rating of the machine.

8.8. Single-phase Machines. Since single-phase power is practically always taken from polyphase systems, single-phase generators are not common. Their use lies in a few special applications. There are, however, many forms of single-phase motors. No single-phase device, though, involves any principles beyond those already discussed for other types of machines. The basic problem with the single-phase motor is to give it starting torque and to insure a given direction of rotation. In general, once a single-phase motor is started it will increase in speed to its normal value.

A single-phase magnetic field is pulsating, but it may be considered to be the equivalent of a field moving in abrupt jumps from pole to pole in either direction or the combination of two equal and oppositely revolving fields each having half value. Accordingly, a single-phase induction motor, once started in either direction, may be said to follow its corresponding stator moving magnetic field. Starting methods employ one of two ideas: (a) the production of a 2-phase revolving field, one phase of which may be much weaker than the other, and (b) a commutator with brushes so as to fix the torque angle at some intermediate position between 0 degrees and 90 degrees. Those motors using the first scheme come under the classification of split-phase machines. Those using the second method are of the repulsion type.

The following tabulation indicates the principle employed in the several forms of single-phase motors:

Synchronous:
- (a) Single-phase synchronous. Small reluctance-torque motors. Generally started as an induction motor.

Induction:
- (a) Single-phase induction. Not self-starting
- (b) Shaded-pole (essentially 2-phase).
- (c) Split-phase.
- (d) Capacitor. Actually a split-phase motor.

Direct-current:
- (a) Series motor. Will run on either d-c or a-c.

Combination Induction and Direct-current:
- (a) Repulsion.
- (b) Repulsion start-induction run.

8.9. Summary. Summarizing the major points of circuit theory as they apply to all machines, the following general statements may be made:

1. Any machine is but a particular application of the coupled circuit involving 2 electric circuits having a mutual magnetic path.

2. One of the electric circuits is the main exciting winding providing magnetization, although excitation may be supplied in part by the second electric circuit. The excitation may be a-c or d-c, or both if supplied by the 2 windings. The volt-amperes required for magnetization are usually but a few per cent of the kva rating of the machine.

3. The second electric circuit on rotating machines is the power, or armature, winding in which electrical power is converted to mechanical power, or vice versa. In stationary machines, electrical power supplied the device remains electrical power at the output terminals. The kva, or equivalent horsepower, carried by the power winding is given on the nameplate of the machine.

4. All machines are built to provide the magnetic system with the same number of magnetic poles on both stator and rotor. The poles are even in number, ranging from a minimum of 2 to perhaps 150. The flux through each pole is determined by the voltage and frequency of the winding through which electrical power to be converted enters or leaves the machine.

5. The mutual magnetic field has relative motion with respect to the power winding and perhaps also with respect to the exciting winding. Either or both of the electric circuits will have an induced voltage because of the variation of mutual flux through them.

6. The magnetic poles created by the 2 electric circuits react upon each other to produce a mechanical force. In rotating machines, the product of the resulting torque and the angular velocity is directly proportional to the electrical power converted to mechanical power, or vice versa.

7. All power windings have a finite frequency which is fixed by the number of poles and the relative speed of that winding with respect to the magnetic field of the exciting winding. The exciting winding has either normal frequency for the machine or the limiting case of zero frequency (d-c).

8. The phase angle between voltage and current in d-c circuits is zero. In a-c machines, the power factor at the terminals is determined by the watts active power and the volt-amperes for magnetization.

9. The torque, or load, or power angle of any machine is the angle between the axes of magnetization of the stator and rotor mmf's. It is fixed for commutator machines, and variable within limits for others.

10. Copper losses accompany current flow in all windings, and heat losses are present in the iron of the magnetic circuit whenever the flux is varying. Rotating machines have, in addition, windage and friction losses.

11. All machines are tested by similar, or equivalent, methods for the same character of information. The tests are normally

(a) Open circuit, giving knowledge on excitation characteristics and the fixed losses, and

(b) Short circuit, giving knowledge on the internal impedance and the variable losses.

12. The quantitative analysis of all machines may be carried out by means of equivalent electrical networks in which shaft loads can be replaced by variable resistances.

Problems

1. Three 100-turn coils are placed at 120° intervals along the circumference of a circle with their axes radial. Electrically they are connected in Y, and carry 10 amp maximum under a 3-phase impressed voltage. Using as a reference that time at which the current in a particular coil is 10 amp, show that the resultant ampere turns of the 3 coils is 1,500. Locate the axis of magnetization. Calculate the resultant ampere turns and locate the axis of magnetization for 30°, 60°, and 90° time intervals with respect to the reference coil.

2. Sketch the general character of the magnetic field within the cylindrical air core of a 6-pole stator with the rotor removed.

3. A 2-pole, 3-phase generator is connected electrically directly to a 6-pole, 3-phase motor. In one revolution of the rotating field structure of the generator, how far has the revolving magnetic field progressed in the motor? Answer the question for a 14-pole generator supplying power to an 8-pole motor. Answer the same question for a 6-pole, 3-phase generator supplying power to a 4-pole, 2-phase motor through a Scott-connected transformer bank.

4. Carry through the expansion of the terms in equation 8.1 to obtain the final result shown there.

5. A soft-iron bar on a spindle is rotated slowly between the poles of a strong 2-pole electromagnet, the plane of motion being horizontal and parallel to the magnetic field. What determines the magnetic polarity of the bar? Does its polarity change during its motion? What position would the bar take if left to swing freely on its spindle? At what points in the complete revolution of the bar is the torque on it zero? In what general positions is the torque a maximum? Would you expect any torque on a cylinder of iron if it were substituted for the bar, its axis being normal to the magnetic field? Why, or why not?

6. A coil carrying direct current replaces the bar of the preceding problem, the coil axis coincident with the axis the bar had. What is the manner of variation of torque on the coil during a complete revolution, moving in the same path as did the bar? How will the coil tend to place itself if left to swing freely on a spindle?

7. Referring to the preceding questions, suppose the coil is now slipped over the bar. What determines the polarity of the bar? Assuming the torque variations to be sinusoidal, and the maximum torque for the coil when carrying current to be four times that for the bar, plot the component and resultant torque-angle curves. If the poles of the electromagnet were to revolve about a point midway between themselves, what type of motor would this assembly represent? If the current to the coil were suddenly reversed, what effect would be observed? Would it be in the forward or reverse direction? What would determine this direction? Would the relative polarities of coil and bar be, in general, predictable?

8. The rotor of a 3-phase, fractional-horsepower motor is removed and in its place is mounted a solid iron cylinder. Will the cylinder have a tendency to rotate under the influence of a revolving stator field? Compare the physical observations you would expect for the solid cylinder with those for a cylinder laminated along its axis. What would be the effect of increasing the number of laminations?

9. A small squirrel-cage made of brass is slipped over each cylinder, in turn, of the preceding problem. Would you expect an appreciable difference in torque to be noted in the two cases? Would the squirrel-cage rotate without either iron core, assuming it properly mounted and free to turn? How would the torque on the cage alone compare with the torques with the iron cores present? What are the functions of the iron core in any machine?

10. What determines the amount of flux in the stator cores of the synchronous and induction machines? How is this flux affected by the length of the air gap between stator and rotor? What mechanical considerations are required on a machine having a small air gap? What general electric and magnetic effects result from a long air gap? Does the length of the gap necessarily control the torque in the synchronous, induction, and d-c machines?

11. In place of the squirrel-cage of problem 9, the laminated iron core is slotted to carry a 3-phase winding similar to that of the stator. If this rotor winding is short-circuited on itself, will the rotor behave as it did with the squirrel-cage around the core? Suppose the rotor winding is brought out through slip-rings to external resistors. What are the general effects of increasing the resistance inserted in each leg of the winding, assuming a constant load torque applied to the shaft?

12. A 3-phase, 220-volt, 5-hp motor of the type discussed in problem 11 has full voltage impressed on the stator terminals. A d-c voltage is applied to 2 terminals of the rotor winding, after the motor has been brought up to speed with the rotor windings initially short-circuited. What effect does the application of direct current have on the machine? What type of motor does the machine now represent? Can this type of machine be driven as a generator?

13. The rotor winding of the motor of problem 12 has 60% of the turns that are on the stator. Both windings are connected in delta. What is the approximate 1-phase secondary voltage if the rotor is blocked and the motor is used as a 1-phase transformer? How is the output voltage affected by turning the rotor? Can the motor be used as an induction regulator? If so, what are the approximate voltage limits? Can the motor be used as a 3-phase transformer? If so, what would be the expected secondary voltage? Would turning the rotor affect its magnitude?

14. Single-phase synchronous machines usually are 3-phase, Y-connected machines with one leg left idle in operation. What is the ratio of the power which the machine can carry 1-phase to that it can carry 3-phase?

15. Two synchronous machines comprise a frequency changing set. They are mechanically fastened together, and electrically are connected to power systems, one to 25-cycle lines, the other to 60-cycle lines. Power thus can be transferred back and forth between the two systems. Give the minimum number of poles of each machine and the speed of the set.

16. A 3-phase, 1,800-rpm 60-cycle synchronous motor is built with the armature winding on the rotor and the d-c excitation is supplied coils on the stator. How many magnetic poles are built into the armature winding? How many projecting poles will be on the stator? What is the direction of motion of the revolving field of the armature with respect to the rotor? At rated speed of 1,800 rpm for the machine, what is the speed of the revolving armature field with respect to the stator? Where would the squirrel-cage winding for starting be placed?

17. Each of the many coils on the rotating armature of a d-c machine carries the same magnitude of current except during the brief time interval the coil is short-circuited by a brush and its current is being reversed. All coils have approximately a 180° span. What should be the position of the brushes relative to the field poles? Where is the axis of magnetization of the armature currents relative to the field poles? What is the approximate torque-angle of the d-c machine?

18. For a given direction of motion for a d-c motor, what is the direction of the generated voltage in a single conductor under a given field pole? What is the direction of the applied voltage? Under what conditions can the generated voltage become greater than the applied voltage? Does the character of the machine change during this time interval? Does the current reverse? Will the motor change in speed? Why, or why not?

19. In making a speed test on a d-c motor the field circuit, containing a rheostat, is connected to a constant-voltage source. The armature circuit is placed across a variable-voltage supply. For constant field current, how does the speed of the motor vary with the armature voltage? If the field current is increased and the test repeated, what position would the latter characteristic have on a graph with respect to the first? If the armature voltage were held constant, what would be the general shape of the curve of speed vs. field current?

CHAPTER IX

MACHINE WINDINGS

9.1. General. Before entering upon the analysis and performance characteristics of specific types of machines, it is quite advantageous to treat briefly the types of windings that they carry, since some knowledge of these features of construction leads to a better understanding of the operation of any machine. There is no desire here, however, nor is there any requirement, to follow details of mechanical construction of a machine, except insofar as these may assist in the knowledge of the electrical characteristics to be considered. Up to this point, a great many of the distinctive features of machines already have been presented. This chapter is intended only as an introduction to the extensive study of windings in general.

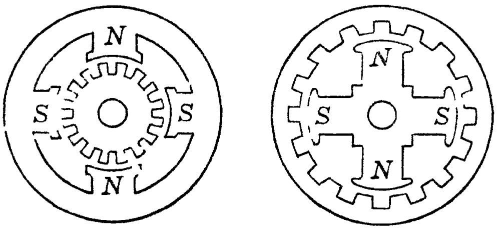
Windings on machines come under two main classifications: armature and field. The armature winding is the seat of the generated voltage and the developed torque. The field winding supplies the excitation for this voltage and the torque. Either winding may be located on the stator or on the rotor, depending upon the type of machine. The armature winding of the d-c machine is on the rotating element, and the field coils are on the stator, or frame. The armature winding of the synchronous machine practically always is on the stator, the field coils being placed on the rotor poles. In the induction motor, the excitation is supplied by the stator winding, hence those coils compose the field circuit. The armature is the rotor, holding either a squirrel-cage or a group of coils similar to those on the stator. The stator and rotor of the induction motor often are called the primary and secondary, respectively, because of the similarity to the transformer.

In reference to machines in general, it often is more desirable to speak of the stator and rotor rather than armature and field, but if a particular type of machine is understood in the discussion it should be immaterial. In this chapter, the comments made on the armature, or stator, winding of the synchronous machine are to be recognized to hold also for the stator, or field, winding of the induction machine. Further, a wound-type rotor winding for the armature of the induction motor is essentially the same as that on the stator.

The armature of a machine is composed of an iron core with slots and teeth, the slots being filled with coils which compose the armature winding. The core is built up of many layers of thin laminations (or punchings) of steel which may be circular or punched as segments. Stacked to a desired length, with spacers placed every few inches along the stacking length to allow for ventilation, and bolted together to form one compact unit, we have the armature core.

In the case of the d-c machine, the core is on the central or rotating body and may be pressed onto the shaft or mounted on a spider which in turn is secured to the shaft. In the synchronous machine, the core is practically always the outer body and is stationary. It forms a cylindrical shell about the field structure. Figure 9.1 shows sketches of these two general types of armature core, and the manner in which the field poles are placed with respect to the armatures.

The rotating body of the d-c machine also carries a commutator placed directly adjacent to one end of the armature stacking. The commutator is in turn a group of copper segments (commutator bars) insulated from each other but connected in a manner to the ends of the armature coils. Brushes resting on the commutator thus permit current exchange between the armature and an external load or source of power. As pointed out previously, the commutator is nothing more than a mechanical rotating switch which maintains a direct voltage and current at the machine terminals while the individual armature conductors have induced in them alternating voltages and currents. Figure 9.2 shows two views of a commutator bar, the bar extending lengthwise of the shaft and tapered in a radial direction. The end notches permit V-shaped clamping rings to bind the assembly tightly together, and the riser carries a slot in the top in which is soldered the ends of two coils.



(a) D-c machine (b) A-c machine

FIG. 9.1. General types of core.

The rotating body of the usual a-c machine carries two slip rings on which rest brushes to permit direct current in the field coils carried by the rotor. If the armature revolves, the over-all appearance of the a-c machine then may be similar to that of the d-c machine with the exception that the slip rings replace the commutator. In the case of the synchronous converter, the same armature winding converts a-c to d-c, or vice versa. The shaft hence carries a commutator at one end of the armature winding and slip rings at the other, connections being made usually at opposite ends of the winding. The rotor of the induction motor carries slip-rings for connecting the rotor, or armature, winding to external resistors when that winding is of the wound type.

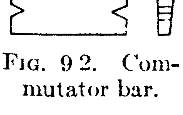


FIG. 9.2. Commutator bar.

9.2. General Features of Windings. The types of winding to which this discussion is limited are the most usual of those met in practice. Although there are variations, they are not of sufficient consequence to warrant treatment here. Further, for the sake of simplicity, it is better to restrict our attention only to those representing by far the greater number actually found on present-day armatures. A knowledge of the essential features of the general winding will permit a ready understanding of the few departures or other details of construction.

We shall treat with those armature coils which are wound on a special form of the shape the coils are to take, or which are wound in a loop and then pulled into final shape by an adjustable spreading machine. The diamond-shaped coil is the most common form, Figure 9.3. The coil may have but a single turn, or it may have several turns. Each turn may have but a single strand, or it may have several strands of copper mechanically and electrically in parallel. Stranding gives greater flexibility in forming the coil, and it decreases eddy-current losses caused by flux variations in the copper when the winding carries load current.

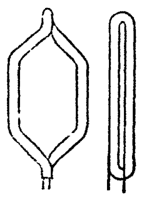


FIG. 9.3. The diamond-shaped armature coil.

The following statements are made by way of definition:

1. Strands of copper, electrically in parallel, compose a conductor.

2. An active conductor here is defined as that length of copper lying in a single slot, and thus a single turn is composed of 2 conductors. The end loops of the coils serve as connections between the conductors.

3. A coil may be a single turn, or it may be composed of many turns, the turns being in series electrically.

4. A coil has 2 coil sides which lie in slots approximately a pole pitch from each other. Looking down upon the coil in its position in the slots, with the leads towards one, the coil sides will be seen to lie in slots approximately one pole pitch distant from each other. Either coil side (left-hand or right-hand) may rest in the top half of its slot, the other coil side then being at the bottom of (or at a lower position in) its slot.

5. A slot will contain at least 2 coil sides (this is equivalent to 1 coil), but may have any even number. A slot for the usual a-c machine will have but 2 coil sides, one above the other. The d-c machine may show other configurations of coil sides such as 3 horizontal and 4 vertical, this being essentially the consolidation of 6 slots of 2 coil sides each.

6. A winding 2 coil sides deep in a slot is a double-layer winding. A-c windings are of this character; d-c windings may have more than 2 layers.

7. The number of coils obviously is a multiple of the number of slots.

8. Whereas coils in general have their coil sides approximately a pole pitch apart, most a-c machines have short-pitch windings in which the coil span may be as low as 90 degrees. Common pitches are $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{9}$, etc., for the a-c machine. Short-pitching reduces the magnitudes of harmonics, improves wave shape, saves copper, and permits end-connections between coils to be made more easily.

9. A coil span (or coil throw) may be given in terms of the electrical degrees between its coil sides or by the numbers of the slots it occupies. A coil lying in slots 1 and 8 of a machine having 9 slots per pole has a throw of 7 slots, a pitch of $\frac{7}{9}$ths, and spans 140 degrees. These statements all say the same thing.

Beginning with any slot, the coils are laid in to fill all consecutive positions occupied by the upper coil sides. In passing around the armature once in this manner, the lower coil sides will have filled the other half of the slot openings.

The winding is then in position for all end connections to be made. It is to the ways of connecting coil ends that we now turn our attention.

9.3. A-c Windings. Treating the subject by way of a special case, consider a 3-phase, 6-pole, 54-slot machine. Eighteen slots in all are given to any phase, *A*, which means, for a symmetrical winding, 3 slots under each pole. Slots 1, 2, and 3 are assigned to phase *A*; 4, 5, and 6 to phase *B* (or *C*); and 7, 8, and 9 to phase *C* (or *B*). The phase windings occupy the higher-numbered slots in the same sequence.

The winding will be assumed to be double layer with 2 coil sides per slot and with a coil throw of 1 to 9. Figure 9.4 shows the positions and connections of the

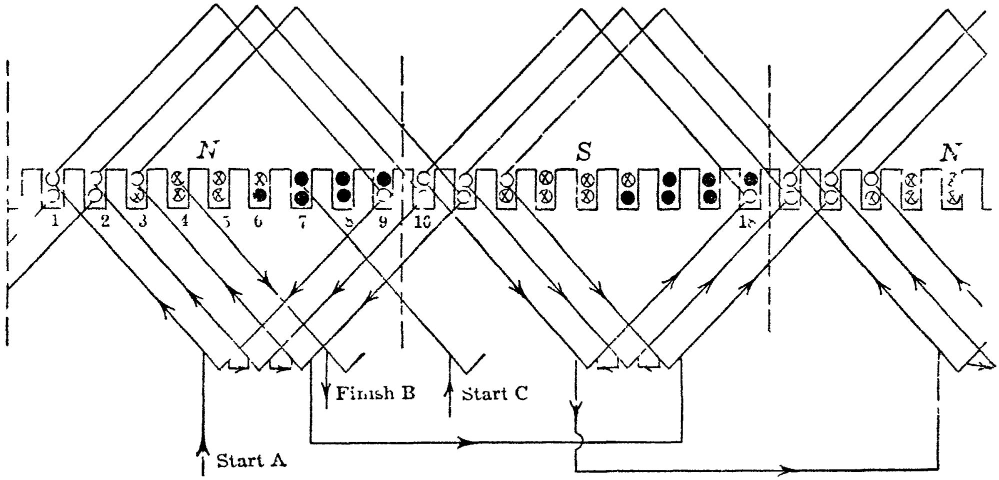


Fig. 9.4. Developed winding for the first 3 polar groups of phase *A* of a 3-phase, 6-pole, 54-slot a-c machine. Short pitch. End terminal of phase *B*, and starting terminal of phase *C* also are indicated.

armature coils for phase *A* which lie under the first 2 poles. Starting at the entrance lead of the first coil, the winding passes through the top half of slot 1 to the back of the armature and thence returns through the lower half of slot 9. After completing the several turns of the first coil, connection is made to the second coil which lies in slots 2 and 10, and then to the third coil. These 3 coils constitute a *polar group* of phase *A*. All other polar groups for the 3 phases are arranged in an identical manner. There will be 18 polar groups in all.

The stator winding must of course be wound for the same number of magnetic poles as are on the rotor, and the polar groups then must be so connected that, when carrying current, they produce alternate north and south poles. This step is accomplished, as shown in Figure 9.4, by connecting the output, or right-hand, terminal of the first polar group to the right-hand terminal of the 4th polar group. The arrows on the coils and their connections show the path traversed. The paths for the 7th and 10th polar groups, and for the 13th and 16th, are exactly similar to that for the first two, whence the left-hand terminal of the 16th polar group becomes the output terminal of phase *A* when all of its groups are in series.

If slots 4, 5, and 6 are assigned to phase *B*, it is observed that the windings of phase *B* are 60 degrees in position from phase *A*, and that phase *C* is 120 degrees from phase *A*. Because of this placement of the coils, the generated voltages of the three phases will appear 60 degrees in time from each other in this order. However, for either Y or delta systems, these phase voltages must appear at the machine terminals 120 degrees from each other, and this step may be accomplished in one or the other of two ways. In the first instance, which is that employed in practice, electrical symmetry is obtained by letting the starting ends of phases *A* and *C*, in slots 1 and 7, remain as they are. The terminals of phase *B* then are reversed in sense to let the coil lead from slot 4 become the output terminal, the other extremity to be its starting point. This is equivalent to reversing the vector voltage of phase *B* 180 degrees on a graph of the voltages of the three phase windings. The mutual connection of the three starting terminals thus gives a Y-connected winding, for example, or with the phases connected start-to-finish the closed delta winding is realized.

The second scheme for a symmetrical 3-phase winding calls for the first polar groups of the three separate phases to be started 120 degrees in position along the armature surface. These three entrance leads then are corresponding terminals without reversal of the terminals of any phase.

What has been said on this particular winding holds in general for other windings. Differences between windings lie mainly in change of pitch or in the number of slots per pole per phase. We have treated here with an integral number of slots per pole per phase, although in many cases this quantity is fractional. For a balanced winding, it is necessary that slots (or coils) phases = an integer. A 48-slot armature, for example, could be wound for 4 or 6 poles. The first case is that of an integral slot winding of 4 slots per pole per phase. The second is a fractional-slot winding of $2\frac{2}{3}$ slots per pole per phase in which the number of slots under the first 3 poles assigned to the respective polar groups is 3–3–2, 3–2–3, and 2–3–3. The next 3 poles repeat, to give in the end a balanced 3-phase winding of 16 coils per phase.

9.4. Circuits—Parallel Paths. In the particular example just discussed, it is seen that the 7th and 10th polar groups in series, and the 13th and 16th polar groups in series, are exactly similar to the series connection of polar groups 1 and 4, all assigned to phase *A*. This suggests the possibility of placing these 3 pairs of polar groups in parallel to give what is called a 3-circuit winding. The input end of phase *A* then is the common junction of the left-hand (starting) terminals of polar groups 1, 7, and 13. The output end of phase *A* is the common junction of the left-hand (finish) terminals of polar groups 4, 10, and 16. The polar groups of phases *B* and *C* are connected similarly. Such a completed winding, either Y or delta, may be said to have 2 poles series, 3 poles parallel. Figure 9.5 is the schematic diagram for the 3-circuit Y winding.

Likewise, all 6 polar groups of any phase can be connected in parallel to give a 6-circuit machine wherein the left-hand terminals of the 1st, 7th, and 13th polar groups, and the right-hand terminals of the 4th, 10th, and 16th polar groups are all connected together as the common input terminal of phase *A*. Again, the first 3 polar groups assigned to a given phase can be series-connected and placed in parallel with the second set of 3 polar groups of that phase to give a 2-circuit winding. The criteria for balanced windings are:

$$\begin{aligned} \frac{\text{poles}}{\text{circuits}} &= \text{integer} \\ \frac{\text{slots (or coils)}}{\text{circuits} \times \text{phases}} &= \text{integer} \end{aligned} \tag{9.1}$$

For the same flux per pole and the same speed of the rotor, the terminal emf of the stator windings decreases inversely with the number of circuits if the machine is a generator, or, if a motor, there is required a correspondingly decreased applied voltage. The line current, however, rises directly with the number of circuits, thus maintaining the same kilowatt or horsepower rating for the machine. Fundamentally, for a given rotor speed and for normal flux and current densities, the volume of a machine enclosed by the air gap and the stacking length represents a certain number of kilowatts or horsepower in output. Changes from Y to delta or in the number of circuits, therefore, should not alter the power rating of the device. Change of circuit permits change of voltage and current rating for a machine, and it is met in practice by way of factory design or in reconnection jobs.

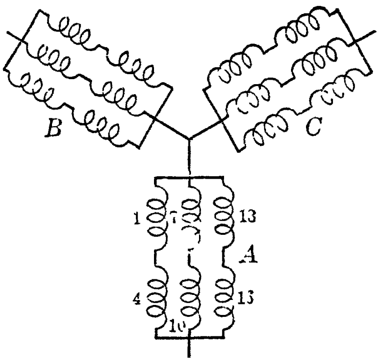


FIG. 9.5. Three-circuit Y-connection for a 6-pole a-c winding.

9.5. The D-c Lap Winding. In the lap-wound d-c armature the coil ends are fastened to adjacent commutator bars; that is, polar groups and phases do not enter now, but the output end of the first coil and the input end of the second coil find a common junction at a commutator bar. Likewise, the output end of the second coil and the input end of the third coil are connected together at the second commutator bar. This process is carried out for all coils, the total number of coils being equal to the total number of commutator bars. Theoretically, any number of coils may be used, with 2, 4, 6, etc., coil sides per slot. An average number for d-c machines may be 10 or 11 slots per pole.

Figure 9.6 shows a 4-pole, full-pitch lap winding of 16 slots and 16 coils. An actual d-c machine will have more than 4 slots per pole, perhaps in the general range of 8 to 12. Also, the case of only 2 coil sides per slot is unusual, and even though this were so, the arrangement of the coil sides in the slot would be vertical rather than horizontal. The assumed conditions and the diagram, however, are for the purpose of instruction, and what is said in this example holds fundamentally for all lap-wound armatures.

The following observations are made which should be verified by the student. For the direction of motion assumed, the induced emf's in all conductors are given by the small arrows. In passing through the winding from commutator bar 1, first in one direction and then in the other, the same direction of induced voltage is followed until one reaches the commutator bars directly under the adjacent poles One half of the total number of coils are thus taken into account.

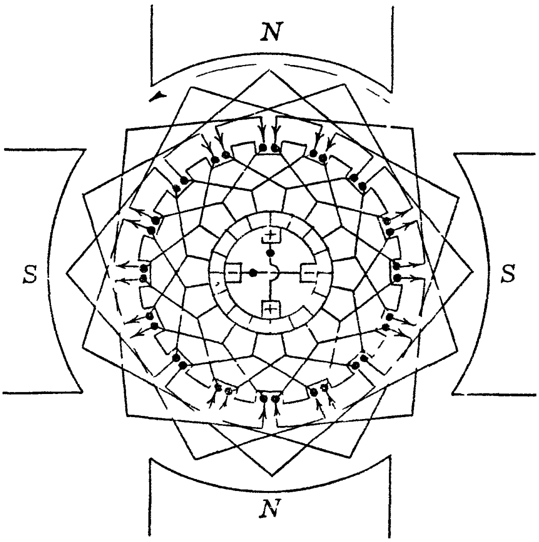


FIG. 9.6. Lap winding for a 4-pole, 16-slot, d-c machine. Full pitch. Polarity shown is for generator action.

Continuing, one arrives at commutator bar 9 by moving against the arrows and will have taken into account the other half of the coils. In passing halfway around the armature of a 6-pole machine, one goes with the arrows for the first span of 180 degrees, then against the arrows over the next pole pitch, and finally with the arrows again for the last 180-degree span.

Therefore, with brushes placed and given polarity marks as shown, it is seen that between positive and negative terminals of the machine there are 4 parallel paths, each including the same number of coils. Further, the brushes short-circuit the coils only when they are in positions of minimum or zero induced voltage and allow reversal of current in the coils as they pass the brushes. Note that it is the arrangement of brushes, coil connections to commutator bars, and motion of the armature that gives automatic switching so as to maintain direct voltages between brush sets even though the coils have alternating voltages induced in them.

Generalizing, for a lap-wound machine having any number of pairs of poles:

1. Coil ends are fastened to adjacent commutator bars.
2. The number of coils equals the number of commutator bars.

3. There are as many parallel paths through the armature winding between the 2 line leads as there are number of poles. The paths lie under adjacent poles.

4. There must be as many sets of brushes (or brush arms) as there are number of poles, in order to utilize the complete winding.

5. Brush position always is such as to short-circuit the coils when they are in the position of minimum induced voltage.

The major mechanical difference between this type of d-c winding and the a-c winding previously described lies in the end connections of the coils. Lap windings are especially adaptable to low-voltage machines because of the parallel arrangement of groups of coils. In this respect, the parallel paths through the d-c winding correspond to the circuits of the a-c winding.

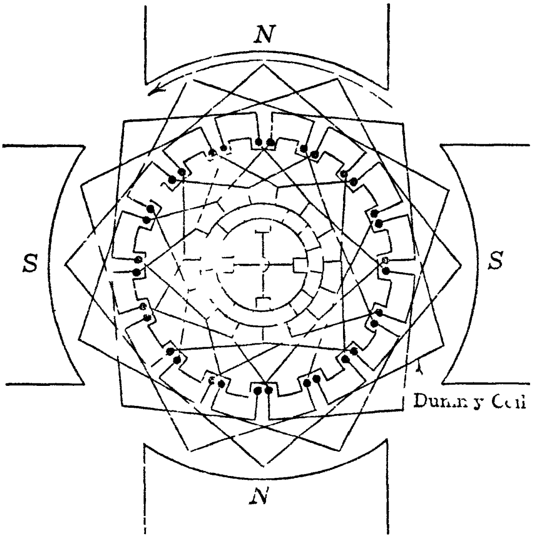


FIG. 9.7. Armature winding for a 4-pole, full pitch, 16-slot, 16-coil, 15-commutator bar, wave-wound d-c machine. Retrogressive. Dummy coil is in position but is not connected to commutator.

Irregularities due to mechanical or electrical construction may, in lap-wound machines, give rise to slightly unequal voltages in the different parallel paths through the armature. This results in circulating currents in the winding, with consequent difficulties in commutation, even at no load. To provide paths for these currents so as to keep them from the brushes, heavy copper straps are connected at the back end of the armature to similar points along the winding. All points so connected together are then brought to the same potential. Large machines usually are thus equipped and will have equalizer rings fastened to every few coils. In this case, the number of coils is divisible by the number of pairs of poles.

9.6. The D-c Wave Winding. On many d-c armatures the coil ends are spread out from each other and are fastened to commutator bars approximately two pole pitches apart rather than to adjacent bars. Such windings are known as wave (or series) windings. Figure 9.7 shows the fanned-out diagram for a 4-pole, full-pitch, 16-slot, 16-coil, 15-bar wave winding. With all coils alike and spanning the same number of commutator bars, stepping once around the armature must bring one to a commutator bar adjacent to that at which he started, either to the left or to the right. Continued passage in this manner ultimately will have included all coils and have closed the winding at the initial commutator bar.

The example chosen illustrates the point that the number of coils fastened to commutator bars (active coils) cannot be a multiple of the number of pairs of poles, else the winding closes the first time around the armature. In this case, with our chosen arrangement of slots and coils, only 15 active coils (and, of course, only 15 commutator bars) are permitted, the 16th coil being a dummy coil. It is like any other coil and mechanically balances the armature, but it never is connected. Some armatures may have more than one dummy coil for a wave winding.

The following general statements concern wave windings and hold for any number of pairs of poles:

1. Coil ends are fastened to commutator bars approximately 2 pole pitches (360 degrees) apart.
2. The number of active coils equals the number of commutator bars. There may be dummy coils.
3. The number of active coils cannot be a multiple of the number of pairs of poles, else the winding closes upon itself the first time around the armature. One less or one more will be satisfactory.
4. There are 2, and only 2, parallel paths between line terminals regardless of the number of poles. Any path is under all poles.
5. Two brush sets only are needed, but the machine carries as many sets as there are poles so as to assist commutation. Brush positions are the same as for lap windings.
6. If, in passing once around the armature, one returns to the commutator bar to the left of that at which he started, the winding is said to be retrogressive; if to the right, the winding is progressive.

Wave windings are used for higher voltages because of the series connection of the coils, that is, the minimum number of circuits.

9.7. Pitch Factors. Consider the general a-c winding. The two coil sides of a full-pitch coil have voltages induced in them in exact time phase. If the coil is short-pitched, however, the two coil-side voltages are then additive at that angle lost by short pitching, and the resultant coil voltage is less than twice the voltage of either coil side. For example, in the case of the a-c winding discussed, full pitch calls for a coil throw of 9 slots whereas the actual pitch is $\frac{8}{9}$ths, or a span of 160 degrees. The 2 coil-side voltages add at 20 degrees, hence

$$E_{\text{coil}} = 2E_{\text{coil-side}} \cos \left(\frac{20}{2}\right)$$

where $\cos \frac{20}{2}$ is the pitch factor for this particular winding, Figure 9.8

In general,

$$K_p \text{ (pitch factor)} = \cos \tfrac{1}{2} \text{ (angle lost by short pitching)}$$

$$= \frac{E_{\text{coil}}}{2E_{\text{coil-side}}} \leqq 1 \tag{9.2}$$

K_p is readily computed for each particular winding.

9.8. Distribution Factors. Again, in the case of a-c windings, the several coils of a polar group add together at their respective slot angles measured from the first coil side. For the particular case considered, the angle between slots is 20 degrees, hence the voltage of the second coil adds at 20 degrees with that of the first coil, and the third coil voltage adds at 40 degrees with respect to the first. The resultant voltage of the polar group is less than 3 times the voltage of any coil because of the distribution of the winding among the 3 slots. Figure 9.9 shows the manner of obtaining the resultant voltage of the polar group.

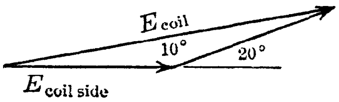


FIG. 9.8. Determination of coil voltage.

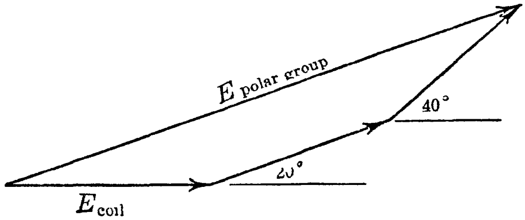


FIG. 9.9. Determination of voltage of polar group.

By definition,

$$K_d \text{ (distribution factor)} = \frac{E_{\text{polar group}}}{nE} \tag{9.3}$$

where n is the number of coils in a polar group and E is the coil voltage. Distribution factors are fixed, depending upon the number of slots per pole per phase. They are given in the following table for 3-phase windings having 3, 6, 9, 12, and 18 slots per pole.

Slots Per Pole Per Phase	K_d	*Slots Per Pole Per Phase*	K_d
1	1.00	4	0.958
2	0.966	5	0.957
3	0.960	6	0.956

9.9. Generated Voltage. The average induced voltage in an armature turn is

$$E_{\text{avg}} = \frac{\Phi}{t} \text{ volts per turn}$$

where Φ is the maximum flux in webers threading the turn and t is the time in seconds in passing from these maximum linkages to zero linkages, or vice versa. If the turn initially is in position to enclose the maximum flux, it will have no flux through it $\frac{1}{4}$ cycle later. Therefore, the time of cutting Φ webers is $1/4f$ seconds, where f is the frequency. Thus,

$$E_{\text{avg}} = 4f\Phi \text{ volts per turn}$$

Remembering that the total turns per phase per circuit, T, contribute to the generated voltage per leg of the a-c machine or per path of the d-c machine, that pitch and distribution factors may modify this voltage, and that the effective voltage of the a-c machine is 1.11 times the average voltage for sine-wave generation, we have

$$E_{\text{a-c}} = 4.44f\Phi TK_pK_d \text{ volts per leg, effective} \tag{9.4}$$

Although equation 9.4 has been developed from the viewpoint of the a-c machine, it is perfectly general and holds for all machines with recognition given the significance of the modifying factors. Observe, for example, that with $K_p = 1 = K_d$, and T becoming the total primary or secondary turns, equation 9.4 is the voltage expression developed previously for the transformer. Likewise, equation 9.4 is true for the d-c machine. Fundamentally, the electrical circuit of the d-c armature is that of the single-phase armature and would give rise to the same a-c voltage between brushes were they permitted to revolve with the commutator. The coil span of the d-c machine is always so nearly 180 degrees that K_p is assumed unity in all cases and hence does not appear in the generated voltage expression. The distribution factor is $2\ \pi$, just as it is for the single-phase winding. Because of the fixed position of the brushes of the d-c machine, they always are in that position to pass to the external circuit only the maximum induced voltage, which for sine-wave generation is $\pi\ 2$ times the average. This numerical factor exactly cancels the distribution factor, and again these terms never appear in the generated voltage equation for the d-c machine. Both factors, nevertheless, physically are present even though not stated explicitly. The form factor 1.11 now does not show in the final result. The turns per phase per circuit of the a-c winding become the turns per path of the d-c winding, and they depend upon the type of winding—lap or wave.

It is usual to develop the generated voltage expression for the d-c machine by observing that a single conductor cuts ΦP webers in one revolution of the armature. Φ is the total flux per pole and P is the number of poles. The lines cut per second are $\Phi PN/60$ where N is the speed of the machine in rpm. With Z conductors total on the armature, arranged in p' parallel paths, the voltage between terminals is

$$E_{\text{d-c}} = \frac{\Phi PNZ}{60p'} \text{ volts} \tag{9.5}$$

The student should show that equations 9.4 and 9.5 are exactly equivalent under the interpretations discussed above on the several terms in the equations. It will be found that $E_{\text{d-c}} = \sqrt{2}E_{\text{a-c}}$. Such a result has an extended significance in that for the same single-phase machine, with slip rings at one end of the armature and a commutator at the other, we now have the conversion ratio between the input a-c voltage and the output d-c voltage, or vice versa. This is the synchronous converter which transforms electrical energy of one form to that of another. Note that we have explicitly assumed sine-wave generation in the entire discussion. Non-sinusoidal voltage generation leads to a form factor different from 1.11, hence alters the magnitude of the a-c voltage and, consequently, the ratio $E_{\text{a-c}}/E_{\text{d-c}}$.

9.10. Field Windings. Any calculations on field coils of machines follow knowledge on the ampere-turns required to drive normal flux through the magnetic circuit. The problem then becomes one of designing the field coils to provide this driving force, taking into account excitation voltage and limitations on heating and space.

Field coils of synchronous machines normally are wound with many turns of relatively small wire, although edge-wound strap copper often is employed. The spool on which the coil is wound is slipped over the pole body before the latter is bolted or keyed in position. For large turbo-generators, the field coils are made of bar copper of large cross-sectional area, are preformed to give concentric groups, and are held in place in longitudinal slots on the rotor by non-magnetic wedges.

Normal excitation voltages for the synchronous machine are 125 and 250 volts, frequently obtained from a d-c generator, called an exciter, mounted on the shaft of the machine. Depending upon the size of the synchronous machine, and the manner in which its excitation is obtained, the field current is controlled by a rheostat in the field of the main unit or in the field circuit of the exciter.

The field coils of the d-c shunt machine, in which the field circuit is in parallel with the armature, contain many turns of small wire. The magnitude of the field current is but a few per cent of the armature current. Control is usually obtained by a rheostat in the field circuit. The series-field coils of series and compound d-c machines carry the armature, or load, current and hence are composed of only a few turns of fairly large wire or bar copper.

PROBLEMS

1. Compute K_d for an 8-pole, 3-phase machine having 120 slots; for a 6-pole machine having 108 slots. Compute K_d for a 6-pole, 2-phase machine having 48 slots.

2. Draw the winding diagram and show the connections for one circuit of a 3-phase, 6-pole, 36-slot machine having a double-layer winding of 4 turns per coil and $\frac{2}{3}$ pitch. Assume the machine to be, first, a 2-circuit, Y-connected alternator, and, second, to be a 3-circuit, delta-connected alternator.

3. An alternator has a double-layer winding of $\frac{5}{7}$ pitch and 12 turns per coil. The pole-face area is 90 sq in. and the flux density is 40,000 lines per sq in. The stator carries 72 slots and is to have a 6-pole, 3-phase, 1-circuit, delta winding. Calculate the terminal voltage for a speed of 1,200 rpm.

4. A 60-cycle, 64-pole, 3-phase, 2-circuit, Y-connected alternator has 576 slots. Each coil of the double-layer winding has 3 turns and a throw of 1 to 7. Calculate the terminal voltage for a flux per pole of 0.04 weber.

5. A synchronous machine has the following data: 4 poles, 60 cycles, 3 phases, 24 slots, 20 turns per coil, coil throw of 1 to 6, 2 circuits, Y-connected. The flux density is 80,000 lines per sq in. maximum at the pole center, with sinusoidal distribution. Armature stacking length is 12 in., and the radius to the coils is 6 in. Calculate the no-load terminal voltage.

6. A 3-phase, 60-cycle, 8-pole, 72-slot alternator has a winding with 24 turns per coil and a throw of 1 to 9. The stacking length is 6 in. and the inside armature diameter is 16 in. What is the required flux per pole if the machine is to generate 440 volts between lines for a coil arrangement of 4 circuits delta? What is the flux density at the pole center if the flux distribution is sinusoidal?

7. A synchronous motor has a 6-pole, 3-phase, 1-circuit, 72-coil, Y-connected winding with 5 turns per coil and a pitch of 75%. Calculate the flux per pole for 440 volts between lines at 1,200 rpm.

8. A 3-phase, 8-pole, 900-rpm alternator has a flux of 0.036 weber per pole. The winding is distributed among 96 slots, is 2-circuit Y, has $\frac{2}{3}$ pitch, and generates 440 volts between lines. Sketch the winding configuration and its connections.

9. For a 6-pole, 3-phase, a-c machine, what numbers of slots between 36 and 72 will give balanced leg voltages? Which are for integral-slot, and which are for fractional-slot, windings?

10. The following stator windings are for 440-volt motors. Select those which can be reconnected for 220 volts; for 110 volts.

Poles	*Slots*	*Connection*	*Poles*	*Slots*	*Connection*
4	48	1 circ. Y	8	90	2 circ Y
6	48	1 circ. delta	10	90	2 circ. Y
6	72	1 circ. delta	10	108	1 circ. Y
8	72	1 circ. delta	10	120	1 circ. delta
8	90	1 circ. delta	14	120	1 circ. delta

11. An 8-pole, 500-kva, 440-volt, 900-rpm, 3-phase synchronous motor has 96 slots. The winding is double layer with 58.3% pitch and 4 turns per coil. The flux is 3.92×10^6 lines per pole. What is the connection of the winding? How should the winding be reconnected if it is to operate on 500 volts approximately?

12. A 3-phase, 10-pole, 880-volt, 600-rpm synchronous motor has 120 slots. The winding is double layer, each coil having 12 turns and a throw of 1 to 9. The flux is 0.0287 weber per pole. How is the winding connected?

13. A 2-phase, 8-pole, 750-rpm motor has 96 slots. The coils have 8 turns each, are of $\frac{2}{3}$ pitch, and are arranged in 4 circuits per leg to give 440 volts per phase. Can this winding be reconnected for 220, or 440 volts, 3-phase, 60-cycles, without changing normal flux more than 10%?

14. A 3-phase, 8-pole synchronous motor has 108 slots. It is connected 4-circuit Y. Each coil has 10 turns of 2 parallel strands 0.16×0.12 in. each. Mean length turn (MLT) is 61 in. Double-layer winding. Calculate the 60° resistance between terminals.

15. The stator of a 500 kva, 3-phase, 500-hp, 8-pole, 550-volt, 375-rpm motor has 96 slots. Each coil of the double-layer winding has 8 turns, with conductors composed of 2 parallel strands 0.17×0.10 in. each. MLT = 85 in. The winding is arranged 4-circuit delta. What is the 60° resistance between terminals of the winding? What is the per cent resistance of the winding?

16. A 3-phase, 6-pole, 250-kva, 500-rpm, 220-volt motor has a double-layer winding arranged 6-circuit Y in 72 slots. Each coil has 4 turns, with conductors composed of 2 parallel strands 0.25×0.10 in. each strand. MLT = 90 in. What is the resistance of the winding between terminals, and what is the per cent resistance? Assume 60°C.

17. The 60° resistance between terminals of a 3-phase, 6-pole alternator is 0.00142 ohm. A spare coil has 2 turns, each turn composed of 2 parallel strands 0.225×0.10 in.

each. MLT = 60 in. If the machine has 108 coils what is the probable connection of the winding?

18. A lap-wound, d-c generator has 720 conductors arranged in coils of 8 turns each. How many commutator bars are on the armature?

19. A 6-pole, 1,200-rpm, lap-wound generator has 64 coils of 4 turns each. What is the flux per pole for a generated voltage of 250 volts?

20. An 8-pole, 1,150-rpm, wave-wound d-c generator is designed for 20 volts between commutator bars. If each coil has 2 turns, what is the flux per pole to give 500 volts? What would be the voltage if the same coils were reconnected for a lap winding?

21. A 4-pole, 1,800-rpm, 125-volt, lap-wound exciter has 36 armature coils. If the flux per pole is 0.0145 weber, how many turns are in each coil?

22. A 4-pole, wave-wound, d-c machine is designed from a standard armature which can carry 48 coils. What number of commutator bars will the new winding have? Give the numbers of the commutator bars, for both retrogressive and progressive windings, touched by the winding in moving twice around the armature.

23. A 6-pole, 54-slot armature is to be wave-wound with 4 coil sides per slot. Give the commutator bar sequence numbers in moving twice around the armature.

24. A 4-pole wave winding is to be placed in a 33-slot armature. Discuss the possibilities of using 1, 2, or 3 coils per slot.

25. The armature winding of a 6-pole, 400-kw, 250-volt, 1,200-rpm, lap-wound, d-c generator is composed of 108 1-turn coils. The cross-section of the copper is 0.50 × 0.21 in. MLT = 70 in. What is the percentage loss in the armature winding at full load? Assume 60°C.

26. A 24-pole, lap-wound, d-c generator has 360 slots with 6 conductors per slot, each conductor composed of 2 parallel strands of rectangular copper 0.08 × 0.50 in. in section. MLT = 120 in. Calculate the 60° resistance of the winding.

27. A 35-hp, 230-volt, 4-pole, 1,000-rpm, wave-wound motor has 81 commutator bars. Each coil has 2 turns, and the conductor section is 0.139 × 0.234 in. MLT = 44.5 in. If the armature efficiency is 89%, what is the per cent armature copper loss at 60°C?

28. A 220-volt, wave-wound, d-c armature has 45 slots and 135 commutator bars. Each slot carries 12 rectangular conductors of 0.06 × 0.40 in. section per conductor. MLT = 52 in. Calculate the 60° resistance of the winding.

29. A 3-phase, 12-pole, 250-rpm, 600-volt synchronous converter has 252 slots, 504 commutator bars, and single-turn, full-pitch, lap-wound coils. What flux per pole is required? Could 250 slots be used? Why or why not? If not, what are the nearest higher and lower numbers that would be satisfactory?

30. Show that $K_d = 2\ \pi$ for a 1-phase winding having a series connection of all coils in the slots over a complete pole pitch.

31. A 6-pole, 400-kw, 250-volt d-c generator at normal excitation requires 5,600 NI per pole for its shunt-field circuit. Each coil has 920 turns. Wire size is 0.072 × 0.072 in., and MLT = 42 in. What is the per cent voltage drop across the field coils at 60°C if the excitation voltage is 250 volts?

32. At normal excitation a 35-hp, 230-volt, 1,000-rpm, 4-pole shunt motor requires 3,320 NI per pole. Each field spool has 1,890 turns of No. 18 wire, 27.8 in. MLT. What is normal field current? Maximum field current? Total weight of copper in the field coils? Assume 60°C.

33. A 4-pole, 250-kva, 750-rpm, 2,300-volt synchronous motor requires 11,500 NI per pole for normal field flux. Each coil has 875 turns of No. 10 wire, MLT = 46 in. What is the normal voltage drop across the field coils at 75°C? What is the maximum field current on 250 volts?

CHAPTER X

SYNCHRONOUS MACHINES

10.1. Introduction. The essential principles underlying the synchronous machine already have been discussed. We have seen that uniform motion of the rotor of the alternator gives rise to a balanced polyphase system of voltages at the terminals of a polyphase armature winding. A similar machine, its terminals connected to those of the alternator, then is driven as a motor by virtue of the revolving field produced in its air gap. As pointed out, the difference between any synchronous machine acting as a generator or as a motor lies in the direction of power flow. The magnitude of the power flow is exhibited by the angular displacement of the rotor with respect to the revolving stator field, and the direction of power flow is shown by the relative motion of the rotor.

We come now to an analysis of the synchronous machine in which a study is made of its steady-state performance as influenced by characteristics of the magnetic circuit. Primarily, the 3-phase machine will be considered. Further, we shall treat as the fundamental and general type the salient-pole machine which has on the rotor definite projecting poles each with its own field coil. In general, all field coils are connected in series, although there are a few instances in which the coils are placed in parallel or in a series-parallel arrangement. Excitation often is obtained from an exciter mounted on the shaft of the main machine. The stator winding, of course, will be wound for the same number of magnetic poles as there are poles on the rotor.

10.2. Types of Rotor Construction. Although the stator construction of the synchronous machine is basically the same regardless of physical size or any of the quantities in the nameplate data, the rotors may differ in form. Practically all synchronous motors have salient poles (see Figure 9.1b) in which the air gap over a pole pitch varies greatly, but generators may, as in the case of turboalternators, approach a more uniform air gap.

The cylindrical-rotor machine is essentially a special case of the general salient-pole type, the interpolar regions now being equally filled with iron. The same magnetic effects are then obtained over those areas as over the axes of the field windings. The closest approach to cylindrical-rotor construction is that in which the rotor is slotted uniformly around the periphery the same as the stator. The rotor appears similar to the armature core of the d-c machine, Figure 9.1a. The rotor winding may or may not be similar to the stator winding, but it carries direct current. A wound-rotor induction motor, for example, can be operated as a cylindrical-rotor synchronous machine if direct current is supplied the rotor winding. Fundamentally, the salient-pole machine is the universal type, but the theory of the cylindrical-rotor machine is simpler, and from the vector diagram for it the diagram for the other is obtained readily.

The turbo-alternator rotor often is said to be cylindrical. However, the rotor body is slotted deeply in the interpolar axes in order to carry the heavy field windings, and the equivalent air gap over the slotted regions may be large in comparison with the air gap over the rotor pole faces. That is, it is not the mere rotor diameter which is of concern, but the equivalent air gap around the periphery of the rotor. The turbo-alternator hence may be looked upon as a salient-pole generator but with much less variation in air gap than that of the rotor of Figure 9.1b.

Rotor construction has a pronounced effect on torque production. As we have seen, reluctance torque exists in the salient-pole machine because flux is confined to fixed rotor areas. The stator mmf wave aligns the rotor so as to give a path of minimum reluctance for the air-gap flux, and the rotor has a very strong tendency to remain in that position with respect to the revolving stator field. There is no reluctance torque, though, in the cylindrical-rotor machine. It can carry load only when field current is present to fix the position of the rotor magnetic poles. Reluctance torque in the salient-pole machine greatly reduces the power angle as compared with the angle for the equivalent cylindrical-rotor machine.

10.3. Viewpoint of Analysis. The effect of the highly-variable air gap around the rotor periphery must be taken into account in the analysis of the synchronous machine. The machine is studied by considering any one of the phases of the armature (the other two are similar), thus reducing the problem to an analysis of a simpler single-phase circuit. By assigning to the armature winding appropriate reactances to account for the magnetic effects of the variable air gap, the performance of the machine then may be given by a vector diagram showing leg voltage, leg current, power factor, rotor angle displacement under load, and excitation voltage which leads to determination of field current for a given condition of operation. This is the scope of the present chapter.

The alternator and the synchronous motor are studied together since, although some variations between them may exist in mechanical design, the basic theory is identical. Essentially, they are the same machine, and either may serve as a generator or as a motor. The underlying philosophy in the presentation is that the machine is but an electric circuit and hence is discussed merely as an extension and application of single-phase circuit theory, the analysis of any phase of the machine holding for the others. Also, it makes no difference whether the machine is Y- or delta-connected. Any general treatment does not require that one type of connection be distinguished from the other, and, in fact, no test at the terminals of a machine can give information as to whether or not the windings are in Y or delta. One can gain such knowledge only by actually tracing out the connections. A delta-connected machine very often, if not usually, is analyzed by way of its equivalent Y circuit, that is, by assuming it Y-connected and treating it as such.

10.4. Armature Reaction and Leakage Reactance. For a single, unloaded alternator, only the flux created by the d-c field mmf is present to generate voltages in the armature conductors, and it is these voltages that appear across the terminals of the machine at no load. When the machine is carrying load, however, the 3-phase armature currents produce an additional rotating field as was explained in Chapter VIII. This field of *armature reaction* rotates at synchronous speed, i.e., at the same uniform speed as the poles carrying the d-c field excitation, and, hence, are stationary with respect to the field poles for any constant value of load. For an alternator, with the armature current wholly leading, the armature mmf is directly along the axis of the field poles and is additive with the d-c field mmf, thereby tending to increase the resultant air-gap flux. If the armature current is wholly lagging (a 180 degree reversal from that of the leading case), the two mmf's are directly subtractive. In the case of the synchronous motor the direction of the armature mmf is in the opposite direction, i.e., if the motor were taking a nearly 90-degree leading current, the armature mmf would directly oppose the field mmf, while for a 90-degree lagging current the two mmf's are directly additive If the armature current is in phase with the voltage generated by the main d-c field, the armature mmf is said to be cross-magnetizing because it acts upon magnetic paths midway between the rotor field poles: this is true for either alternator or motor operation. Simply expressed, armature reaction is the effect of the armature mmf acting upon the magnetic circuit of the machine, the combination of armature and field mmf's leading to a resultant flux which, in turn, yields a resultant air-gap voltage.

The effects of armature reaction and leakage reactance are to be discussed now, to the extent that they may be taken into account in a vector diagram for either the alternator or the synchronous motor. The flux of armature reaction in the machine, at least the component acting along the pole axis, corresponds to the mutual flux in the transformer in that it exists in the magnetic path mutual to the field and armature windings. Corresponding to leakage flux of the transformer, there is some flux produced by the armature mmf which links with that winding alone and does not reach the field coils. One component of leakage flux encircles the armature conductors by crossing the slots and closing its path through the iron below the bottom of the slots. Another component may link the conductors of several slots in this way. Another component encloses the coils by passing from tooth to tooth in the air gap, or from tooth to pole face and back to a tooth, returning through the iron below the slots. And yet another component encloses the coil ends which project beyond the armature stacking. Armature leakage reactance produces a voltage drop in the machine, when it is carrying current, that leads the current by 90 degrees as is the case for any inductive reactance carrying current.

If saturation does not exist (it is neglected in this discussion), one may think of component fluxes produced in the air gap by the field and armature windings. One also may go a step further and speak of the induced voltages caused by the component fluxes. The induced voltage of armature reaction will be 90 degrees out of phase with the armature current in the same way that the induced voltage in any coil follows the flux variation by 90 degrees in time. Since the induced voltage of armature reaction in any leg of the machine has the same direction and phase relation with respect to the leg current as the voltage drop across an inductive reactance inserted in the current path, it is treated in the synchronous machine by substituting for it an equivalent reactance. This is called the reactance of armature reaction.

By means of a diagram like that shown in Figure 10.1, one may indicate the several voltages that exist in each leg of the machine. The circuit is applicable to both the alternator and the motor. The following tabulation defines and describes the quantities shown in the figure:

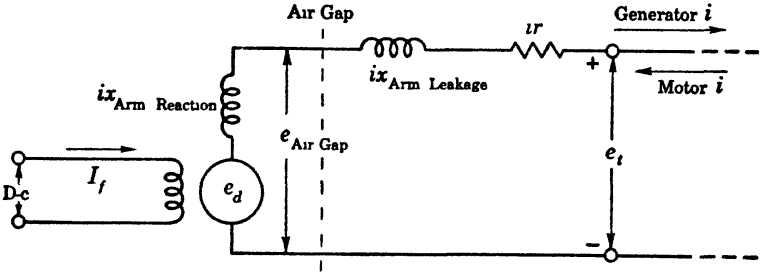


FIG. 10.1 Voltage components in the synchronous machine

I_f = d-c field current conceived to generate e_d.

e_d = armature induced leg voltage caused by the main field flux. With saturation neglected, it is proportional to I_f. In the alternator, it corresponds to that voltage which would be read at the machine terminals if the external circuit were opened.

i = armature and line current.

$ix_{\text{arm. reaction}}$ = voltage induced in the armature coils due to the air-gap flux produced by the armature current i.

$e_{\text{air-gap}}$ = voltage generated by the net air-gap flux, the net air-gap flux being that which is created by the resultant field and armature mmf's.

$ix_{\text{arm. leakage}}$ = voltage drop of armature leakage reactance.

ir = voltage drop of armature resistance.

e_t = terminal leg voltage. For the alternator it is the load voltage and for the motor the applied voltage.

Since the values of ir and $ix_{\text{arm. leakage}}$ are relatively small, the terminal and air-gap voltages are, to a first approximation, equal. To this degree of approximation, it follows that the net air-gap flux, for constant terminal voltage, does not vary with load.

10.5. Components of Armature Reaction. In the case of the cylindrical-rotor machine, the armature mmf acts on a path of uniform reluctance throughout the entire air gap, but with the salient-pole machine the armature mmf meets an air gap which is minimum at the middle of the field poles and maximum midway between the poles. As an aid to visualizing the effects resulting therefrom, consider Figure 10.2 in which an armature mmf of the same magnitude acts first on the *direct axis* passing through the middle of the pole and, second, on the *quadrature axis* midway between the poles. The flux waves of Figure 10.2 are typical for the two paths of greatly different reluctance, and the fundamental frequency components in general are in the ratio 2:1 for the direct and quadrature axes. Since the vector diagram can include only those quantities having the fundamental frequency, harmonics are neglected.

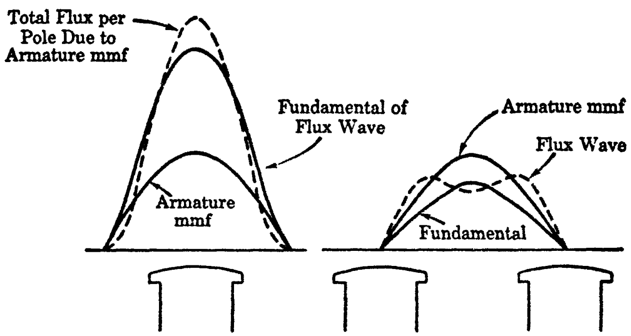


FIG. 10.2. Armature reaction flux in direct and quadrature axes of a salient pole machine.

Because the armature mmf of a machine may have any position between these two extremes, depending upon the power factor, the stator voltage which ultimately is induced by it also will vary between limits set by the air gap. If the mmf lies over the poles, the flux, induced voltage, and consequent reactance of armature reaction are greatest. If the mmf lies over the quadrature axis, these quantities are a minimum. We are thus led to the scheme of analysis first proposed by Blondel in 1899 in which the armature mmf is resolved into two components, one acting directly over the poles and called the *direct-axis component*, the other acting over the axis midway between the poles and called the *quadrature-axis component*. Such an artifice is entirely conceivable because any sine wave can be resolved readily into two sinusoidal components of different magnitudes but at 90 electrical degrees with each other. The IR and IX_L drops of a coil, for example, are simply the 90-degree components of the applied voltage. Thus, each of the two complementary systems, the direct and the quadrature, 90 degrees apart and revolving in the same direction at synchronous speed, has its own set of currents, mmf's, reactions, reactances of armature reaction, and reactive voltage drops within the machine. The armature mmf, then, regardless of position with respect to the field poles, will be split into two components which are referred to the direct axis of the poles and the quadrature axis midway between them. The scheme is very convenient because, as we shall see, these two components of reactance of armature reaction are included in reactances which are readily obtainable. These reactances form part of the general design or test information on the synchronous machine. No resolution of these quantities into components is necessary, of course, for the cylindrical-rotor machine because it is simply the special case of the salient-pole type in which the direct- and quadrature-axis components are equal to each other.

10.6. Synchronous Reactance and Impedance. Since the effect of the armature mmf reacting back on the armature winding can be expressed in terms of an equivalent reactance, armature leakage reactance and that of armature reaction are combined into a single constant for the machine and called the *synchronous reactance*. The terminology merely means that the total effect is that of a lumped reactance in the current path and that its value is to be determined at synchronous speed (rated frequency) for the machine.

Corresponding to the direct and quadrature reactances of armature reaction, there will be, then, the *direct synchronous reactance*, x_d, and the *quadrature synchronous reactance*, x_q. Either of these two reactances usually is treated in whole; that is, it is seldom separated into its components from which we have formed it because of the difficulty in doing so. Test, as discussed in the next section, gives the synchronous reactance directly in ohms or in per cent.

If the resistance of the armature also is considered, there will result the *direct and quadrature synchronous impedances*. However, armature resistance for the usual machine is very small and, in general, will be only 1 to 5 per cent of the value of the direct synchronous reactance. The larger the machine, the smaller is the ratio of resistance to that reactance. The direct synchronous reactance drop in the usual machine, at rated current, will be 60 to 150 per cent of rated voltage, the quadrature synchronous reactance 50 to 100 per cent, and the resistance only 1 or 2 per cent. For this reason, since the percentage armature resistance is so small, and since synchronous reactance and synchronous impedance are practically equal to each other in magnitude, armature resistance is generally neglected in all but very small machines. When desired, armature resistance may be found by a d-c test, although the value so obtained will be low because the d-c test cannot include the eddy-current losses in the copper and surrounding iron created by the flux of the changing current. Such losses normally are assigned to armature resistance by giving it a slightly higher value and calling it the effective or a-c resistance, so that the I^2R loss then calculated will include these eddy-current losses as well.

It should be apparent that, because of the iron in the magnetic circuit of the machine, saturation will affect the fluxes and consequently alter the synchronous reactances. Saturation effects, however, are beyond the scope of this discussion and hence will be neglected. The circuit quantities with which we treat will be given by their unsaturated values, and our ultimate results will therefore be correct within the limits imposed by saturation. For many machines, the errors may be as great as 10 per cent, but only by detailed analyses can they be eliminated. Standard practice, in any case, is to state for any machine the unsaturated values of its constants, to calculate results on this basis, and later, if deemed necessary, to shade those results in view of the degree of saturation of the machine at its operating point.

Armature resistance, armature leakage reactance, the reactances of armature reaction, the synchronous reactances, and the synchronous impedances may be expressed either by their equivalent ohmic values or by percentages. The percentage designation means that the voltage drop of the quantity in question, when the machine is carrying rated current, is that percentage of rated voltage. This scheme of stating quantities is the same as that employed in giving percentage values to the equivalent resistances, reactances, and impedances of a transformer. For the machine, of course, rated current and voltage are those for any leg of the machine, Y or delta.

10.7. Test Determination of the Synchronous Impedances and Reactances. The direct synchronous impedance is readily measured by test. Figure 10.3 shows on the same graph the saturation curve for a machine and what is called the synchronous impedance characteristic. Both are obtained at synchronous speed, the second with the armature completely short-circuited at its terminals and an ammeter introduced to read the short-circuit current. The curves correspond exactly to the open-circuit and short-circuit tests on a transformer. If saturation were not present, the magnetization curve would follow the air-gap line, the abscissa of which represents the field current required to force the flux through the magnetic circuit of the machine against the high reluctance of the air gaps and the much smaller reluctance of the unsaturated iron. The short-circuit curve usually is a straight line up to 150 or 200 per cent of rated current.

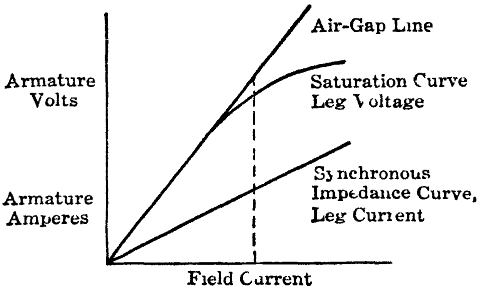


FIG. 10.3. Saturation and synchronous impedance curves of a synchronous machine.

Interpretation of the curves shows that any particular field current will give rise to a certain open-circuit voltage as read from the air-gap line (no saturation), and to a certain corresponding short-circuit current as read from its straight line characteristic. Neglecting saturation, then, the ratio of the voltage on the air-gap line to the armature current, both taken for the same field current, will be the unsaturated value of the direct synchronous impedance or the direct synchronous reactance x_d, if resistance is negligible. Since we are taking the ratio of points on two straight lines, the ratio is a constant for any value of field current. Only the direct synchronous impedance or reactance can be found in this way because, with negligible resistance, on short circuit the armature current lags the induced voltage by 90 degrees approximately, and the effect of armature reaction then is wholly in the direct axis over the poles as we already have pointed out. Further, since the mmf of armature reaction is in direct opposition to that of the d-c field, the net air-gap flux will have a low value and the magnetic circuit will be in a completely unsaturated condition. The *net* airgap voltage need be only sufficient to drive the short-circuit current through the resistance and leakage reactance of the armature circuit. The induced voltage produced by the main d-c field, e_d, is consumed in forcing the current against the direct synchronous impedance of the armature.

The following method, referred to as the "slip method," affords the best practical means of obtaining x_q and also may be used to obtain x_d. We shall here assume resistance negligible, otherwise we have the synchronous impedances from which resistance must be taken in order to obtain the synchronous reactances. The machine to be tested is left unexcited and a reduced 3-phase voltage is applied to its armature. The machine is coupled to a driving motor of sufficient capacity to overcome the reluctance torque at the reduced voltage and is slowly passed through synchronism. In this manner, the poles are slipped slowly past the stator mmf wave, and the stator magnetizing current then is a function of the rotor position with respect to the revolving field. Corresponding readings of voltage and current can be taken in the supply lines of the synchronous machine and the resulting reactances plotted as a function of time, or better, of pole position. In some cases, it may be desirable to take oscillograms of voltage and current on the same film rather than to use instruments.

If the speed of the driving motor can be controlled so that the synchronous machine is brought very slowly through synchronism, there will be no difficulty in following the swings of the voltmeter and ammeter needles. The pointers of both instruments will swing, the amplitude of the ammeter needle being much greater than that of the voltmeter and perhaps having a variation of 2:1 for maximum and minimum readings.

When the poles line up with the axes of the mmf wave, the current will be a minimum, and the ratio of the voltage to that corresponding current will be the direct synchronous reactance in ohms. Similarly, when the poles are midway between the axes of the mmf wave, the current will be a maximum and the ratio of the voltage to that current will be the quadrature synchronous reactance in ohms. It will be observed that the reactances x_d and x_q will be the maximum and minimum values, respectively. This is true because the less the reluctance of the magnetic circuit, the greater will be the flux, the greater will be the corresponding reactance, and the less the current for that position. The path reluctance for the armature circuit is a minimum when the poles are directly under the axes of the stator mmf; hence, x_d is obtained at this position.

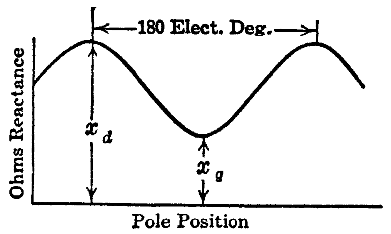


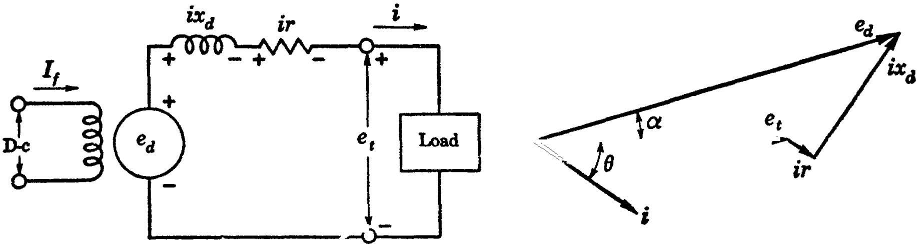
FIG. 10.4. Synchronous reactance curve for a synchronous machine obtained by a slip test.

Figure 10.4 shows the plotted synchronous reactance curve for a slip test. It is obtained as the ratio of corresponding voltages and currents as read from an oscillogram. It simply happens that the design of the usual synchronous machine is such that the variation in reactance as a function of pole position is a second harmonic of the electrical angle measured along the armature periphery.

10.8. Vector Diagram for the Cylindrical-rotor Machine. From the discussion of machine reactances, one logically goes next to the vector diagrams which illustrate certain characteristics of the machine under normal steady loading. Although never so stated, the use of the vector diagram graphically to represent the machine has come about because of the simplifying steps shown in the preceding sections of this chapter whereby the actual physical processes going on within the machine may be translated in terms of certain reactance coefficients Fundamentally, the machine is a circuit—a balanced one—and, in its treatment, is calculated on an equivalent line-to-neutral basis, whether Y- or delta-connected. Each phase winding is subjected to the same influences as the next one, although at a different time, and what holds for one is true for all. Therefore, by reducing the alternator or motor to an equivalent circuit containing resistance and reactance, it may be dealt with according to the principles of elementary a-c circuits, the analysis of the circuit to represent faithfully the machine under steady-state conditions.

The cylindrical-rotor machine will be considered first because it is the simpler, and because the vector diagram for it is needed later in the construction of that for the salient-pole machine. Furthermore, most of the basic principles and characteristics of the general synchronous machine can be explained in terms of the cylindrical-rotor machine and, for this reason, many texts present only the treatment that is applicable to this type.

Figure 10.5 shows the equivalent circuit and vector diagram for the alternator. The load power factor is lagging. The effect of the armature mmf is accounted for by the single synchronous reactance x_d. The other quantities are as defined for Figure 10.1. The conventions adopted in Chapter IV have been used in placing the polarity marks on the circuit diagram. The vector diagram simply shows e_d as the vector sum of e_t and $i(r + jx_d)$. At no load, e_d and e_t are one and the same voltage. Under load, however, they are separated by the angle α and, in general, have unequal magnitudes. Since e_d is that voltage assumed to exist by virtue of the flux which the field ampere-turns produce in the direct axis of the poles, α is then that angle by which the poles have moved forward from their no-load position when the alternator carries load. It is the torque angle, or power angle, or load angle of the machine. These angular displacements may be observed and very accurately read by means of a stroboscope illuminating a graduated disk fastened to one end of the shaft.



(a) (b)

FIG. 10 5. Equivalent circuit and vector diagram of the cylindrical-rotor generator.

Having found e_d from the vector diagram, it is taken to the air-gap line from which the corresponding value of I_f is obtained. I_f is the field current required to satisfy the given conditions of load. Conversely, if the load is removed and saturation is absent, this value of field current will cause the terminal voltage to rise to the value e_d. Actually, at no load and because of saturation, the terminal voltage will not be so great but will be approximately that read on the saturation curve directly below the air-gap line value of e_d. The percentage voltage regulation always is

$$\frac{e_{\text{no-load}} - e_{\text{full load}}}{e_{\text{full load}}} \times 100 = \text{per cent voltage regulation} \qquad (10.1)$$

where the no-load voltage is taken from the saturation curve.

Except in determining the no-load voltage, saturation can, in general, be neglected, and this method of calculating voltage regulation gives very close results for the usual machine. Under load conditions, e_d should always be carried to the air-gap line and not the saturation curve because the combination of field and armature ampere-turns never results in a net mmf sufficient to produce saturation to any extent.

In all vector diagrams and examples shown in the text material, voltages and voltage drops are expressed in terms of percentage values or their decimal equivalents. Normal terminal voltage and normal line-to-neutral voltage, therefore, are both given by $e_t = 1.0$ or 100%. Of practical importance is the fact that on a percentage basis the vector diagrams of voltages are exactly similar to those of Figure 10.5b, where 100 per cent (or unit, 1.00) voltage is the rated voltage of the machine, and 100 per cent current is the rated value. The percentage values of armature resistance and the reactances are defined as were the constants for the transformer; 100 per cent field current is that which corresponds to rated voltage on the air-gap line. If the vector diagram for the machine is drawn in terms of percentages, e_d gives directly the percentage field current necessary. If, in any calculation or vector diagram, voltage quantities are expressed in volts, rather than in percentages, *leg* values must be used.

The equivalent circuit and vector diagram for the synchronous motor are shown in Figure 10.6. The vector diagram for the synchronous motor is constructed in a manner similar to that of the alternator, with the exception that all voltage drops are reversed 180 degrees. For the motor, e_d always lags e_t; α is the angle by which the rotor drops back because of the load torque. With no load connected to the shaft of the motor, the torque angle, α, is practically zero. As load is applied to the shaft, the rotor continues to turn at synchronous speed, but falls behind its no-load position as was explained in Section 8.5. The in-phase component of motor current is set by the shaft load requirement. Adjustment of the d-c field current, which, in turn, sets the value of e_d, controls the value of the out-of-phase component of armature current. Thus, it follows that the power factor of a synchronous motor can be controlled by varying its d-c field current. In general, the range of control is sufficient so that the motor can be made to operate at either lagging, unity, or leading power factor. This characteristic of the machine and, in particular, the ability to operate at a leading power factor is one of its more useful features since it can be used to control partially the power factor of a plant. These relations are further discussed in Section 10.13.

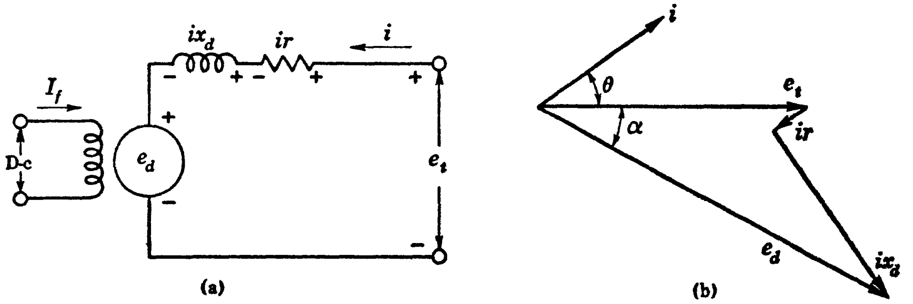


FIG. 10.6. Equivalent circuit and vector diagram of the cylindrical-rotor synchronous motor. Power factor is leading

10.9. Vector Diagram for the Salient-pole Machine. The vector diagram for the salient-pole machine is somewhat different from that for the cylindrical-rotor machine because of the presence of two components of current and their respective voltage drops. These direct and quadrature quantities are introduced because of the unlike magnetic reluctances over the poles and in the interpolar regions. The vector diagram for the salient-pole machine is the general one, that for the cylindrical-rotor machine being but a special case, but the first is constructed from the diagram for the cylindrical-rotor machine. We shall draw first the vector diagram for the salient-pole alternator and, later, show the manner in which it is obtained.

Consider Figure 10.7, for the alternator. Starting with the terminal voltage e_t as before, the load current is drawn at an assumed lagging power-factor angle, θ. This total current is resolved into two components—one which leads to flux production in the pole axes and the other which yields flux in the quadrature axes. They, therefore, have components in phase and out of phase with e_d. i_d is that component of armature current that results in an mmf directly along the axis of the field poles and, hence, is associated with the direct reactance, x_d. To the extremity of e_t is added the ir drop, not split into components because r is the same for both i_d and i_q. At right angles with i_d is drawn i_dx_d, and next, 90 degrees ahead of i_q, is shown i_qx_q. The diagram is completed by e_d, the nominal voltage for the salient-pole generator, and which, carried to the air-gap line, determines the field current. The torque angle is α.

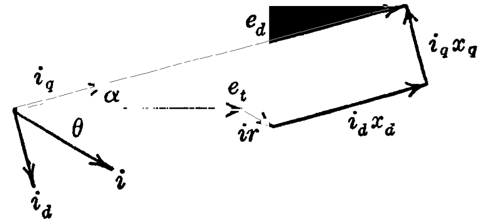
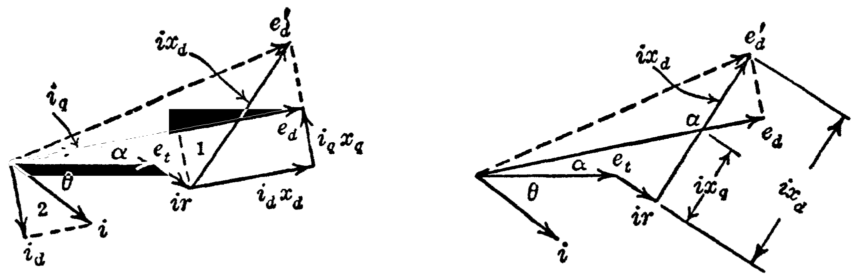


Fig. 10.7. Vector diagram for the salient-pole alternator.

In constructing the diagram for the salient-pole alternator, the direction of e_d must be known if the components of i are to be obtained. There is a simple and direct method for doing this. Consider Figure 10.8a which shows superposed diagrams for a machine. It is viewed first as a salient-pole generator, and next



(a) Superposed diagrams for salient-pole and cylindrical rotor alternators

(b) Construction of salient-pole diagram from that for the cylindrical-rotor machine

Fig. 10.8. Construction of vector diagram for salient-pole alternator.

as the equivalent cylindrical-rotor generator having the same value of x_d. Load conditions, of course, are not to change. The full lines of the figure are for the salient-pole machine, and the dotted lines are for the equivalent cylindrical-rotor machine. From the geometry of the figure, triangle 1 is similar to triangle 2, whence that part of ix_d between the outer end of ir and the vector e_d is ix_q

There is thus afforded a simple method for constructing the diagram for the salient-pole machine. To do so, the graph for the equivalent cylindrical-rotor machine is drawn as in Figure 10.8b. On the vector ix_d is located the point *a*, distant from the end of ir by the length ix_q. The line *oa* thus gives the direction of e_d for the salient-pole machine, and its length is determined by dropping a perpendicular to it from the outer end of the nominal voltage vector, shown now as e_d', for the cylindrical-rotor machine. e_d is the projection of e_d' on the line *oa* extended, as can be proved by geometry from Figure 10.8b. The excitation and torque angle for the salient-pole machine thus are found directly without recourse to current components.

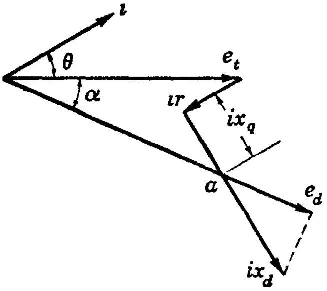


FIG. 10.9.

The vector diagram for the salient-pole synchronous motor carrying current at a leading power factor is shown in Figure 10.9. It is constructed in the same manner as that for the generator, but with all impedance drops reversed. From the superposed diagrams, it is clearly shown that the torque angles for the two types of machine may have widely different magnitudes. If x_q is about 50 per cent of x_d, the angle for the salient-pole machine will be approximately 50 per cent of that for the equivalent cylindrical-rotor machine. This may be of concern in problems involving pulsating loads, oscillatory conditions, transient conditions, and calculations on natural frequencies of sets in which synchronous motors are directly connected to reciprocating machinery. Practically all synchronous machines are of salient-pole construction and should so be treated.

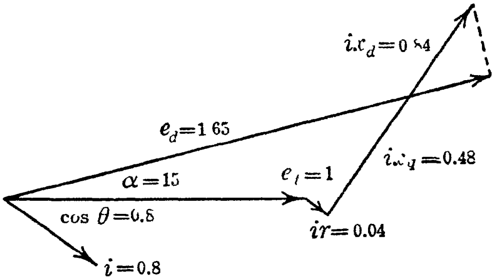


FIG. 10.10. Numerical example for a salient-pole alternator.

10.10. Example. Figure 10.10 shows the vector diagram for a 25-kva, 220-volt, 1,200-rpm, 60-cycle alternator carrying 20 kva at 0.8 pf lagging. The machine constants are 5% r, 105% x_d, and 60% x_q. A voltage of 220 volts on the air-gap line requires 3.2 amp field current.

The quantities on the diagram are given in the unit system as decimal fractions of normal leg values on an equivalent line-to-neutral basis. The voltage drops in the machine hence are 80 per cent of normal. By completing the diagram, e_d is found to be 165 per cent, whence, from the air-gap line, the required field current for this particular load condition is 5.28 amp. The load angle is 15.3 degrees.

As we have just seen, 5.28 amp field current is required under load in order to overcome, primarily, the effects of armature reaction and to maintain 220 volts at the terminals of the machine. The field current was found from the air-gap line because saturation under load is limited as a result of the opposing action of the field and armature ampere-turns. At no load, however, the field current is taken to the saturation curve (not shown), and 5.28 amp give an open-circuit terminal voltage of 253 volts. The voltage regulation of the generator, for the particular load and power-factor specified, then is

$$\text{voltage regulation} = \frac{253 - 220}{220} \times 100 = 15\%$$

10.11. Power-angle Relations. The vector diagrams give the power-angle equations from which can be determined the rotor displacement as a function of load, the maximum power which can be delivered and the angle at which it occurs, and the synchronizing torque which is the torque per unit of angular displacement and is in turn the slope of the power-angle curve. Beyond this, the analytical result permits the interpretation of the manner in which certain parameters influence the power or the angle.

Consider Figure 10.6, the vector diagram for the cylindrical-rotor machine. Neglecting resistance, we have from trigonometry,

$$\frac{ix_d}{\sin\alpha} = \frac{e_d}{\sin(180 - 90 + \theta)} = \frac{e_d}{\cos\theta}$$

from which

$$i = \frac{e_d \sin\alpha}{x_d \cos\theta}$$

Therefore,

$$\text{power} = e_t i \cos\theta = \frac{e_t e_d}{x_d} \sin\alpha \tag{10.2}$$

With saturation and resistance neglected, the power-angle curve thus is sinusoidal, with maximum power delivered when the angle between the terminal and nominal voltages is 90 degrees. This is, of course, the limiting angle. From zero to 30 degrees the characteristic is practically linear and is the usual operating region. Insulation requirements set the terminal voltage of a machine, field heating limits e_d, and x_d is fixed once the machine is built. The power of a machine, therefore, is set within limits by these conditions.

For the salient-pole machine, Figure 10.7, neglecting resistance,

$$\begin{aligned}\text{power} &= e_t i \cos\theta \\ &= e_t i_q \cos\alpha + e_t i_d \sin\alpha\end{aligned}$$

Also from the figure,

$$e_d = e_t \cos\alpha + i_d x_d$$

and

$$i_q x_q = e_t \sin\alpha$$

Solving for i_d and i_q from these last two equations, and substituting their values in the equation for power, we obtain

$$\text{power} = \frac{e_t e_d}{x_d} \sin \alpha + \frac{e_t^2(x_d - x_q)}{2x_d x_q} \sin 2\alpha \tag{10.3}$$

Equation 10.3 is the power-angle expression for the salient-pole machine. It will be seen immediately that the first term of the equation is that which was developed for the cylindrical-rotor machine. The second term has no dependency upon the excitation, but exists only because of the salient-pole construction. It is the reluctance power and, for most machines, depending upon excitation, may be a large part of the total power. Note that by letting $x_d = x_q$ (cylindrical rotor) the reluctance term disappears, as it should. For general-purpose, high-speed motors (514 rpm and above), the torque angle at rated load is approximately 34 degrees for 1.0-pf machines, and about 28 degrees for 0.8-pf machines. These values assume an average x_d of 80 per cent and an x_q of 65 per cent, and excitation satisfying nameplate conditions of loading.

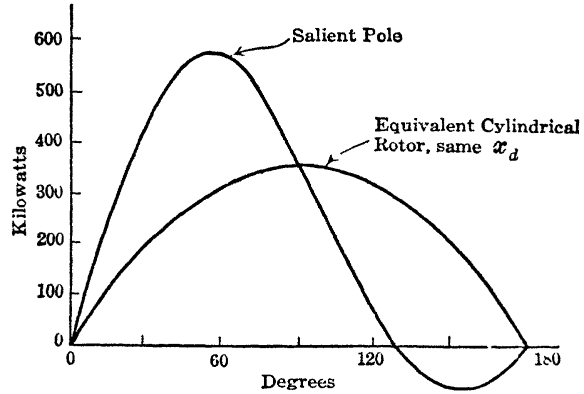


FIG. 10.11. Power-angle curves for a salient-pole synchronous motor and the equivalent cylindrical-rotor machine.

Figure 10.11 shows the power-angle curves for a 6-pole, 435-kva, 0.8-pf, 1,200-rpm, 2,300/4,000-volt, 3-phase synchronous motor. The comparison is pronounced. Both curves were plotted from the last power equation above in which only the first term is used to represent the machine as of equivalent cylindrical-rotor construction, that is, same x_d. The particular values used were $e_t = 1$, $e_d = 1$, $x_d = 1.18$, and $x_q = 0.45$, all on unit basis. The power equation from these numbers is

$$\text{power} = 0.847 \sin \alpha + 0.687 \sin 2\alpha$$

and is expressed as a decimal fraction of rated kva (435 kw at 1.0 pf). The maximum load (pull-out) for the given figures is 585 kw and occurs at an angle of 55 degrees. The displacement at normal load of 348 kw is 22 degrees.

If voltages and ohmic values of reactances are used in the torque-angle equation, they should be leg quantities. The calculated power then is that per leg, which, when multiplied by three, will give the total power carried by the machine. If percentages are used, the calculated power is expressed directly as a decimal fraction of the kva rating of the machine.

Writing the power-angle equation in the manner

$$P = A \sin \alpha + B \sin 2\alpha$$

it can be shown readily that the angle for maximum power is a function of the ratio A/B. Figure 10.12 shows the general relation between pull-out angle and this ratio. On the same graph also is shown the percentage reluctance torque as a function of A/B. The reluctance torque shown in the figure is a percentage of the total torque the machine carries at pull-out.

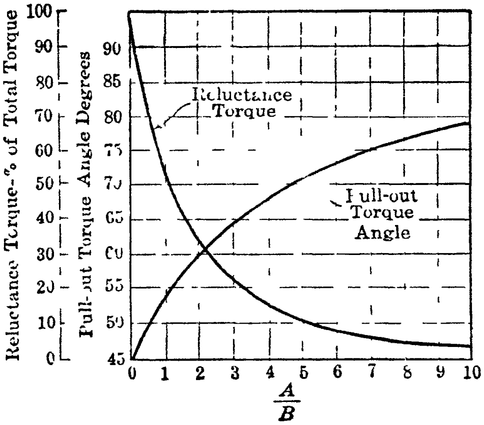


FIG. 10.12. Pull-out torque angle and reluctance torque as a percentage of the total torque versus A/B.

10.12. Synchronizing of Alternators: Parallel Operation. Synchronizing is that process whereby an alternator or a synchronous motor is brought up to speed and then connected in parallel with the other machines on a power system. Briefly, the following conditions must be observed for satisfactorily synchronizing a machine: (a) the incoming machine must have the same voltage as the power system, (b) the frequencies of machine and system must be the same, (c) the phase sequence of both must be the same, and (d) the voltages of both must be in phase when the machine is closed on the line. These requirements should be apparent in order that the incoming machine be placed in parallel with the others at zero or a minimum disturbance of existing conditions.

A discussion of the dark lamp method of synchronizing will serve to illustrate the principles involved. Figure 10.13 represents a machine brought up to proper speed and voltage and connected to a power system through lamps, the sum of the voltage ratings of each set of lamps being sufficient to take twice the voltage of either the machine or the system. A voltmeter can be used to check the two voltages.

The lamps flicker at the frequency difference of the machine and the system, hence no flicker is observed when both have exactly the same frequency. If the phase sequence is correct, all lamps flicker in unison at the difference frequency. The lamps are brightest when the machine and system voltages are additive across the lamps; they are darkest when the two voltages are equal and are in time phase with each other. The line switches, therefore, may be closed when the two voltages are equal, and when all lamps become dark together in slowly passing through this dark period. In practice, an alternator is almost always synchronized using a synchroscope, perhaps with lamps in addition as a check.

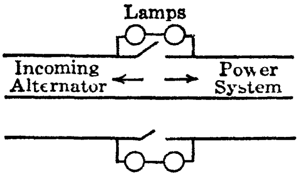


FIG. 10.13. Dark-lamp method of synchronizing an alternator.

The prime movers of alternators in parallel should have the same speed-load characteristics and these should be drooping in order that the sets operate stably together; that is, an increase in load on one should not permit it to attempt to gain speed, thus to pick up more load, with cumulative results. All alternators on the same line must run at the same frequency, but an attempt by the prime mover of one to gain speed for that set will result in an increase in load angle for its alternator, a direct measure of the alternator output. An alternator can change load only by a change in power input to its prime mover which is determined by the governor setting of the latter. Although only the governor setting determines the kw output of the alternator, the field excitation controls the reactive kva which it will deliver, either to the load or by way of circulating current among the other synchronous machines on the same system. The over-all power factor and the resultant reactive kva of the system is set, of course, by the system load. The power factor and the resultant reactive kva of a single generator, in parallel with others, are determined by the amount of field current as well as by the kw output. The field current of each generator in parallel with others hence should be so adjusted that the alternator will take its share of the wattless current required by the system.

10.13. Power-factor Correction, Voltage Control, and Synchronous Condensers. From the discussion on vector diagrams for the synchronous machine, it is recognized that given load conditions of voltage, current, and power factor require a certain value of field current to produce the nominal voltage e_d. If the power factor alone is varied, the in-phase current therefore remaining constant, the field current required for the motor becomes greater and greater as the power factor passes from a low lagging value to a low value leading. Figure 10.14, for the cylindrical-rotor motor of zero armature resistance, serves to illustrate this point, the above statement holding for the salient-pole motor as well. On the other hand, if the shaft load is fixed and the field current is varied, the power factor of the motor automatically is changed in accordance with this principle. The out-of-phase component of current is just sufficient in its magnetizing or demagnetizing action to provide that decrease or increase in excitation lost or gained by the field current. Variation in armature current with respect to field current may be shown by V-curves, Figure 10.15, obtained at various shaft loads. The minimum points on the curves correspond to 1.0 pf.

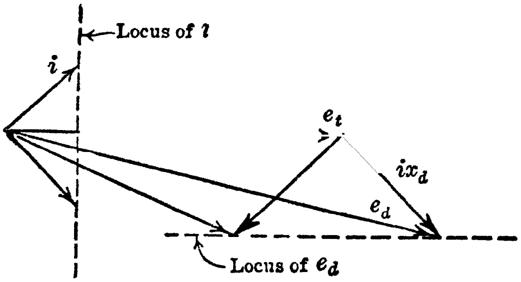


Fig. 10.14. Relation between field current (e_d), and power factor for the synchronous motor.

In view of such action, the synchronous motor may therefore be operated overexcited (higher field current than needed for 1.0 pf) to draw leading reactive current, as well as in-phase current for its load. The machine thus is said to function partly as a synchronous condenser, and in so doing it can assist in power-factor correction by compensating for the lagging reactive current taken by neighboring inductive apparatus. Most general-purpose synchronous motors are designed for 80 per cent leading power factor for exactly this form of operation. They also can be run as synchronous reactors, drawing lagging current when the excitation is below that required for 1.0 pf. Field heating limits the leading power factor of general-purpose motors.

Many large machines are designed to function solely as synchronous condensers or reactors. They have smaller bearings, since shaft load is absent, draw only a small energy component of current to supply their running losses, and are said to "float" on the line. Under such operation, however, they act primarily as voltage regulators. If the field current is set for a given terminal voltage at a load center and with no circulating current ($e_d = e_t$), any increase in line voltage automatically becomes an underexcited condition for the machine. It then draws lagging current through the power lines from the supply, introducing a reactive drop in the lines which serves to maintain the initial load voltage. It will be recalled from studies on 1-phase circuits that a lagging current drawn through an inductive reactance lowers the voltage at the load end of the line, whereas a leading current drawn through the reactance can raise the load voltage to a value above that at the generating end. These principles are

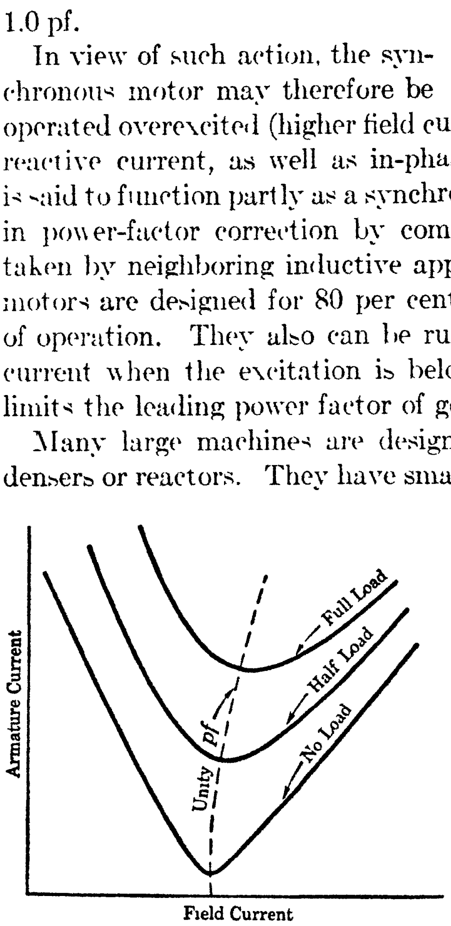


Fig. 10.15. V-curves for a synchronous motor.

utilized in synchronous condenser or reactor operation. A 25,000-kva synchronous condenser, for example, can carry from 25,000 kva leading (overexcited) to 15,000 kva lagging. Under this latter condition, the field current is zero and the machine runs on reluctance torque.

10.14. Starting Synchronous Motors. Synchronous motors do not have inherent starting torque. A synchronous motor may be brought up to speed by an auxiliary machine, synchronized as an alternator, and the auxiliary machine then disconnected from its power supply. Most synchronous motors, however, are equipped with squirrel-cage windings for starting, these grids surrounding and being rigidly connected to the field structure. Such a winding is composed of many brass or copper bars placed in slots along the pole faces and running parallel with the shaft. Copper or brass rings at both ends of the rotor completely short-circuit the winding.

Because of the revolving field of armature reaction, voltage applied to the stator gives rise to induced voltages and currents in the squirrel-cage winding. The resulting torque thereby produced in the air gap forces the rotor to follow the revolving stator field, thus bringing the machine to a speed near synchronism. If field current then is applied to the motor, it will pull into step with proper polarity between stator and rotor magnetic poles. Whether a motor may be started on full voltage or must be started on reduced voltage depends generally upon the size of the machine, the inertia of all moving parts, any connected shaft load, and limitations on fluctuation of line voltage caused by high starting currents. In some cases, shaft loads can be reduced during starting, an advisable procedure when possible since the starting torque is usually much less than the torque that can be produced at synchronism.

Small motors may pull into synchronism at reduced voltage with no shaft load, or at full voltage with some shaft load, on reluctance torque alone without the application of d-c to the field coils. As indicated previously, reluctance torque is due solely to the induced magnetism in the rotor poles by virtue of the stator revolving field. If the rotor is sufficiently close to synchronous speed, the stator mmf may have time to magnetize the rotor with fixed polarity and force it into step immediately. Once in synchronism, it is an even chance that the applied d-c excitation will give the same polarity to the rotor structure as it had previously by induction from the stator. If not, the rotor will "slip a pole," that is, will drop back one pole position and then run in synchronism. Obviously, a synchronous motor will not have to slip a pole if brought into synchronism by field excitation.

During starting, the field circuit carries no d-c current. If the coils are closed on the excitation system, a high induced field current will flow because of transformer action and may prevent the revolving field from reaching the rotor. At starting, the field circuit is usually closed through its own discharge resistor. A high induced voltage that otherwise would exist at the field terminals is thereby eliminated, and, further, the field circuit also can contribute some torque along with the squirrel cage, this component of torque increasing as the slip decreases.

The maximum speed reached during the starting period by torque action of all closed rotor circuits depends upon the shaft load and the inertia of the moving system. When excitation is applied to the field circuit, the motor must have a "pull-in torque" greater than the reactive shaft torque in order that the machine synchronize. Pull-in torque is defined as the maximum constant torque under which the synchronous motor will pull into step at normal voltage and frequency when d-c field current is applied. Connected loads, such as fans, that increase considerably in their torque requirements as speed is increased will necessitate higher induction-motor speeds before field excitation is applied so that the pull-in torque of the machine will be sufficient to bring it into synchronism.

Both the starting torque and the pull-in torque of general-purpose, high-speed synchronous motors designed for 1.0-pf operation are to be a minimum of 110 per cent of full-load torque. For 0.8-pf motors the minimum value for these torques is 125 per cent. It is understood that starting is to be at rated voltage and frequency, and that normal field current is to be applied at the time of pull-in. In general, the values given apply to all sizes of motor. Slow-speed motors have a starting torque in the region of 40 per cent, and about 30 per cent pull-in torque.

10.15. Hunting of Synchronous Machines. The starting windings of the motor also serve to reduce, or to eliminate in many instances, oscillations of the rotor, or hunting. The causes of oscillations may be change of load, pulsations produced by the type of load (reciprocating machinery, for example), electromechanical characteristics of the machine, or line disturbances. The oscillations are superimposed upon the average running speed, and hence lead to induced voltages and currents in the squirrel-cage winding which react to reduce the amplitude of the rotor swings. Because of their action, these grids are often called amortisseur, or damping, windings. Oscillations also may be reduced within limits by increasing the moment of inertia of the rotor (greater WR^2), perhaps through the addition of a flywheel.

Alternators sometimes hunt because of poor governor control on the prime mover. Prime movers, such as gas engines, which produce periodic impulses require properly damped and not overly sensitive governors to minimize rotor swinging. Although generators may carry damping grids to reduce oscillations, the amortisseur windings usually are not so strong as those for synchronous motors because they do not have to be used in starting. In addition to the damping action of pole-face windings, iron losses in the pole faces of the synchronous machine, caused by the relative motion of flux across them under oscillations, also lead to some damping effect.

10.16. Application of Synchronous Motors. Synchronous motors have been built for commercial application since about 1893. The first machines had revolving armatures, but since 1898 that type has given way to those having revolving fields. Although the first motors were not self-starting, induction-motor starting windings quickly came into use, and all synchronous motors now are equipped with them. Machines for general use are available in standard line designs from about 20 hp to 7,500 hp. The largest yet built have ratings of 10,000 hp, or thereabout, but 62,500-hp synchronous motors are proposed for the pumping installation at Grand Coulee, Washington. Synchronous condensers, of course, have been built for many years with ratings as high as 60,000 kva, but they carry no shaft loads and, because of their particular application, are apart from the discussion here on the synchronous machine as a driver. Standard frequencies are 25, 40, 50, and 60 cycles, and standard voltages range from 220 to 13,200 volts, in general. Only a few motors of higher voltage have been built because of insulation problems and cost. Only three and two phases are found. There are no single-phase synchronous motors, unless one includes the tiny reluctance-torque machines.

Factors of concern in applying synchronous motors are starting requirements, speed, overload capacity, power factor, and efficiency. Starting torque demands vary widely, from 30 to 200 per cent of full-load torque, depending upon the particular job to be motorized. The starting torque of the motor must be sufficient to overcome static friction and to accelerate the rotor. It also may have to provide for an inertia load or to bring up to speed an active load. As already stated, the general-purpose motor produces both a starting torque and a pull-in torque somewhat above 100 per cent of full-load torque, the motor being brought to about 95 per cent of synchronous speed before d-c excitation is applied. A few applications—fans, for example—may require a much lower slip before the motor can pull its load into synchronism.

Low-speed synchronous motors develop much less starting and pull-in torque than do high-speed machines and will draw from two to three times normal current when started on full voltage. They are started usually on full voltage, if the inrush current is permissible for the system, because of their relatively low starting torque. The power factor at starting also is low, perhaps 25 per cent. Some motors have amortisseur windings of double squirrel-cage construction so as to give them a better developed torque.

At synchronism, the synchronous motor will carry an increased shaft load up to a maximum called the "pull-out torque" at which the motor stalls. Pull-out torque is the maximum sustained torque the motor will produce for one minute under normal voltage, frequency, and excitation. For the general-purpose, high-speed motor, the pull-out torque is a little better than 150 per cent for 1.0-pf motors and is about 200 per cent for 0.8-pf machines. Many motors have higher values, and it should be fully realized that in any application the motor will be designed for the requirements of the job. Values given here are merely of a representative nature. The power angle at pull-out is approximately 80 degrees for all general-purpose machines.

Speeds of synchronous motors vary from 72 rpm or somewhat less, depending upon the number of poles and the frequency, to 3,600 rpm. Because of the difficulties in rotor construction, the cost of the high-speed motor is considerably higher than that of those having low speeds. Commercial machines are not built for 3,600 rpm below 250 hp in size. A great number of loads are inherently of low-speed character, and the synchronous motor has a distinct advantage here over other types of drive. Absence of gearing and belting decreases cost, saves space, and gives a better over-all efficiency. Again, the synchronous motor is truly a constant-speed machine and is used for drives in which no speed regulation is desired. The maximum output of many types of load is obtained at rated speed, and in these cases synchronous motors lead to an increased output.

For machines having high horsepower ratings and low speeds, the cost of the synchronous motor, even including the cost of the exciter, may be less than that of the squirrel-cage induction motor. The synchronous motor takes none or but a part of its excitation from the a-c lines, whereas the induction machine draws all of its excitation from the stator supply. The synchronous motor, therefore, utilizes to a greater extent than the induction motor the material built into it, and the ratio of power to volume is higher. The rotor structure of the synchronous motor, however, is more complex than that of the squirrel-cage machine.

The power factor of the synchronous motor is controllable by field current adjustment, and hence the machine can correct power factor as well as carry shaft load. Standard motors are designed for either 1.0 or 0.8 leading pf. although a greater reactive kva can be carried by decreasing the active load. A motor of higher rating is purchased if a given horsepower load is to be carried and the power factor is to be less than 0.8 leading. Power-factor correction results in savings on the power bill, releases kva of the system for active energy, means a lower investment in apparatus and wiring, and yields better voltage regulation. Efficiencies of synchronous motors are high—in the range of 90 to 96 per cent. In general, they are better than those of other types of driving machines and represent considerable savings.

Synchronous motors find application in a great many fields, and others are continually appearing. In a large number of instances, because of better efficiency and because of power-factor control, they are displacing other types of motors. In many cases, they are replacing other motors so as to release capacity in generating, transmission, and distribution equipment. Without going into details on specific applications within the fields of use, they are found in ball, steel, lumber, pulp and paper, flour, and rubber mills, and in cement plants. One of the first applications, that of driving compressors of various sorts, has been extended and they are now connected to fans, blowers, mixers, crushers, grinders, and pumps as well. They are a standard drive for shovel sets, and they always have had wide use as motors in motor-generator sets because of their constant speed.

Small reluctance-type synchronous motors, from $\frac{1}{2}$ to 15 hp, have salient-pole rotor bodies with squirrel-cage windings. They start as induction motors, pull into synchronism on reluctance torque, and pass to induction motor operation when the load becomes high. Commercial application is limited, and they are used chiefly for governor control on water-wheel generators.

PROBLEMS

1. A Y-connected, 100-hp, 440-volt, 1,200-rpm, 60-cycle, synchronous motor gives the following test data at full load: 440 volts, 106.5 amp, 1.0 pf, stator core loss of 2,270 watts, d-c field excitation of 8.3 amp from a separate 220-volt supply. $r_a = 0.084$ ohm per leg.

(a) Determine the windage and friction losses. Draw a power flow diagram (refer to Figure 11.1) and on it show the values of input, output, and each loss. Calculate the efficiency including the field power as part of the input power.

(b) A no-load test is made with the field excitation adjusted for unity pf. Calculate the a-c power input and the line current (assume the no-load core loss is 80% of the value at full load).

2. A 3-phase cylindrical-rotor synchronous motor is rated 100 hp, 440 volts, 131 amp, 60 cycles, 1,200 rpm, 0.8 pf (leading). A field current of 6 amp produces rated voltage on the air-gap line. $x_d = 120\%$ and $r_a = 1.6\%$. Calculate the following: number of poles, efficiency for full load at 0.8 pf (exclude field power), x_d and r_a in ohms, the armature copper loss for half rated current.

3. For the motor of problem 2, draw the vector diagram for rated conditions (0.8 pf lead) and determine the torque angle and the required field current. Express the torque angle in mechanical degrees. Repeat for a line current of 65 amp at 0.8 pf lagging, calculate the individual readings of two wattmeters used to measure input power. Repeat for a line current of 52 amp at unity pf.

4. The motor of problem 2 is carrying no shaft load (losses may be neglected in this problem). The field current is set at 9 amp. Calculate the line current, kva, and the power factor. Repeat for a field current of 4 amp.

5. A Y-connected cylindrical-rotor machine has a rating of 100 kva, 440 volts, 1,200 rpm, 60 cycles. The synchronous reactance, x_d, is 2.13 ohms, and the armature resistance is negligible. A d-c field current of 10 amp produces 440 volts between terminals on the air-gap line at no load. Calculate x_d in per cent. For generator operation, with terminal voltage of 440 and 90 amp line current, draw the vector diagrams and determine the required field excitation for each of the following power factors unity, 0.8 leading, and 0.8 lagging. Calculate the field current required to produce rated current on a short-circuit test.

6. The machine of problem 5 is used as a synchronous motor and is carrying 98 amp at 0.8 pf leading. Draw the vector diagram and from it determine the torque angle and field excitation. Calculate the power output (consider machine losses negligible) using both expressions of equation 10.2. For the motor drawing 50 kw and a field excitation of 15 amp, calculate the torque angle; draw the vector diagram and from it determine the line current and power factor.

7. A synchronous machine requires 12.2 amp field current for rated terminal voltage on the air-gap line at no load. 14.4 amp field current produces rated armature current on short circuit. Calculate x_d in ohms and in per cent.

8. An oscillogram for a slip test on a Y-connected, 3-phase, 25-kva, 220-volt, 1,200-rpm, 60-cycle alternator shows the following results as rms values:

$$e_{\max} \text{ line-to-line} = 203 \text{ volts} \qquad e_{\min} \text{ line-to-line} = 183.5 \text{ volts}$$
$$i_{\min} = 57.5 \text{ amp} \qquad i_{\max} = 91 \text{ amp}$$

Calculate x_d and x_q from this analysis. Express in ohms and per cent.

9. A 3-phase, 440-volt, 40-hp, 44.5-amp, 1,200-rpm, 60-cycle synchronous motor has an armature resistance of 7.1%, an x_d of 104% and an x_q of 56.5%. Calculate the ohmic values of these quantities.

10. A 3-phase, Y-connected, 220-volt, 7.5-kva, 1,200-rpm, 60-cycle, salient-pole alternator gives the following test results (voltage is measured between terminals):

(a) 220 volts, i = 21.4 amp, 1.0 pf, motor operation.
(b) 220 volts, i = 18.2 amp, 0.8-pf lagging motor.
(c) 220 volts, i = 22.2 amp, 0.8-pf leading motor.
(d) Repeat (c) for a terminal voltage of 176 volts.
(e) 220 volts, i = 17.1 amp, 0.8-pf leading generator.
(f) 220 volts, i = 22.4 amp, 0.8-pf lagging generator.

r_a = 0.288 ohm per leg, x_d = 5.35 ohms per leg, x_q = 3.0 ohms per leg. 3.32 amp field current produces 220 volts between terminals on the air-gap line at no load. Compute the excitation and torque angle for these loads.

11. The alternator of problem 10 is carrying a 1.0-pf generator load at rated voltage and with a 30° torque angle. What is the field excitation and the generator output?

12. What is the load and what is the torque angle for the alternator of problem 10 if it is carrying an 0.8-pf leading load at rated voltage with minimum field current?

13. A circulating pump is to be driven by a 3-phase, 2,300-volt, 500-hp, 125.5-amp, 0.8-pf, 900-rpm synchronous motor so as to allow for power-factor correction during the daytime. The pump load in terms of motor input is 400 kw, and operation is 24 hours per day. 1.0 pf is held at night and 0.8 pf leading during the day. Over what range in ohms must the external field resistance be varied if the source of field voltage is 125 volts? 2,300 volts at no load on the air-gap line requires 12 amp field current. The field coil resistance is 4.5 ohms and x_d = 110%. Neglect r_a and x_q.

14. A synchronous motor has an x_d of 80% and an x_q of 60%. What is the effect on the motor if the field-circuit resistance is doubled while the motor is carrying full load at 1.0 pf?

15. As a motor, the machine of problem 10 is carrying 19.8 amp at 0.8 pf leading.

(a) What is the armature current if the load is dropped? What is the kva?
(b) What is the armature current if the load is dropped and the field switch opened? What is the kva?
(c) What change in excitation is required from no load, 75% kva, 0 pf lagging to no load, 50% kva, 0 pf leading?

Neglect r_a in this problem and make calculations as though the motor had a cylindrical rotor.

16. Why does the overexcited synchronous machine have a smaller torque-angle and a higher break-down torque than the 1.0-pf machine?

17. What maximum power can the machine of problem 10 carry if the field current is fixed at 4 amp? What will be the torque angle, the armature current and the pf? Neglect r_a.

18. Calculate the torque angle for the case (b) of problem 10 with r_a neglected and using the calculated value of excitation. Assume the same power input.

19 A 500-hp, 1,200-rpm, 440-volt, 60-cycle, 0.8-pf synchronous motor has an x_d of 105% and an x_q of 60%. What is the synchronizing torque per mechanical radian displacement at full load?

20. In a book on electric motors, the statement is made that "an 0.8-pf synchronous motor operating at rated load will supply more reactive kva to the system than will a 1.0-pf motor at partial load or even no load." Assuming cylindrical-rotor construction, and x_d = 80%, calculate the leading kvar (reactive volt-amperes) drawn at no load by each of these two types of motor. Express this kvar as a percentage of rated kva. From the comparison, would you consider it advisable to purchase a 1.0-pf motor to correct power factor at light loads?

21. While operating under load, the field current of a synchronous motor is increased 20%. State the effect upon the following quantities, and give the reasons:

(a) speed
(b) load
(c) power factor
(d) line current
(e) torque angle
(f) efficiency

What is the effect on the above quantities of decreasing the shaft load on the motor? Of decreasing the frequency 5%?

22. A synchronous motor driving a pump and running at 75% rated hp load is to have its power factor adjusted to 0.8 leading. State the manner in which you would make the adjustment, using only the switchboard ammeter for line current to indicate the correctness of your steps. What precaution must be observed if the motor is to be run underexcited?

23. What is the effect in attempting to start a synchronous motor with the field-circuit energized? Would the magnitude of the field-circuit resistance qualify your answer? What are the effects on starting a synchronous motor with the field circuit open, with it short-circuited upon itself, or closed through a discharge resistor? What conditions require reduced-voltage starting?

24. Under what conditions of load would the addition of a flywheel to a synchronous motor be an advantage? Give one or two applications of this. Give one or two applications in which a flywheel would be a disadvantage. What problems are attendant upon the use of a flywheel with a synchronous motor?

25. A plant has a present load of 360 kw at 60% pf lagging and is to be enlarged by the addition of 100 kw carried by synchronous motors running at 0.8-pf leading. Will the new kva taken by the plant be within the initial value? What added kva in capacitors are required to bring the over-all power factor to 85%? If 0.8-pf synchronous motors are added to the initial plant load to the extent that they bring the over-all power factor to 85%, what kva in capacitors will be required to keep the plant within the initial kva? What kva and kw will be carried by the synchronous motors?

26. The transformer bank supplying a plant carries rated kva for an average loading of 800 kw at 70% pf lagging. It is desired to add motor loads of 250 hp to care for new requirements in the plant. If synchronous motors having an average efficiency of 93% and running at 0.8 pf leading are used as the driving machines, what further step must be taken so as not to overload the supply system? What added kw load could the supply transformers carry, and what would be the associated cost at $12 per kva of capacitors installed?

27. A 500-hp load of induction motors is operating at 70% pf lagging on a 4,000-volt, 3-phase circuit. The average efficiency is 85%. Give the voltage, current, and kva rating of the synchronous condenser required to raise the power factor to 85%.

28. What kvar are required to correct a 360-kw plant load from 60% to 85% pf? Assume an 0.8-pf synchronous motor having an x_d of 100% is installed to correct the power factor to 85% when carrying no shaft load. What should be the kva rating of the motor? Sketch the kva locus of the motor from this power factor angle of 90° leading to full kva at 1.0 pf. To what maximum value can the motor correct the plant power factor, and what kw are carried by the motor at that point? What maximum kw can the motor carry and correct the plant power factor to 85%?

29. A 200-hp induction motor costs $1,550 list price. An 0.8-pf synchronous motor which can replace the induction machine costs $2,200. What is the approximate investment in dollars kvar for the synchronous motor? What advantages or disadvantages does the leading-power factor synchronous motor have with respect to the induction motor and static capacitors?

30. A 15,000-kva synchronous condenser carries 10,000 kva at rated terminal voltage and with zero field current. What is the percentage x_d for the machine? State quantitatively the effect of increasing the field-circuit resistance 25% with the machine initially operating under nameplate conditions.

31. A 25,000-kva, 11,000-volt synchronous condenser has a ratio of no-load field current at rated voltage to full-load field current of 0.4 to 1. During light loads on the system it is desired to run the condenser at lagging power factor. What maximum lagging kvar can it carry if the terminal voltage is 12,000 volts? Neglect saturation.

32. When an alternator, which is to be connected to a system to run in parallel with other generators, is first installed, it is "phased out" so that its phase sequence at all times in the future will be that of the system. What schemes can you suggest to check the phase sequence? State the manner of each form of test, sketch the circuits, and list the test apparatus required for each.

33. In synchronizing a 3-phase alternator, using a 3-blade switch with a set of lamps across each blade and associated clip, what is the ratio of the maximum effective voltage on each set of lamps to the system voltage? Assume generator and system voltages to be equal. What is the maximum percentage effective voltage across each set of lamps shown in Figure 10.13 of the book? For these cases, state your conclusions on the voltage ratings of the lamps you would select.

34. What are the conditions requiring the use of 3 lamp circuits in synchronizing a 3-phase alternator, and those permitting the use of 2 lamp circuits or only one lamp circuit? Using 3 lamp circuits, what is the effect of interchanging the leads to 2 of these circuits, either on the generator side, or on the system side, of the switch? Are there any advantages in such a scheme?

What is the inherent disadvantage of synchronizing using lamps when but one or 2 lamp circuits are used?

Can voltmeters be used in place of lamps? If not, why not? If so, are there any advantages or disadvantages?

35. Under what condition can a single-phase synchroscope be employed in synchronizing a 3-phase alternator? Which of the requirements in synchronizing will be indicated by the single-phase synchroscope, and which will not?

In practice, a single-phase synchroscope together with one set of lamps usually is used in synchronizing 3-phase machines. What is the advantage of using both synchroscope and the set of lamps?

For a 3-phase alternator, employing a single-phase synchroscope together with one set of lamps, sketch the circuits when instrument transformers are required because of high system voltages. Sketch the circuit in which 2 transformers with one lamp can be used. Is the knowledge of transformer polarity essential in connecting this circuit? Can you suggest the manner of building a special single transformer with only one lamp for indications on synchronizing? (This transformer would have to be the equivalent of 2 transformers.)

36. Two alternators having identical ratings are operating in parallel. If the speed regulation of one prime mover is 4% and that of the other is 6%, what will be the relative kw load on each machine for a given total load?

37. Two alternators in parallel supply the charging current of a long transmission line. Will there be any change in terminal voltage of the remaining generator if one of them is disconnected from the line? Why or why not?

38. What are the effects of varying the field current of a single alternator that is supplying power (a) to a group of non-motor loads? (b) To a single synchronous motor having a fixed field excitation? (c) To a single synchronous motor in which the excitation can be altered at will? (d) To combined non-motor and synchronous-motor loads?

Follow through the reasoning for the case of 2 alternators working in parallel and supplying the above types of load. Consider the cases of an increase in excitation on both generators, a decrease in excitation on both, and an increase of excitation on one but a decrease of excitation on the other. With several alternators in parallel and no connected load, what is the effect of changing the field current on any one of them?

For the types of load given above, what factor or factors determine the power factor of a single generator? The power factor of a single generator working in parallel with another or with several others? The over-all power factor of a system between 2 or more parallel generators and the combined loads?

39. On many ships, a single 3-phase alternator supplies the power to a single synchronous motor for propulsion. What basic problem is involved in connecting the motor to the generator and bringing it up to speed? Suggest the manner of performing this operation as smoothly as possible.

40. A manufacturer of electrical and mechanical equipment wished to make efficiency tests on a line of turbogenerators rated 500 kw, 230/400 volts Y, 1,000 rpm, 50 cycles. Two wattmeters were used for the electrical measurements, each instrument rated 2.5/5 amp/100/200 volts, with 500 watts full-scale graduation and a multiplying factor of ½ for simultaneous connections to both low ranges. The wattmeters had a 200% maximum rating for the current coils and a 150% maximum rating for the potential circuits. One wattmeter, having resistances of 4,592 and 9,184 ohms for its potential circuits, was used with a 300/600-volt multiplier having a resistance of 3,100/9,308 ohms. The second wattmeter, having resistances of 4,958 and 9,917 ohms, was used with a multiplier with a resistance marking of 3,125/9,375 ohms. For rated kw at 400 volts, the load being a water rheostat, what should be the watts indication on these two instruments? The current transformers had 5-amp secondaries.

41. At a cost of $15,000, a paper manufacturing plant substituted synchronous motors for some induction machines, and replaced the squirrel-cage rotors in others by salient-pole rotors in order to improve power factor. For a yearly saving of $1,200 because of decreased losses, and a decrease of $3,000 per year because of better power factor, calculate the percentage return on the investment.

42. What factors can you mention which might influence a decision in correcting the power factor of a plant or a system to a given value? What factors can you mention which might be weighed in considering capacitors or synchronous condensers for power-factor improvement? How do the number of hours of yearly operation of a plant affect the investment return on changes made or equipment purchased to improve power factor?

43. An increase in plant load of an industrial concern calls also for 250 kva of corrective apparatus in order to keep within the rating of the supply transformers. For the following data, compare the cost of capacitor and synchronous-condenser installations on the basis of a 20-year life: $17 per kva for capacitors, $8 per kva for the synchronous condenser, losses of ¼% and 5%, respectively, 0.7 cent per kwhr energy charge, 3,500 hours per year operation, 10% fixed charges not including amortization of equipment.

44. Because of extremely low power factor throughout the system, 1,800 kva total of corrective capacity is required by a small power company to improve its operation. What major factors stand in favor of capacitors at the load centers rather than synchronous condensers, or rather than a single synchronous condenser at the generating station?

Assume that the cost per kva is $15 for capacitors, $7 average for several small synchronous condensers, and $5 for a single 1,800-kva unit; that losses are 0.2%, 5%, and 4%, respectively; that energy costs 0.4 cent per kwhr; that the corrective equipment is on the line 24 hours a day; and that the yearly savings in transmission losses alone is $2,000. By what annual amount is one of these investments favored over the others? For a fixed charge of 15%, how do the capitalized savings of the favored investment compare with the difference in purchase prices? What additional factors, not yet considered, make power-factor improvement of greater value than shown by any of the above calculations? If these last factors represent a yearly saving of $400, what is the yearly return of total savings on the purchase of capacitors?

If power-factor improvement adds 1,000 kw to the generating apparatus, what is the cost per kilowatt of this increased system capacity?

45. A 400-hp, 2,300-volt, 1,200-rpm, 60-cycle synchronous motor drives a d-c generator supplying 250 kw at 500 volts to a resistance load. If the motor is disconnected from its supply, in what approximate time will the set drop to 20% speed? The set has a WR^2 of 2,500 lb-ft^2, the generator carries constant field current, and 40 kw of normal rotational losses of the set are assumed to vary as the square of the speed. What electrical angle will the motor have lost with respect to the supply system in 0.25 sec? What is the per cent slip of the motor at the end of 0.25 sec? Would you expect the motor to pull into synchronism if the supply switch were closed at this time?

46. A 3-phase, 250-hp, 120-rpm, 2,400-volt, 60-cycle, 1.0-pf synchronous motor drives a compressor. $x_d = 120\%$, $x_q = 80\%$, and full-load efficiency is 92%. Assuming 3% of the losses within the stator, calculate the synchronizing torque in the air gap of the motor in pound-feet per mechanical radian displacement of the rotor. If the squirrel-cage windings produce 40% of rated torque at 95% synchronous speed, calculate the damping torque in pound-feet per mechanical radian per second slip of the rotor with respect to the stator revolving field. Calculate the polar moment of inertia in mass units for the set if the total WR^2, including a large flywheel, is 250,000 lb-ft.^{2} Write the general torque equation for the set with displacement angle α as the dependent variable. From the coefficients of the left-hand members of this equation, calculate the natural frequency of oscillation of the rotor. Assume that rotor oscillations have no effect on line frequency or voltage.

47. If damping torque of the amortisseur windings is neglected, show that the approximate natural frequency of oscillation f_n of a synchronous motor connected to a large system is

$$f_n = \frac{588}{N}\sqrt{\frac{P_s f}{WR^2}} \text{ cycles per second}$$

where N is rated speed in rpm, f is line frequency, and P_s is the synchronizing power in kilowatts per electrical radian displacement at the operating point.

CHAPTER XI

POLYPHASE INDUCTION MOTORS

11.1. Introduction. In 1888, Ferraris of Italy found that a copper cylinder suspended between two coils placed at right angles with each other was set into rotation if alternating currents of the same frequency but of different phase were sent through the coils. The explanation that he gave was that the resultant field of the two currents revolved around the common center-line of the coils and, by means of eddy currents created in the copper, dragged the cylinder after it. At about the same time, independent investigations by Tesla in the United States led to similar results, his practical development being the induction motor.

The induction motor is an application device, its function being almost wholly that of a driver for general-purpose use. It is the most serviceable of all electrical machines and, because of simplicity, reliability, ruggedness, and low cost, it long ago took the lead in the field of motive power for needs other than voltage generation. Industrial requirements are such that polyphase induction motors alone are built in sizes ranging from $\frac{1}{2}$ hp to the large 22,500-hp drivers installed in some airplane carriers. Ratings of induction units in frequency sets have reached 37,500 kva.

The enormous total amount of electrical energy which this type of motor utilizes has made it one of the greatest factors in the development of the a-c system. The invention of the induction motor was the practical fulfillment of the desire for an a-c machine that would compete with the d-c shunt motor and which would give, because of its power requirements over the middle of the day, a more uniform electric power demand on generating stations. The universal employment of the induction motor and the several outstanding applications of physical principles which it embodies make a brief study of the machine of considerable value, particularly to one interested in application of electric circuits and machines.

11.2. Elements of Construction. Three-phase machines are by far the most common of polyphase induction motors, hence only this type will be discussed here. All are practically alike in performance, and what is said of the 3-phase motor applies to the others except for a few details. Mechanically, the induction motor consists of a cylindrical body called the rotor separated by a relatively small and uniform air gap from the surrounding stator. The rotor core is similar to the core of the d-c armature, and the stator is like that of the synchronous machine. Both rotor and stator are built of thin laminations of sheet steel so as to reduce iron losses.

As with practically all machines, the induction motor has two electrical cir-

cuits—the stator and rotor windings—and one magnetic circuit common to them. What has been said previously on armature windings for the a-c machine applies to the induction machine. With a 3-phase voltage impressed, the winding gives rise to a revolving field in the air gap, identical to that in the synchronous motor. An interchange of any two line leads reverses the direction of rotation. Instead of fixed rotor poles following the stator field, however, induced poles on the rotor provide the force action.

Rotor windings may be of either two types—squirrel-cage or wound. In the squirrel-cage type, heavy bars of copper, brass, or aluminum are placed in longitudinal slots in the rotor iron, the ends of the bars being joined by end rings. The starting, amortisseur, or pole-face windings of the synchronous motor are merely variations of the induction motor squirrel cage; they are lighter, however, because they are not required to carry running loads. In the second type (wound-rotor), the rotor winding is composed of coils made similar to those on the stator. The winding of the wound-rotor motor must have the same number of poles as the stator and usually is wound for the same number of phases although the latter is not necessary. It may be connected Y or delta, the first predominating. The three winding terminals are brought out to slip rings so that a variable resistance can be inserted in the circuit, as will be discussed later.

All rotor windings must be closed, either upon themselves or through some small external resistance, in order that currents flow and torque be developed. The rotor has no electrical connection to the stator, and no voltages are applied to the rotor windings under usual operation. Electrically, the squirrel-cage and wound-rotor windings are equivalent. The squirrel-cage machine is more rugged, is more adaptable to service in which dust or fumes may be present, and costs less because of the absence of slip rings and because of the lesser requirements in rotor construction.

11.3. Rotor Motion: Slip. We already have discussed somewhat the manner in which the revolving stator field causes the rotor to follow it. Referring to Figure 8.8, the moving air-gap flux induces voltages in the rotor bars in such directions that the resultant currents are into the paper over those regions under the stator north poles, and out of the paper under the stator south poles. Hence there will be a decrease of flux on the forward side of the rotor conductors, and an increase of flux on the far side. Motor action, therefore, takes place, and the rotor moves in the direction of the stator field.

A second explanation, again coming from first principles in physics, is that the rotor currents will produce alternate magnetic poles on that body but in the regions between the stator poles. As can be observed from Figure 8.8, the rotor poles will lie ahead of the stator poles of like polarity. Mutual force action between rotor and stator poles thus results in counterclockwise motion of the rotor of Figure 8.8. As already indicated, induction-motor action is analogous to the action of the friction clutch or the fluid drive. The induction motor is one of the outstanding practical applications of induction and, equally, of the revolving field.

The actual speed of the rotor always is less than synchronous speed, the difference being called the slip. Slip is expressed either in rpm or as a decimal fraction (or equivalent percentage) of synchronous speed; that is,

$$\text{per cent slip} = \frac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}} \times 100$$

$$= \frac{N - S}{N} \times 100 \qquad (11.1)$$

where the symbols represent, respectively, the quantities in the first expression. The slip of a 6-pole, 60-cycle induction motor running at 1,140 rpm under load is 60 rpm, 0.05 as a decimal, or 5 per cent.

The drop in speed from synchronous speed allows torque to be developed of such magnitude that it will be sufficient to care for the shaft load and the rotor losses. The greater the slip, the greater will be the rotor induced voltages and currents, and the greater the developed torque. The relation between load and slip is not linear, nor, in general, will torque and slip increase together to 100 per cent slip, which is zero speed (standstill). For the usual motor, slip increases with load to a maximum torque called the pull-out, or breakdown, torque at which the motor stalls. This maximum torque is perhaps from 200 to 300 per cent of rated torque for most machines. The motor is self-regulating and, as shaft torque is altered, will adjust its speed for required loads over the range from no load to pull-out. The slip can be controlled by design and is generally within the range of 2 to 5 per cent at full load for squirrel-cage motors. Over this range, the speed-load characteristic is practically linear. Wound-rotor motors have a greater speed regulation, perhaps as high as 15 to 20 per cent for some applications.

11.4. Power Relations: Efficiency. From elementary principles in mechanics, the power input to the rotor is directly proportional to the product of the air-gap torque T and the synchronous speed N. The synchronous speed is the speed at which the power is leaving the stator and crossing the air gap to the rotor. Similarly, the rotor output is proportional to the product of that torque and the rotor speed S. The difference between the power input to the rotor and its power output, therefore, is proportional to the product of the torque and the slip s and corresponds to losses within the rotor. Assuming windage and friction losses of the machine and the small rotor iron loss to be part of the rotor output (that is, treated as part of the shaft load), the rotor losses then are copper losses entirely. Thus, we have

$$\begin{aligned}\text{power input to rotor} &= \frac{2\pi TN}{33{,}000} \times 746 = KT \text{ watts}\\ \text{power output of rotor} &= K(1 - s)T \text{ watts (developed power)}\\ \text{rotor copper loss} &= KsT \text{ watts} \qquad (11.2)\\ &= \text{slip} \times \text{rotor input}\\ \text{rotor efficiency} &= (1 - s) \times 100, \text{ in per cent}\end{aligned}$$

The complete power relations for the induction motor are conveniently set forth by a power flow diagram, Figure 11.1. In computing the stator copper loss, the machine is considered to be Y-connected, and the resistance per leg is, therefore, $\frac{1}{2}$ the value measured between terminals.

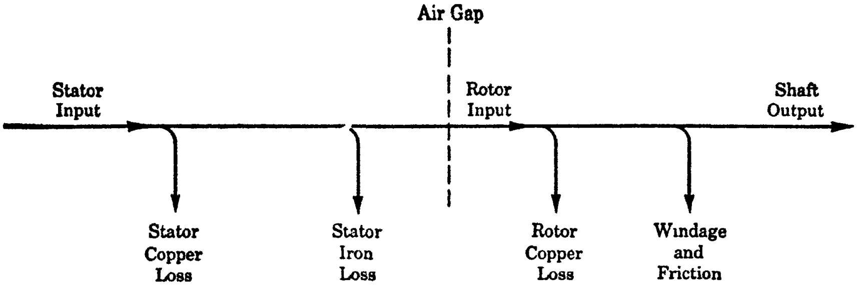


FIG. 11.1. Power flow diagram for the induction motor.

The no-load readings for an induction motor include a small stator I^2R loss, the stator iron loss, windage and friction, and the very small rotor iron loss. Because of the small slip, the rotor copper loss at no load can be neglected. The rotor iron loss always is relatively small because the rotor frequency is proportional to the slip and the iron losses vary with frequency as do those of the transformer. The sum of the stator iron loss and windage and friction constitute the fixed loss (relatively independent of load) of the machine. The fixed loss is obtained by subtracting the stator copper loss from the no-load input. The components of the fixed loss cannot be separated without further detailed data.

The question then arises on the manner of taking the fixed loss into account, whether it should be assigned to the stator, to the rotor, or partly to each. Because the results obtained are considered adequate in general, it is often the practice to subtract the fixed loss, as well as the stator copper loss, from the stator input in calculating the input to the rotor.

The following example shows the manner in which calculations on the induction motor are carried out in accordance with the above statements. The method is applicable to those cases in which input quantities may be obtained readily, but where running torque is difficult of measurement. Test data on a 3-phase, 7.5-hp, 220-volt, 6-pole, 60-cycle induction motor are

running light: 236 volts, 6.5 amp, 610 watts, 1,197 rpm

under load: 235 volts, 18.4 amp, 6,700 watts, 1,164 rpm

resistance between stator terminals is 0.54 ohm

The problem is to determine the power factor, output, and efficiency of the motor when loaded.

From the above readings, neglecting rotor copper loss at no load,

$$\text{fixed losses} = 610 - 3 \times 6.5^2 \times 0.27 = 576 \text{ watts}$$

$$\text{stator } I^2R \text{ under load} = 3 \times 18.4^2 \times 0.27 = 274 \text{ watts}$$

$$\text{rotor input} = 6{,}700 - (576 + 274) = 5{,}850 \text{ watts}$$

$$\text{rotor } I^2R = 0.03 \times 5850 = 179 \text{ watts } (3\% \text{ slip})$$

$$\text{motor output} = 5{,}850 - 179 = 5{,}671 \text{ watts, } 7.59 \text{ hp}$$

$$\text{efficiency} = \frac{5{,}671}{6{,}700} = 84.7\%$$

$$\text{power factor} = \frac{6{,}700}{\sqrt{3} \times 235 \times 18.4} = 89.3\%$$

Should the fixed losses be included in the rotor input and then later subtracted after the rotor I^2R has been calculated, the output is 7.58 hp and the efficiency is 84.4 per cent. From a Prony brake test on the motor, the efficiency was 84.1 per cent for 7.5 hp output. The calculated efficiencies are high, as they should be, for there are additional losses (mainly eddy-current losses in the copper and surrounding iron) present under load that do not appear at no load. These may affect the efficiency $\frac{1}{2}$ to 2 per cent.

11.5. The Induction Motor as a Circuit. Essentially the induction motor is a transformer having a revolving secondary kept within the field of the primary. The mutual action of force between primary and secondary coils of the transformer now becomes one of torque in the motor. With the rotor stationary, the squirrel-cage machine becomes a short-circuited transformer. The wound-rotor machine may be short-circuited or may have a small external resistance in the rotor circuit. It is of interest that wound-rotor induction motors having suitable turn ratios between stator and rotor windings are occasionally, and very conveniently, used as static transformers and as induction regulators. With a d-c voltage applied to the rotor, they also have been used as cylindrical-rotor synchronous machines.

Because of the similarity between the induction motor and the transformer, it is possible to represent the motor by the same equivalent electrical circuit as that for the transformer. The treatment of any leg of the motor thus permits immediate reduction of the machine to an equivalent, simple, single-phase circuit easy of calculation and lends itself to an added qualitative understanding of the motor by way of its physical interpretation. The major difference between the static transformer and the induction motor is that the latter is a rotating device, thereby calling for the inclusion in the equivalent circuit of the relative velocity of the rotor with respect to the stator mmf wave. The transformation for the circuit diagram, however, is obtained very simply in terms of slip.

Consider, first, the motor with the rotor at standstill and open-circuited.

Actually, of course, the rotor circuit of the squirrel-cage machine always is closed. That of the wound-rotor motor can be open, in which case it will not run. Because of the electrical equivalence between squirrel-cage and wound-rotor motors, the complete rotor winding of either type may be assumed to be a group of Y-connected coils of which we shall treat only one leg. Each stator leg is thus only an inductance coil having a high ratio of reactance to resistance. With E_1 volts impressed per leg, an exciting current I_0 flows, giving rise to a revolving field. The input to the stator is the sum of the copper loss $I_0^2r_1$ and the stator and rotor iron losses.

The total flux of the revolving field has two components, leakage and mutual These components are exactly similar to those in the transformer and in the synchronous machine. The mutual flux is that which crosses the air gap to the rotor iron and corresponds to the flux of armature reaction discussed in the preceding chapter. E_1 will be consumed in small part by the stator resistance r_1 and leakage reactance x_1 and, in a much greater part, by the reactance of the winding due to the large component of the air-gap flux. Letting E_2 be the counter emf generated by the synchronously moving field in the air gap, we have the equivalent circuit for the stator, Figure 11.2.

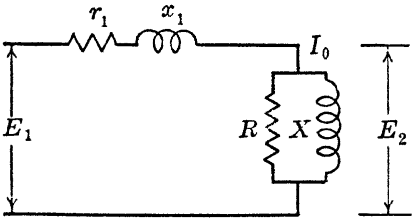


FIG. 11.2. Stator circuit of the induction motor.

The parallel branches of R and X form the exciting circuit for the motor. With voltage impressed on the machine, an exciting current always is present in the stator, whether the machine is at rest or is carrying load. The two components of I_0 which flow through R and X represent, respectively, the current that cares for the iron losses and the current that produces the total flux. The exciting circuits for the induction machine and the transformer are exactly similar, although I_0 was so small in the transformer it usually could be neglected in calculations and was not introduced in our treatment. It is much larger for the induction motor, however, and must now be taken into account. Further, under running conditions, R is adjusted to include windage and friction as well as iron losses. I_0 then is the no-load, or running-light, current and includes an increased power component for windage and friction.

At standstill and with the rotor circuit closed, the induced voltage in the rotor gives rise to a current flowing through the resistance and the leakage reactance of that winding. Treating the induction motor as a transformer, these impedance quantities may be expressed in terms of equivalent ohmic values, r_2 and x_2, as viewed from the stator terminals, thus giving a 1:1 ratio of transformation. At standstill, E_2 is then that component of the applied voltage consumed by the impedance of one leg of the rotor. The rotor frequency now is the rated frequency of the machine, and x_2 consequently is a maximum.

When the rotor is moving under load at speed S (slip s), the rotor-induced voltage is sE_2, the rotor frequency is sf (which leads to low iron losses), and the rotor leakage reactance is sx_2. The rotor current, therefore, is

$$I_2 = \frac{sE_2}{\sqrt{r_2^2 + (sx_2)^2}}$$

$$= \frac{E_2}{\sqrt{(r_2/s)^2 + x_2^2}} \text{ amperes per leg} \tag{11.3}$$

Examination of the above relation thus reveals the possibility of treating the rotor of the loaded motor as an equivalent electrical circuit comprising a constant leakage reactance, x_2, and a variable resistance, r_2/s, on which is impressed a constant voltage, E_2. Note that the resistance term alone includes the actual motion of the machine.

Thus, in the completed equivalent circuit of Figure 11.3, r_2/s is the total apparent resistance of the rotor circuit. It includes the rotor resistance as well as the equivalent load resistance which is to represent shaft output. This latter quantity, hence, is

$$\frac{r_2}{s} - r_2 = r_2\frac{(1-s)}{s} = \text{equivalent load resistance} \tag{11.4}$$

These results are in agreement with those obtained from the former analysis of the motor in which

$$\text{total power input to rotor} = KT = \frac{3I_2^2r_2}{s} \text{ watts}$$

$$\text{total actual copper loss} = KsT = 3I_2^2r_2 \text{ watts} \tag{11.5}$$

$$\text{total rotor power output} = K(1-s)T = \frac{3I_2^2r_2(1-s)}{s} \text{ watts}$$

where $K = N\ 7.05$ (equation 11.2). It is to be noted that equations 11.5, which are for total power, are three times the power for any leg of the motor.

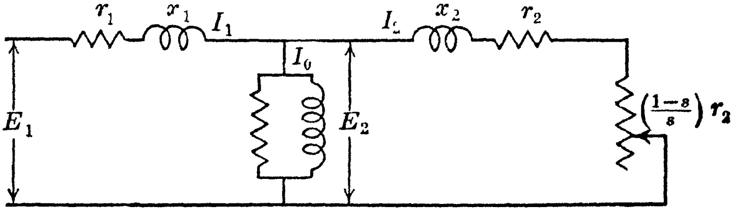


FIG. 11.3. Equivalent circuit for the induction motor.

It is by way of reasoning from the circuit of Figure 11.3 that the induction motor often is said to be equivalent to a transformer supplying a resistance load. The outstanding point of interest is the concept of placing the rotor speed only in the resistance term, thereby eliminating it in considerations on rotor induced voltage and leakage reactance. It should be stressed that r_2 is the actual rotor resistance referred to the stator, and that the total actual copper loss in the rotor is $3I_2^2r_2$ only. The resistance $r_2(1 - s)/s$ is purely fictitious and is used solely to represent the load taken from the motor shaft. There is no loss within the machine that corresponds to it.

The equivalent circuit has many advantages. In the first place, it permits the ready determination of machine performance by way of expressions involving the machine constants. From these relations, the motor may be analyzed both qualitatively and quantitatively. Secondly, the circuit allows all changes in the constants that represent the machine and its performance to be handled easily. Again, it enables the quick and accurate comparison between characteristics of different motors. The one disadvantage of the method is that it does not permit as readily as other devices the visualization of motor performance that may be desired.

Because of the value of the equivalent circuit, it is used greatly as the basis for design calculations, and the A.I E.E. accepts power-factor and efficiency guarantees obtained in this way. The equivalent circuit represents the loaded motor quite faithfully, and power delivered to the load resistance, for example, will be exactly that power delivered to the load through the moving motor shaft. It should be pointed out, however, that commercial design requires slight modifications in some of the machine constants and their resulting calculations, but recognition of these modifying factors is not given here. One simplifying assumption adopted for the transformer, but now not so applicable to the motor, was to neglect the exciting current and its circuit. However, whereas the no-load current of the transformer is but a small percentage of rated current, the no-load induction motor current may be from 20 to 40 per cent of normal value. The error in neglecting the exciting current hence is much greater for the motor because of the larger impedance drop in the stator produced by the no-load current I_0.

11.6. Interpretation of the Equivalent Circuit. From first principles in physics, the air-gap torque in any machine is directly proportional to the product of the flux, the current in the conductors which lie in that field, and the cosine (pf) of the angle between the flux and the current. Thus, for one phase winding of the motor,

$$T \propto \Phi I_2 \cos\theta_2$$

$$\propto E_2 I_2 \cos\theta_2$$

$$\frac{1}{K}\,\frac{E_2^2}{[(r_2/s)^2 + x_2^2]} \cdot \frac{r_2}{s} = \frac{1}{K} I_2^2 \frac{r_2}{s} \tag{11.6}$$

which is the same result shown by equations 11.5 under total rotor power input The rotor voltage E_2 is proportional to the flux, and the final factor of proportionality is $1/K = 7.05\,N$.

In terms of the impressed voltage and the circuit constants,

$$\begin{aligned} E_2 &= E_1 - I_1(r_1 + jx_1) \\ &= E_1 - I_0(r_1 + jx_1) - I_2(r_1 + jx_1) \\ &= e_1 - I_2(r_1 + jx_1) \end{aligned} \qquad (11.7)$$

where $e_1 = E_1 - I_0(r_1 + jx_1)$ and is an equivalent reduced impressed voltage to account for the impedance drop in the primary due to the no-load current. It might be pointed out that I_0 is directly proportional to E_1 if saturation is neglected, whence e_1 and E_1 are directly proportional to each other and both are independent of load. For a given impressed voltage, $e_1 = E_1(1 - k)$ and is constant; k may be of the order of 10 per cent.

Solving equation 11.7 for the magnitude of E_2^2 and substituting it in equation 11.6, we have

$$T = \frac{7.05}{N} \frac{e_1^2 r_2/s}{[(r_1 + r_2/s)^2 + (x_1 + x_2)^2]} \text{ pound-feet per leg} \qquad (11.8)$$

Differentiating equation 11.8 with respect to slip, the developed torque is found to be a maximum when

$$s = \frac{r_2}{\sqrt{r_1^2 + (x_1 + x_2)^2}} \qquad (11.9)$$

and hence the maximum developed torque will be

maximum developed torque =

$$\frac{7.05}{N} \frac{e_1^2}{2[r_1 + \sqrt{r_1^2 + (x_1 + x_2)^2}]} \text{ pound-feet per leg} \qquad (11.10)$$

Interpretation of the last three equations shows that

(a) The developed torque is proportional to the square of e_1. In view of other statements given above, it also is approximately proportional to the square of the impressed voltage.
(b) The slip at which maximum torque is obtained varies directly with rotor resistance.
(c) The maximum torque is independent of rotor resistance.
(d) Starting torque ($s = 1$) can be increased by increasing r_2, and it will be equal to the maximum torque when

$$r_2 = \sqrt{r_1^2 + (x_1 + x_2)^2}$$

The practical application of this information lies in the fact that the rotor resistance can be adjusted to give maximum torque at any desired slip and that the starting torque can be controlled in the same way. The squirrel-cage motor, once built, has a fixed rotor resistance and, hence, control is lost over any change of maximum torque with respect to slip. Its starting torque can be changed only by a change of the impressed voltage. The wound-rotor motor, though—and this is the reason for its type—retains these advantages in that resistance can be inserted in the rotor circuit to alter the slip (or speed) at which maximum or any desired torque is to be obtained. The starting torque also can be increased by added rotor resistance.

Figure 11.4 shows a group of speed-torque curves for a wound-rotor motor with a variable resistance in the rotor circuit. As the total rotor resistance r_2 is increased, the maximum (pull-out, or breakdown) torque occurs at lower values of speed (higher slip). The motor can be made to start with maximum torque, the rotor resistance being gradually reduced as the motor speeds up until all the external resistance is out of the circuit. Graphically, from Figure 11.4, the motor is made to operate successively on one speed-torque curve after the other until it reaches the final characteristic shown by the top line. It should be pointed out that a high rotor resistance, while giving high starting torque, means a high slip at normal load and hence a high rotor loss with correspondingly decreased efficiency. A large speed change with load can be obtained only at the expense of a lower efficiency. The region of stable operation is above the point of maximum torque.

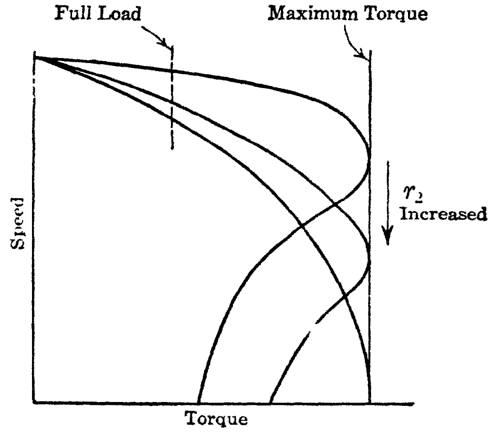


Fig. 11.4. Induction motor speed-torque curves

11.7. Effects of Motor Design on Characteristics. The operating region of the induction motor is over the first portion of the speed-torque curve which is practically linear. As has been discussed, an increase in rotor resistance lowers the speed at which a given torque is obtained, increases the starting torque, and lowers the motor efficiency. It decreases the starting current and increases the power factor, the power factor increasing faster than the starting current decreases, thus leading to better starting torque. In a squirrel-cage motor, the value of rotor resistance will be a compromise between the opposing requirements of good starting torque and good speed regulation. It can be increased for a given motor by reducing the cross-sectional area of the end-rings. Normally, the resistance of the squirrel-cage motor, referred to the stator, is less than that of the wound-rotor machine of the same rating. The stator resistance should be as low as possible so as to reduce stator copper losses and the stator impedance drop. An increase in r_1 lowers the maximum torque.

The leakage reactances are affected by changes in air gap and in slot openings. As the air gap is increased, these reactances are decreased, but a higher exciting current flows as a consequence, and at a lower power factor. Open slots give the same effect. Increase in reactance also lowers the pull-out torque. Induction motors are built with as small an air gap as possible (calling for heavier shafts, with ball bearings very often used) so as to reduce I_0. General practice also is to design motors for semi-enclosed slots, many having totally closed slots on the rotor. Again this is a compromise in that partially or totally closed slots decrease I_0 and the tooth losses but increase the reactance, and they reduce the power factor, the starting torque, and the pull-out torque.

Some squirrel-cage induction motors are built with very deep slots or with slot configurations as shown in Figure 11.5a and are called double-squirrel-cage machines. The effect is to give the high starting torque of the wound-rotor motor and the good running characteristics of the single-squirrel-cage machine. The effects of the double-squirrel-cage type, or its equivalent, may be analyzed in the following way. At starting, the rotor frequency is highest, being equal to line frequency. The reactance—and therefore the impedance—of the conductors or the lower portions of them more deeply embedded in the rotor iron is highest. The current flow consequently is greater in the outer conductors that have lower reactance and that also may be made of material having greater resistance. The motor thus is said to start on the outer winding with its higher resistance and better power factor due to the increased ratio of resistance to reactance.

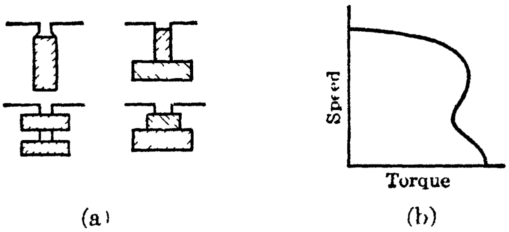


FIG. 11.5. Slot configurations (a) and speed-torque curve (b) for double-squirrel-cage induction motors.

Under running conditions the rotor frequency is the slip frequency and the above phenomena now are reversed. The impedance of the lower winding is less than that of the outer one, it carries the greater current, and its ratio of resistance to reactance is less, giving the effects of a low-resistance squirrel-cage motor. The resistance at starting may be several times that under running conditions. When up to speed, the motor is said to run on the inner squirrel cage. A typical speed-torque curve for a double-squirrel-cage motor is shown in Figure 11.5b.

An analysis of the double-squirrel-cage machine requires a modification of the equivalent circuit of Figure 11.3.

11.8. The Circle Diagram. From the equivalent circuit of Figure 11.3,

$$
\begin{aligned}
I_2 &= \frac{E_2}{\sqrt{(r_2/s)^2 + x_2^2}} \\
&= \frac{E_2}{x_2} \cdot \frac{x_2}{\sqrt{(r_2/s)^2 + x_2^2}} \\
&= \text{a constant} \times \sin\theta \qquad (11.11)
\end{aligned}
$$

which is the polar equation of a circle, the locus of I_2. The stator applied voltage E_1 and the no-load current I_0 may be shown graphically as in Figure 11.6.

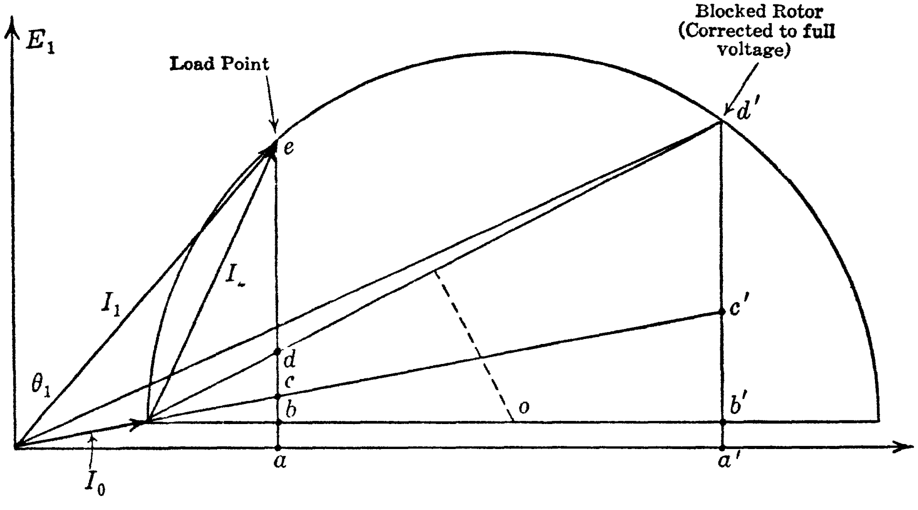


FIG. 11.6. Circle diagram for the induction motor.

I_2 and, consequently, I_1 ($I_1 = I_0 + I_2$, vectorially) then follow a circle, one end of the diameter of which is at the tip of I_0. Since two points will determine the circle, the second may be found most conveniently by blocking the rotor so that it cannot turn and by reading the applied voltage, the current, and the power input.

Although the blocked-rotor test may be taken at reduced voltage, the corresponding current and power input can be adjusted readily to conform to a full-voltage test. The diagram is based throughout on normal voltage. The blocked rotor current is laid off at its known power-factor angle with respect to E_1. The intersection o, which the perpendicular bisector of the line joining the extremities of I_0 and the blocked-rotor current makes with the horizontal drawn from the tip of I_0, thus fixes the center of the circle. The diagram is constructed by treating one leg only of the motor, that is, all voltages, currents, and power components shown on the diagram are leg quantities. Note that both points used in locating the circle are obtained by tests which correspond exactly to the open-circuit and short-circuit tests on the transformer.

The total in-phase component of the blocked-rotor current is divided further into components which represent the various losses. That is, referring to Figure 11.6,

$$\begin{aligned} 3E_1(a'd') &= \text{total blocked-rotor input} \\ 3E_1(a'b') &= \text{no-load loss of motor} \\ 3E_1(b'c') &= \text{total stator copper loss at blocked rotor} \\ 3E_1(c'd') &= \text{total rotor copper loss at blocked rotor} \end{aligned} \tag{11.12}$$

For any load current I_1, the diagram gives the power factor at the machine terminals, the equivalent rotor current I_2 referred to the stator, the component losses, and the output in watts, all per leg of the motor. The vertical components of current between the rays drawn from I_0 give these respective values of losses and power output per leg when multiplied by the leg voltage E_1. At no load, I_1 and I_0 are one and the same current, and the power factor is a minimum. As load is increased and I_1 moves along the circle, the power factor is raised, the limiting value being determined by the tangent to the circle. Note that the power factor of an induction motor can never be leading, as with the synchronous motor, nor can it even be so very close to unity. The normal working range is below the point of tangency.

Summarizing the above statements, we have for the assumed load point e of Figure 11.6

$$\begin{aligned} 3E_1(ae) &= \text{total motor input} \\ 3E_1(ab) &= \text{no-load loss} \\ 3E_1(bc) &= \text{total stator copper loss} \\ 3E_1(cd) &= \text{total rotor copper loss} \\ 3E_1(de) &= \text{motor output} \\ \text{slip} &= \frac{cd}{ce} \\ \text{delivered torque} &= 3E_1(ce)\,\frac{7.05}{N}\ \text{pound-feet} \\ &= 3E_1(de)\,\frac{7.05}{N(1-s)}\ \text{pound-feet} \\ \text{max. motor input} &= 3E_1 \times \text{max. value of } ae \\ \text{max. rotor input} &= 3E_1 \times \text{max. value of } ce \\ \text{max. motor output} &= 3E_1 \times \text{max. value of } de \end{aligned} \tag{11.13}$$

Calculations based upon the circle diagram may be fully as accurate as those obtained from the equivalent circuit, and both methods are employed in practice. Both require, however, for commercial design and resulting efficiency guarantees, certain modifications of these basic methods, but a discussion on them is beyond the scope of this treatment.

11.9. Starting. The induction motor at standstill is purely an R—X circuit of low impedance, and at starting on full voltage the current therefore is very large and at a low power factor because of a high ratio of X to R. The inrush current may be from 4 to 8 times normal current. Although the motor itself is generally quite able to withstand line starting, it often is advisable or necessary to start the motor at a reduced voltage This will depend upon the nature of any connected load, the frequency of starting duty, the degree of acceleration desired or permitted, heating during the accelerating period, and voltage fluctuations produced on the supply system that would affect lights or other apparatus.

Either resistance or reactance in the motor leads will serve to limit the starting current because of the increase in impedance. As the motor gains in speed, the impedance may be reduced in steps or in whole. Automatically, as the speed rises and the current falls, the voltage at the motor terminals increases.

Resistance in the rotor circuit to lower starting current and to raise the starting torque and power factor already has been discussed. This method is applicable, of course, only to the wound-rotor machine.

Y-delta starting reduces the initial current to 33 3 per cent and the input power to 33.3 per cent of the values obtained with the delta connection. It is equivalent to starting the motor on the 57.7 per cent taps of a starting compensator. It can be used for a motor in which all 6 of the leg terminals are available and the stator designed for normal voltage with delta arrangement of the windings. The method is not in common use in this country.

The autostarter, or starting compensator, is most usually used for motors requiring a low voltage at starting. It is simply an autotransformer, connected either open-delta or Y, with one or more sets of taps available. The compensator is automatically disconnected from the lines when the motor is finally thrown on full voltage. A motor that takes 500 per cent normal current on full-voltage starting and develops 150 per cent of full-load torque would draw 325 per cent current at its terminals and develop approximately 63.4 per cent rated torque when started on the 65 per cent taps of a compensator.

11.10. Calculation of Starting Torque. Under normal running loads, the power transferred across the gap to the rotor provides for the shaft output, windage and friction, the rotor copper loss, and a very small rotor iron loss. At standstill, the total rotor input is consumed only in losses. This power, dissipated as heat in the rotor copper and iron, is productive of torque to accelerate the machine. Because the rotor frequency is normal at standstill, the iron losses at usual starting voltages are considerably greater than at rated slip. Because of their manner of variation, for approximate results it often is assumed that the rotor input and, thus, the starting torque vary as the square of the stator impressed voltage which essentially determines the air-gap flux. Hence, if the starting torque is known for any particular voltage it then can be calculated fairly closely for any other voltage. This is not true when saturation is present.

A more exact calculation on starting torque, based on running-light and blocked-rotor tests, takes certain of the losses into account. For a given different starting voltage, the stator current and power input are computed directly from the blocked-rotor data. The stator copper loss then is subtracted from the input, as in the power flow diagram. Next would be subtracted the stator iron loss, but its magnitude is unknown. A practical approximation is to substitute for it the entire fixed loss of the motor after that loss has been modified by the square of the ratio of the starting voltage to full voltage. The starting torque T_{st}, in pound-feet, is then

$$T_{st} = \frac{7.05}{N} \times (\text{rotor input in watts}) \tag{11.14}$$

The resulting rotor input may appear somewhat pessimistic, but the total fixed loss of the motor is but a very small part of the motor input at starting, even at a reduced voltage of 50 per cent. Moreover, actual test results show starting torque somewhat and, in many cases, appreciably less than the calculated value. The reason lies in the fact that the flux wave is non-sinusoidal, as assumed. The fundamental, which is the only component productive of torque, is lower than might otherwise be expected. Again, lock-in torque between stator and rotor teeth, depending upon the position at which the rotor last came to rest, may require an increase of several per cent in starting torque over a calculated value. Consequently, a computation that is pessimistic to a slight degree has a practical value.

Starting torque usually is calculated in pound-feet, or is stated as a percentage of rated torque. Rated torque is given indirectly in the horsepower and speed ratings of the motor. Often, however, starting torque is expressed in terms of synchronous watts or synchronous horsepower, which is simply the power input to the rotor at starting. The word "synchronous" merely means that that is the speed at which power is given the rotor; hence, it is the speed to be used in the horsepower equation when calculating the torque in pound-feet. The minimum starting torque at full voltage for 2- and 4-pole squirrel-cage motors has been set at 150 per cent of full-load torque. This value of normal starting torque varies progressively downward to 105 per cent of full-load torque for 16-pole motors. All are to have a minimum breakdown torque of 200 per cent.

11.11. Example. A 3-phase, 15-hp, 440-volt, 17.5-amp, 1,155-rpm squirrel-cage motor drives a pump in a mechanical engineering laboratory. Calculate the starting torque of the motor at 65 per cent rated voltage from the following test information:

no load: 462 volts, 9.2 amp, 816 watts, 1,196 rpm

blocked rotor: 223 volts, 74.8 amp, 16.0 kw

resistance between stator terminals = 0.93 ohm

At 65 per cent rated voltage (286 volts), the stator current is 95.9 amp and the motor input is 26.3 kw. The stator copper loss is 12.84 kw.

Fixed losses are 698 watts, which modified by the square of the ratio of the starting voltage to the test voltage are 268 watts. Hence, the rotor input at 65 per cent rated voltage is

$$26{,}300 - 12{,}840 - 268 = 13{,}192 \text{ watts (13.19 synchronous kw, or 17.69 synchronous hp)}$$

$$\text{starting torque} = \frac{13{,}192 \times 7.05}{1{,}200} = 77.4 \text{ lb-ft}$$

$$\text{normal torque} = 68.3 \text{ lb-ft}$$

$$\text{starting torque} = 113.5\% \text{ of rated torque at } 65\% \text{ voltage}$$

11.12. Speed Control. Speed changes in induction motors are accomplished in the following ways:

(a) Inserting resistance in the rotor circuit
(b) Changing the number of poles.
(c) Changing the frequency.
(d) Concatenation.
(e) Introducing an emf into the rotor circuit.

Method (a) has been treated. It should be remarked, however, that this method is primarily one for increasing starting torque even though it will effect control of speed. It leads to lowered efficiency when used for speed control.

Some very large motors, for example, for ship propulsion, are designed with stator coils having 50 per cent pitch and the terminals of the polar groups brought out to a throw-over switch permitting reconnection of the coils. In this way, it is possible to obtain a 2:1 speed change by doubling or halving the number of poles, as the case may be. Motors having 2 independent stator windings, each arranged for pole changing, will have 4 synchronous speeds for the stator mmf.

Changing frequency requires a controlled supply which, by speed variation of the prime mover, gives a frequency change over a desired range. The method is limited in that all motors on that supply must change speed to the same degree.

If the rotor leads from a wound-rotor motor are carried to the stator of a second motor rigidly connected to the first, the 2 are said to be connected in cascade or to be in concatenation. The speed of the set will be equal to that of a motor having the number of poles equal to the sum of the poles of the 2 machines if they are so connected as to run in the same direction alone. If they are connected so as to run alone in opposite directions, the speed of the set will be equal to that of a motor having the number of poles equal to the difference in numbers of poles of the 2 machines.

Method (e) is discussed in Section 15 of this chapter by way of the BTA motor.

11.13. The Induction Generator. As long as the stator of the motor is connected to supply lines, a revolving field is maintained in the air gap. If, now, the rotor is driven above synchronous speed (negative slip) by a prime mover, the rotor winding will have voltages and currents induced in it in directions opposite those for positive slip. The machine then functions as a generator with power flow from mechanical to electrical. Because the rotor carries no exciting current, as does the synchronous machine, it is necessary that the revolving field, which is the exciting flux, be produced by the stator winding. This requirement is met by having one or more synchronous machines of any type always connected to the same lines.

The load carried by the machine as a generator is dependent upon its negative slip and, for a given percentage slip, will be about the same as the load it would carry under motor action. Whereas the power factor of the motor always is lagging, that of the generator is always leading. Any lagging components of current required by a load must be supplied by other synchronous apparatus on the same system.

Applications of induction generators are few, but they are worth noting. In railway locomotives, induction motors often are driven overspeed as the trains move downgrade, thus giving rise to generator action. Mechanical energy lost in descent is converted to electrical energy and given back to the power system. Regenerative braking thus is a factor in safety, saves power, and greatly reduces the wear met in mechanical braking. The induction generator also forms a very good constant-speed, high-torque load for testing purposes, for example, Diesel and gas engines. Further, it is used as a stand-by unit in power stations. A small turbine, direct-connected to pumps, normally drives the pumps with the motor idling. The motor, however, can carry the pump load should the turbine be taken out of service, and it can function as a house generator if trouble occurs on the main lines. Means of excitation, of course, must be available.

11.14. Applications of Induction Motors. The polyphase induction motor has always been the standard all-around, general-purpose motor and has a field of application which is almost unlimited. It is the simplest machine built as an electrical driver and has been used in practically every field in which an electric motor can be employed. These motors are built with ratings from $\frac{1}{2}$ hp to many thousand horsepower. Commercial sizes are listed up to 15,000 hp, but not at all speeds. At 3,600 rpm, the largest are about 3,000 hp; at 1,800 rpm, they are about 5,000 hp. Frequencies, voltages, and number of phases follow those of the synchronous motor.

As with the synchronous motor, requirements of the application determine the characteristics for which the induction motor is designed. The load requirements to be ascertained include knowledge on starting torque, breakdown torque, and any desired speed regulation under running load. Such demands on the motor are not necessarily of concern in the order given. The minimum starting torque of single-squirrel-cage motors lies between 150 and 105 per cent of full-load torque, as previously stated, these values applying only to full voltage at rated frequency. High-rotor-resistance, high-slip, squirrel-cage motors have starting torques of the order of 275 to 300 per cent. The starting torque of double-squirrel-cage motors is from 200 to 250 per cent, but they may be designed to give 300 per cent of rated torque at starting. Since the rotor resistance of these machines is high, the double-cage motor has a somewhat higher slip at full load than the usual single-cage motor.

The starting torque of wound-rotor motors can be varied by changing the rotor-circuit resistance by means of the associated controller, and maximum, or breakdown, torque can be obtained at starting. Maximum starting torque of perhaps 250 per cent may be produced with only 350 per cent or less rated current. The power factor is increased, and the inrush current decreased, with increasing rotor-circuit resistance, but beyond the optimum value the torque will decrease. A primary advantage of the slip-ring motor over the squirrel-cage machine lies exactly in its high starting torque and low starting current, this latter factor sometimes limiting the use of the squirrel-cage machine on certain systems because of voltage fluctuation Again, if accelerating time and consequent rotor heating are excessive for a squirrel-cage motor, the wound-rotor motor would be selected for the drive.

It is to be remembered that the starting current is proportional to the applied voltage, and that the starting torque is approximately proportional to the square of the voltage. Where starting currents do not fall within an accepted maximum value, or where the character of the load or factors affecting the motor dictate reduced voltage at starting, it may be necessary to employ compensators, Y-delta starting, or resistance or reactance in the motor leads. The first method is the most common scheme for starting both synchronous and induction motors when low-voltage starting is required. On 60 per cent voltage, for example, the motor current is 60 per cent that at rated voltage, and the line current to the compensator is 36 per cent. If high starting torque is required, but the starting current is limited, squirrel-cage motors generally are not applicable, and the wound-rotor machine then is the best to use.

The pull-out torque of most induction motors is at least 200 per cent of full-load torque and may be as high as 300 per cent. This maximum torque occurs at about 20 per cent slip for squirrel-cage motors and wound-rotor motors having their slip rings short-circuited. The slip at which maximum torque is developed in the wound-rotor machine is controlled by rotor resistance. The maximum torque of the double-squirrel-cage motor is developed at starting, and the motor usually will not stall unless loaded to this maximum torque. If the maximum torque exceeds that for the upper part of the speed-torque curve (Figure 11.5b), though, the motor will drop greatly in speed.

The induction motor is classed as a constant-speed machine, but it does not have the invariable speed of the synchronous motor. Its speed-load characteristics are similar to those of the d-c shunt motor, but the speed regulation of the induction motor is a little better, particularly that of the squirrel-cage machine. Normal slip is from 2 to 5 per cent. Applications in which flywheels are used or in which a considerable flywheel effect is present, for example, punch presses and shears, may call for a full-load slip of 10 or 12 per cent. The flywheel thereby is allowed to receive or give up energy by change in speed, and current pulsations of the motor are greatly reduced. Motors driving elevators, cranes, hoists, etc., have a higher slip yet, perhaps 15 to 20 per cent, so as to minimize jerking as load is picked up. Voltage variation affects the slip of induction motors to some extent.

Slip-ring motors are designed for speed control between maximum speed and 50 per cent of synchronous speed. They are applicable to those jobs in which speed variation frequently is desired, for example, conveyors, stokers, and compressors. Speed control is obtained for them, however, only at the expense of a decrease in efficiency. Furthermore, large changes in torque may mean a corresponding large speed regulation. For applications requiring definite fixed speeds, widely separated, 2- and 4-speed squirrel-cage motors can be purchased. Throw-over switches reconnect the stator windings in many instances, and in others the number of poles is halved or doubled, as the case may be. Low-speed induction motors are not advisable because they have low starting and pull-out torques and poor efficiency and power factor.

The efficiency of the induction motor generally is good, but it is lower than that of the synchronous motor. The squirrel-cage machine designed for high starting torque usually has a higher slip at rated load with a consequent increased rotor loss and poorer efficiency. The efficiency of wound-rotor motors having resistance in the rotor circuit is very low, but it is not a factor of importance in some applications if the duty is intermittent, for example, elevators, hoists, and cranes, or if low speed is not required for long periods. Average efficiencies range from 80 per cent for 5-hp motors to a little over 92 per cent for 500-hp machines. High-speed motors are slightly more efficient than those having low speeds.

The power factor of induction motors is inherently lagging and is a maximum usually at a considerable overload. It varies with speed—the higher the speed, the higher the power factor for motors having the same horsepower rating. The power factor at full load for 1,200-rpm motors varies from about 82 per cent for 5-hp machines to about 92 per cent for 500-hp machines. The corresponding variation in power factor for 600-rpm motors is from about 55 per cent to 88 per cent. At light loads, the power factor is very low. Actual instances have arisen in which a plant, a lumber mill, for example, may have an average power factor over a period of time of only 50 or 60 per cent because of a great many large induction motors running at light loads much of the time. Power-factor correction then becomes a serious problem.

Where normal starting torque is satisfactory and low slip is desired, the single-squirrel-cage motor is applicable. Such loads, essentially constant speed, include fans, blowers, compressors, machine tools, conveyors, wood-working machines, line shafts, motor-generator sets, etc. Double-squirrel-cage motors having high starting torque are applied to conveyors that start under load, large compressors, crushers, mixers, punch presses, shears, elevators, cranes, hoists, and loads having considerable flywheel effect to start. Wound-rotor motors are applicable to many of the above services of the double-squirrel-cage machines, particularly fans, compressors, conveyors, hoists, cranes, and centrifugal and plunger pumps. In these last instances, high starting torque with low starting current, and moderate speed regulation, may be required.

General advantages of the induction motor are (1) it is easily started, (2) has a high starting torque, (3) requires no synchronizing, and (4) does not hunt. If overloaded, it will regain speed when the excess load is removed. Further, in the case of wound-rotor motors, control is obtained over speed.

11.15. Polyphase Commutator Motors. There are a few polyphase commutator motors that employ principles of several types of machines. Discussion is given here on 2 of them since they follow the general treatment for the induction motor.

The Schrage, or General Electric BTA, motor is a brush-shifting machine. There are two separate rotor windings: the primary circuit fed through slip rings and taking the place of the stator winding of the usual induction motor; and the second rotor winding similar to that of the d-c shunt machine and having a commutator on which rest several sets of brushes. The stator holds a polyphase winding, the ends of each phase being connected to a set of brushes which can shift with respect to each other It now is the secondary of the motor. The voltage between the two sets of brushes for a given phase depends upon brush separation along the commutator surface. Since the commutator voltage is generated at slip frequency, it can be introduced into the stator windings, which also have a voltage of slip frequency induced in them, so as to assist or oppose the emf of the secondary. Speed control is obtained in this way, and the motor can run above synchronous speed.

The BTA motor has a high starting torque, a low starting current which depends upon brush position, and a speed range of 3:1. Speed control is uniform. The machine has good efficiency and some small adjustment of power factor. With the two brush sets of each stator winding resting on the same commutator bar, the motor acts as a typical squirrel-cage machine. Applications of the motor are with blowers, pumps, compressors, etc., which require wide changes in speed. It has had an extended use as a driver for printing presses and similar applications.

The Fynn-Weichsel motor has a rotor with windings like those of the BTA machine. The stator is wound 2-phase, one of the windings being in series with the commutated winding on the rotor. It becomes the d-c field for the motor at synchronous speed. The second stator winding is closed upon itself through a variable resistance and gives the machine the characteristics of the wound-rotor motor at starting.

The motor normally runs as a self-excited synchronous motor with 150 per cent pull-out torque. When that limit is exceeded, the machine then operates as an induction motor up to 300 per cent pull-out torque. It will return to synchronous speed from induction motor operation if sufficient load is removed. The motor can correct power factor, motion of the brushes giving power-factor control by varying the current in the stator exciting winding. It has a general application, but it is specifically of advantage in power-factor correction of induction motors on the same system.

PROBLEMS

1. Calculate the slip for a 6-pole, 60-cycle induction motor running at 1,148 rpm; for an 8-pole, 25-cycle motor having a full-load speed of 362 rpm. A pump is to be driven at 1,100 rpm by a 60-cycle induction motor. How many poles should the motor have?

2. At one point of a load test on a 3-phase induction motor the instrument readings were 223 volts, $W_1 = 1.16$ kw. and $W_2 = -0.41$ kw. The wattmeters had 25-amp current coils and 75/150-volt potential coils with 300-volt multipliers. Full scale was 3 kw for 25 amp and 150 volts. What was the average of the line currents for this particular load?

3. A brake test on a 10-hp. 220-volt, 6-pole, 1,140-rpm, 3-phase induction motor gives the following data for one load point:

217 volts, 22.8 amp. 7.37 kw, 1,151 rpm, 26.8 lb scale weight. 1.3 lb tare weight,
1.5 ft lever arm. Resistance between stator terminals = 0.45 ohm.

Calculate the output, efficiency, and power factor for this load. Sketch the power flow diagram and place numerical values on each arm.

4. A 75-hp, 4,000-volt, 1,150-rpm, 6-pole, 3-phase induction motor gave the following results on test:

no load: 4,000 volts, 3.5 amp, 3.56 kw, 1,193 rpm
load: 4,000 volts, 11.8 amp, 71.52 kw, 1,141 rpm
blocked rotor: 1,330 volts, 16.2 amp, 9.09 kw
resistance between stator terminals = 2.70 ohms, Y-connected

What are the output, efficiency, and power factor of the loaded motor? Sketch the power flow diagram. Solve the problem two ways, that is, by assuming the fixed losses (a) as stator losses and (b) as rotor losses.

5. A 10-hp, 4-pole 60-cycle, 220-volt, 3-phase induction motor is drawing 7,140 watts from the line. The core loss is 310 watts, friction and windage 230 watts, stator copper loss 290 watts, and rotor copper loss 360 watts. Compute the motor efficiency and the running torque.

6. A 3-phase, 6-pole, 60-cycle wound-rotor motor gives the following data on test:

no load: 220 volts, 9.2 amp, 640 watts, 1,195 rpm
load: 220 volts, 27.4 amp, 9.02 kw, 1,145 rpm
blocked rotor: 69.4 volts, 27.4 amp, 2.2 kw
resistance between stator terminals = 0.41 ohm

Calculate the output, efficiency, power factor, and running torque of the motor under load.

7. A 3-phase, 6-pole, 60-cycle induction motor gives the following data on test:

no load: 220 volts, 10 amp, 530 watts, 1,197 rpm
load: 220 volts, 30 amp, 10 hp output, 1,140 rpm
windage and friction = 200 watts
resistance between stator terminals is 0.34 ohm

What is the motor efficiency for the load given?

8. Carry through the calculations on the power flow diagrams for the motor of Section 11.4 so as to check the values given in the last paragraph of that section.

9. What components exist in the running-light current of the induction motor? How do they compare with those for the transformer on open circuit? Would you expect the no-load power factor of the motor to be better or worse than that of the transformer on open circuit?

10. What is the major difference between the induction and synchronous motors in the manner in which they receive their excitation? Is the range of power factor for the induction motor therefore necessarily limited? How would you expect increasing shaft loads on the induction motor to affect its power factor in general? Compare the general range of power factor for the transformer, induction motor, and synchronous motor. What determines primarily the power factor of each?

11. A 440-volt, 1,200-rpm, 60-cycle squirrel-cage motor has 72 slots and a coil throw of 9 slots. The stator winding is modified for 440-volt, 60-cycle, 1,800-rpm operation of the machine. What will be the minimum value of the new exciting current as a percentage of that for 1,200 rpm? What factor will increase the exciting current beyond this magnitude? What factors will increase the no-load motor current?

12. Thinking of the induction motor as 2 sets of moving magnetic poles, why should you expect the developed torque of the machine to be approximately proportional to the square of the stator impressed voltage?

13. Why should you expect the slip of the induction motor to increase, for the same developed torque, with increasing stator impedance?

14. A variable resistor, a fixed resistor, and a fixed reactance are connected in series across a single-phase voltage. Show that the condition for maximum power taken by the variable resistor is the same as given in the equation of conclusion (d) of Section 6 of this chapter.

15. What is the effect on motor heating of frequent starting or reversing, or of starting high-inertia loads? What simple electric circuit can represent fairly closely the general variation of current and the storage of kinetic energy in the induction motor during the starting period? Reasoning from this circuit, what is the relation between the heat produced in the motor during starting and the stored energy of rotation at full speed? Would starting requirements such as those in the first sentence above influence the character of conducting material in squirrel-cage motors? Why and in what manner?

16. Derive the relation $1\ K = 7.05\ N$ given in Section 11.6.

17. The 6 terminals of the 3 phases of an induction motor are available for Y and delta connection. Give the ratios of starting current, starting torque, and kw input at starting when delta connected to the corresponding values when Y connected. Assume same line voltage.

18. A 20-hp, 60-cycle, 3-phase, 1,150-rpm, 440-volt induction motor develops 175% of rated torque and draws 600% of rated current when thrown directly on the line. What would be the line current, motor current, and starting torque if the motor were placed across the 60% taps of a starting compensator?

19. The motor of problem 4 is started on the 40% taps of an autotransformer. Compute the starting torque in pound-feet, in per cent of full-load torque, and in synchronous watts.

20. Calculate the starting torque for the motor of problem 6 when it is started on full voltage.

21. A squirrel-cage motor draws 75 amp from the line and develops 40 lb-ft torque when started on the 40% taps of a compensator. What would be the line current and starting torque if the motor were started on the 60% taps?

22. A 220-volt, 5-hp, 4-pole, 1,720-rpm, 3-phase, squirrel-cage motor takes 550% of rated current at starting and develops 175% of full-load torque with full voltage impressed. What should be the applied voltage if the motor is to develop rated torque at starting? What current would an autotransformer take for this condition?

23. The following test information is obtained on a 220-volt, 40-hp, 1,155-rpm, squirrel-cage motor:

no load: 220 volts, 18.5 amp, 1.80 kw, 1,196 rpm
blocked rotor: 80 volts, 183 amp, 11.4 kw
resistance between stator terminals = 0.046 ohm

The motor is to receive its power from a 90-kva, 3-phase transformer bank having 4.6% Z. What approximate starting voltage should be applied to the motor to limit the voltage drop on the load side of the bank to 5% when the motor is started? Assume the input voltage to the bank remains constant. What is the starting torque of the motor at this reduced voltage?

24. Refer to the motor of problem 4. Compare the line currents at starting when the motor is started by means of a compensator, by resistance in the lines, and by reactance in the lines, all methods to give 65% rated voltage at the motor terminals. Assuming full-load efficiency and power factor for the motor are both 92%, what is the percentage starting current on 65% voltage? What is the percentage starting torque? What ohmic values of line resistance and reactance are required under these two methods of starting? What advantages does the compensator have over the other methods of starting? What disadvantages?

25. A 5-hp, 220-volt, 6-pole, 60-cycle, wound-rotor motor develops 9.4 lb-ft starting torque on 42% rated voltage. The resistance between rotor terminals is 0.84 ohm, and the ratio of stator to rotor turns is 1.65. What minimum blocked-rotor current will the motor draw at full-voltage starting? Why will the actual line current be greater than your calculated value?

26. The following data were obtained by test on a 5-hp, 220-volt, 15.6 amp, 1,140 rpm, 3-phase, 60-cycle, wound-rotor motor:

blocked rotor: 92 volts, 24.3 amp, 1,530 watts, rotor short-circuited with no external rotor resistance
stator resistance between terminals = 0.474 ohm
rotor resistance between terminals = 0.410 ohm, windings only
turn ratio of stator to rotor = 1.65

What approximate resistance should be added to each leg of the rotor circuit for maximum torque to be developed at starting? What will be the approximate percentage starting current and starting torque on full voltage for this condition?

27. A 6-pole squirrel-cage motor has the following nameplate rating: 7.5 hp, 230 volts, 20.4 amp, 3 phase, 60 cycles, 1,140 rpm. Full-load pf is 82.8%. At no load, the motor draws 6.5 amp at 20% pf. If the sum of the rotor iron loss and windage and friction is 180 watts, what is the rotor resistance in ohms per leg referred to the stator?

28. Refer to Figure 8.8 which shows the elements of the 2-pole induction motor. If the rotor reactance were zero, what would be the torque angle of the motor? Would the angle vary with load or slip? How does rotor reactance affect the torque angle? What is the torque angle at starting with maximum developed torque?

29. While a slip-ring motor is carrying a steady load, the rotor-circuit resistance is increased suddenly by a small amount. Give the sequence of resulting actions within the motor until it returns to steady operation.

30. The voltage applied to an induction motor is increased 10%. What are the effects on exciting current, stator current, rotor current, speed, efficiency, and power factor? Give the approximate changes in per cent of rated values. Shaft load remains constant.

31. A 5-hp, 4-pole, 50-cycle, 220-volt induction motor is to be supplied from a 220-volt, 60-cycle source. What will be the effects on exciting current, stator and rotor currents at rated output, speed, efficiency, power factor, starting torque, and maximum torque? If the 50-cycle eddy-current and hysteresis losses are 150 and 100 watts, respectively, what will be the new iron loss at 60 cycles?

32. A customer contemplates using a 40-hp, 440-volt, 3-phase, 60-cycle, 1,140-rpm induction motor on 25 cycles. If this were to be done, what should be the new rating of the machine? What would be the effects on efficiency, power factor, starting torque and pull-out torque?

33. Why are high efficiency and high power factor mutually opposed in an induction motor?

34. What is the effect of the length of the air gap on the efficiency and power factor of the induction motor? Does the gap length influence the type of bearings?

35. For the same horsepower, voltage, and frequency, why will a high-speed induction motor have a better efficiency and power factor than a low-speed motor? For the same voltage, frequency, and speed, why will a motor with a higher horsepower rating have a better efficiency and power factor than one with a lower horsepower rating?

36. As a percentage of full-load torque, would you expect the starting torque of the induction motor to increase or decrease, in general, as the number of poles is increased?

37. For squirrel-cage motors, what would be the general shape of the rotor slots and the relative resistance of the conductors to give:

(a) Normal starting torque and normal starting current,
(b) Normal starting torque and low starting current,
(c) High starting torque and low starting current, and
(d) High starting torque and high slip?

38. A 3-phase, squirrel-cage motor has the following rating: 10 hp, 220 volts, 26.9 amp, 60 cycles, 1,148 rpm.

no-load data are: 220 volts, 9.8 amp, 670 watts, 1,196 rpm
locked rotor: 110 volts, 96 amp, 9.5 kw
resistance between stator terminals = 0.284 ohm

Construct the circle diagram for the motor. Show the point of normal operation. Determine the maximum power factor and the maximum torque.

39. Data for the circle diagram are to be obtained on a 15-hp, 440-volt, 17.5-amp, 3-phase, 60-cycle, 1,150-rpm, squirrel-cage motor. The wattmeter to be used is a polyphase instrument having 5-amp current coils and 150-volt potential circuits with multipliers for 600-volt ranges. Sketch the circuit diagrams for the tests, showing the positions of all instruments and their polarity markings. List all instruments and auxiliary measuring equipment required for the tests.

40. What instruments and auxiliary measuring equipment would you order for indicating the line voltage and current of a 350-hp, 2,300-volt, 3-phase, 1,160-rpm induction motor to be installed permanently?

41. What is the general torque requirement on a motor driving a punch press? How does this influence the character of the motor selected for the drive? What type of squirrel-cage motor would you recommend? Would a wound-rotor machine have any advantages over a squirrel-cage motor for this application?

42. What is the torque requirement on a synchronous motor in starting a fan load and in bringing it up to speed? What type of squirrel-cage winding should the motor have? Answer these questions for a synchronous motor driving a compressor, a blower, a centrifugal pump, as part of a motor-generator set, and when connected to a high-inertia load.

43. The driving machines of a lumber mill are induction motors which run at greatly reduced loads most of the time. Discuss the effect upon the economical operation of the plant.

44. The average yearly load on a plant is 2,500 kw, at a power factor of 0.65 lagging because of induction motors. The plant is in operation 275 days a year, 8 hours a day. Electrical energy costs one cent per kwhr, with a penalty of $\frac{3}{4}$% of the energy bill for each 1% decrease in power factor below 85%, and a bonus of $\frac{1}{2}$% of the energy bill for each 1% increase in power factor above 85%. If capacitors cost \$10 per kva installed, what kva of corrective equipment will yield in savings during the first two years an amount equal to the investment cost?

45. A 125-hp, 2,200-volt induction motor for driving an electric shovel in an open-pit mine has a starting torque of 150%, a pull-out torque of 280% at 15% slip, and full-load torque at a slip of 2%. Sketch the general speed-torque curves for this motor for 100%, 90%, and 80% rated voltage. Does this motor have any advantage over one having the same horsepower but a 440-volt rating? What is the approximate minimum continuous kva in supply transformer capacity required per motor horsepower? If several shovels obtain their power from the same transformer bank, will the ratio of bank kva to horsepower of connected load be affected?

46. A 2,200-volt, 3-phase, squirrel-cage motor draws 1,200 kva in starting on 65% voltage. The stator winding is 1-circuit delta, with 12 slots per pole and 4 conductors per slot. What is the maximum value of the conductor current? Assuming sinusoidal distribution of the mmf over the pole pitch, what are the maximum ampere-turns per pole? What maximum current will flow in the end rings of the rotor winding?

47. The average load on an induction-motor driver is to be 40 hp with overloads of short duration up to 275% of normal torque. The slip at these overloads is not to exceed 5%. What should be the rating of the motor? Would you recommend a squirrel-cage or a wound-rotor machine?

48. The frequency was changed from 50 to 60 cycles over a large part of the industrial area of a city. To satisfy job requirements, a 15-hp, 220-volt, 716-rpm, 50-cycle squirrel-cage motor had to produce 100% of full-load torque when starting on rated voltage at 60 cycles. A blocked-rotor test on the motor showed 80 volts, 41.8 amp, 2,870 watts, 50 cycles. Resistance between stator terminals was 0.38 ohm. Was the motor satisfactory? Neglect iron losses.

49. In a large oil refinery, the rotor of a 1,600-hp, 3,600-rpm steam turbine is direct connected to a 1,250-hp, 2,300-volt squirrel-cage motor, a boiler feed pump, and a booster pump, all on the same extended shaft. The primary purpose of the turbine is to drive the pumps, hence normally the motor lines are open and the machine idles, the turbine driving through it. What two functions can be fulfilled by the induction machine, and under what condition? What approximate kvar is carried by the induction machine at rated slip?

50. An induction motor in substation A is fed by cable from substation B some distance away and drives a d-c generator for charging a large battery. One evening the cable at station B was opened as usual, but the operator in station A failed to disconnect the battery from the generator. Station A was unattended during the night, and in the morning the battery was found discharged. Further, a recording voltmeter showed a continuous voltage at the motor terminals throughout the night higher than normal. Why did this voltage exist at the motor terminals, and does this fact provide the major reason for the discharge of the battery?

51. A 3-phase, 30-hp, 60-cycle, 8 pole, 220-volt induction motor is to drive a traction elevator in an office building. The elevator is to be accelerated at an average rate of 4 ft per sec per sec to a normal speed of 250 ft per min. The equivalent WR^2 of all moving parts is 120 lb-ft² referred to the motor shaft. Show that the required starting torque is

$$T = \frac{WR^2 \times \text{synchronous speed}}{307 \times \text{seconds accel. time}} \text{ pound-feet}$$

Calculate the starting torque as a percentage of the torque obtained from rated horsepower and synchronous speed.

CHAPTER XII

SINGLE-PHASE MOTORS

12.1. Introduction. The discussion thus far has been limited to polyphase machines; there are, however, many fields of application in which only single-phase machines are found. In sizes of 1 hp and greater, they are used generally where 3-phase power is unavailable, but there is no similar factor controlling the use of the fractional-horsepower motor. Because of the character of many loads, the single-phase motor is more adaptable to the service than is a polyphase motor. It is easier to build very small single-phase machines than it is to build them for polyphase operation, and the extremely high speeds required in many applications dictate use of the single-phase motor.

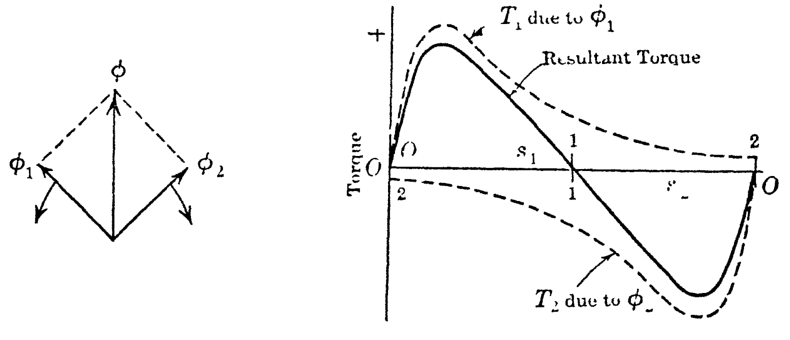
General-purpose, single-phase motors are usually found in the fractional-horsepower group, although there are a great many built with ratings from 1 to 15 hp. There has been no definite limit set on the size of a single-phase motor which can be built, and standard ratings reach as high as 40 hp. They seldom exceed 15 hp, though, because of the voltage fluctuations they cause at starting and because the polyphase motor is better adapted to service in sizes greater than a few horsepower. In many instances, power companies limit the size of single-phase motors which can be connected to their lines.

Fractional-horsepower motors are built with ratings as low as $\frac{1}{250}$ hp, although by far the greater number are in the range from $\frac{1}{20}$ hp to 1 hp. Tiny reluctance-torque motors are listed according to torque, of a few ounce-inches, at a given speed, which usually is very low. Voltages are generally 110 and 220 volts, but a few motors, having relatively high ratings, are designed for 440 volts. Small universal motors run on either a-c or d-c, with voltages as low as 6 volts. Frequencies are standard and in the range of 25 to 60 cycles, although there are a few applications outside these values. General applications call for speeds in the range of 900 to 3,600 rpm, but, again, the small universal motor may have an idling speed of 16,000 rpm and develop its maximum output at 5,000 to 8,000 rpm. The load requirements for single-phase motors are fully as severe as for the large polyphase machines, and the design, construction, and application of the single-phase motor have created an industry that is a large part of the engineering field.

Distinct from the enormous number of small machines for general use, there are many large single-phase motors and generators built primarily for traction duty. Some motors are of the series type and vary in size from a few horsepower, for service such as with cranes and hoists, to very large machines in locomotive drives. Single-phase synchronous motors, used in motor-generator sets on locomotives, have ratings of a few thousand horsepower. Single-phase alternators are normally found only in railway service, but they also are built for special work such as testing. In general, single-phase synchronous machines are simply 3-phase Y-connected machines with one leg left idle in operation, and their kva ratings usually are relatively high.

Practically all single-phase motors are designed so that they will be self-starting, although with but a single winding they do not have the revolving field that is fundamental to the polyphase motor. As a consequence of the methods by which many single-phase motors are started and brought up to speed, they carry names indicative of the starting circuit or device. Others take their names from the manner of operation, and hence we have the split-phase, capacitor, repulsion, series, shading-coil, etc., types of single-phase motor. All of them are built with various ratings along certain standard lines that aim to satisfy general and special starting and load requirements.

12.2. Single-phase Induction Motors. This type of motor is discussed first because it introduces the problem of starting single-phase motors. In the simplest form, the machine has a squirrel-cage winding on the rotor like that of the polyphase motor. The stator also is similar in mechanical construction but has only a single-phase winding, usually of fractional pitch and not necessarily filling all the slots. This single-phase winding does not give the motor an inherent starting torque because the stator mmf is confined to one axis and cannot produce a revolving field. Unless the motor is started by some external means it will not come up to speed, but once started in either direction it will continue to run. The first single-phase motors were of this character and were started mechanically, the speed increasing until the motor could develop sufficient torque to accelerate itself to normal no-load speed. In like manner, single-phase operation can be obtained from the polyphase induction motor. After the machine has



(a) Two oppositely rotating fields

(b) Two opposing torques together with resultant

Fig 12.1. The single-phase induction motor.

reached its no-load speed, in either direction of rotation, it will continue to run if one line lead is opened. Normal load cannot be so great as before, the machine will be noisier, the efficiency and power factor will be poorer, the speed regulation will be greater, and the pull-out torque will be less.

The explanation for the ability of the motor to accelerate when started and to carry load is not simple, and more than one theory has been advanced for the operation of the machine. For simplicity we may consider the single-phase pulsating field of the stator winding to be equivalent to 2 equal and oppositely revolving fields, Figure 12.1a. Either field alone may be thought to give rise to one of the typical torque-speed curves, Figure 12.1b, of the induction motor. The dotted lines are carried over the range from zero to 200 per cent slip, a slip of 2 meaning synchronous speed in the reverse direction with respect to the rotating field. The resultant torque for both fields is given by the full line; hence, once started in either direction, the motor will follow that stronger field. Starting schemes were soon applied to the motor, the first of which is split-phasing.

12.3. Split-phase Motors. Because the single-phase motor has no true revolving field, it is started by special means. In the split-phase motor, Figure 12.2, there are 2 coils on the stator, 90 degrees apart, with resistance usually inserted in series with one of them. Reactance is used sometimes. The basic idea is to obtain unequal ratios of resistance to reactance for the 2 parallel circuits so that the mmf of one will rise to its maximum value at a time different from the other. A time-phase angle of 30 to 45 degrees is obtained normally by using resistance in one branch. The motor is not a balanced 2-phase machine because the turns and wire sizes of the coils are not the same, but an imperfect revolving field does result. As a consequence, the rotor moves in the direction of the field. Near full speed, the auxiliary winding usually is disconnected by a centrifugal switch. Split-phase motors have been built since the early 1890's, although the first ones were not too successful, primarily because of mechanical difficulties in developing a good throw-out switch for the auxiliary winding.

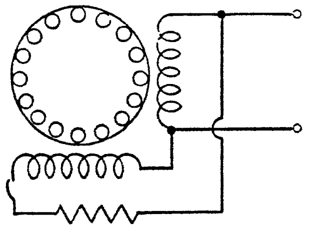


FIG. 12.2. The split-phase motor.

The starting torque of the split-phase motor is low, and the starting current relatively high, several times the normal full-load current. For this reason, it is built only in small sizes, perhaps no greater than $\frac{1}{2}$ hp. In some localities the size may be limited further so that voltage fluctuations due to starting will not be troublesome. The motor has application in home appliances such as machines for washing clothes and dishes, ironers, small grinders, fans, blowers, oil burners, and air conditioners. It is used on farms and in dairies for driving equipment having low starting requirements. Further applications are with office machines, sewing and winding machines, cloth cutters, machine tools, drills, and advertising devices.

12.4. Capacitor Motors. A much better method of phase-splitting is to use a condenser in place of the resistance in the auxiliary winding of the split-phase motor. This second type of machine is called a capacitor motor, Figure 12.3, in distinction to other forms of single-phase motors. It is usual to have 2 condensers in the second circuit, a large one to secure desired starting characteristics, and a smaller one to give good power factor under load. The large condenser is disconnected by a centrifugal switch as the motor nears full speed. A single condenser can accomplish the results of the 2 just discussed if it can be varied in capacity as the motor comes up to speed. The equivalent effect of this is obtained by connecting the condenser across an autotransformer in the auxiliary circuit, a centrifugal switch changing taps on the transformer as the motor nears its no-load speed.

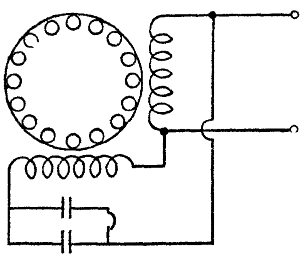


Fig. 12.3. The capacitor motor.

The function of the auxiliary condenser circuit is to convert the single-phase supply to 2-phase power in the motor, thus obtaining a revolving field. The separate fields of the 2 windings can be designed to have the same strength and to appear 90 degrees in time with each other. The motor then runs as a 2-phase machine with constant air-gap torque. The power factor can be made unity at starting, and unity or leading under load by using the proper capacitance. Starting torque is several times normal torque, with a line current about 70 per cent that of a corresponding 2-phase motor. The efficiency is much higher than that of the split-phase or repulsion-induction motors, and this factor makes the motor desirable for applications in which load is carried for long periods of time.

Capacitor motors are built in sizes about $\frac{1}{30}$ hp to 10 hp or more. They may be capacitor start-capacitor run or capacitor start-induction run. Although developed primarily for refrigerators and oil burners, the capacitor motor satisfies almost any job requiring a small-motor drive. Two important uses are for pressure pumps, $\frac{1}{4}$ to 2 hp, on farms, and air compressors, up to 5 hp but usually 2 or 3 hp, in automobile service stations. They have added uses with pumps, stokers, compressors, cream separators, small concrete mixers, and general household appliances. Whereas 110- and 220-volt stators are usual, the larger motors may be wound for 440 volts.

12.5. Repulsion Motors. The repulsion motor, Figure 12.4, is the simplest a-c commutator machine. It consists of a single-phase stator, a d-c armature, and a pair of short-circuited brushes on the commutator. By virtue of transformer action, currents are induced in the rotor conductors and torque is developed. The axis of the mmf of the rotor currents is confined to the brush axis and is stationary in space regardless of speed. The torque hence is

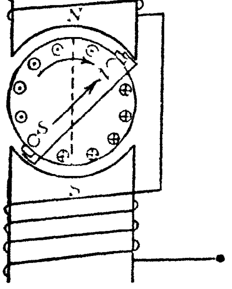


Fig. 12.4. The repulsion motor.

a function of the angle between the axes of the brushes and the stator winding, the angle being about 20 degrees for normal operation. The direction of rotation is reversed by shifting the brushes to the opposite side of the main field axis.

The motor has a very high starting torque with low power factor and a high running speed. Some motors of this type are adjustable in speed by having a brush-shifting mechanism movable by hand, and they can be reversed quickly from full speed in one direction to full speed in the other. Changes in load produce large changes in speed, but a second set of short-circuited brushes at 90 degrees with the first will give constant-speed characteristics. A great advantage of the repulsion motor is the independence of the rotor circuit. The rotor winding is designed generally for low voltage, but the stator can be wound for several thousand volts. The motor has been adapted to railway and crane service in past years, but has given way to other types of machine for these applications and is seldom seen as a straight repulsion motor.

12.6. Repulsion-induction Motors. In addition to a commutated winding, this variation of the repulsion motor has a squirrel-cage winding placed below the other. The two are separate, and both are active in carrying load. The motor has the high starting torque of the straight repulsion machine and now has also the constant-speed performance of the single-phase induction motor. The squirrel-cage winding eliminates a high speed regulation. The motor is similar to the d-c compound motor in the combination of starting and running characteristics. Being somewhat deeply embedded in the rotor iron, the squirrel-cage winding has the desirable running performance of the double-squirrel-cage induction motor. The reactance is high and the resistance is low when up to speed.

Repulsion-induction motors are built in sizes $\frac{1}{2}$ hp to 15 hp. They are heavy-duty machines and are well adapted to service requiring high starting torque, for example, deep-well pumps, compressors, blowers, stokers, and conveyors.

12.7. Repulsion-start Induction-run. If the commutated winding of the repulsion-induction motor is removed from service as the motor comes up to speed, the machine operates then only on the squirrel-cage winding and runs as a single-phase induction motor. In most instances, though, only the commutated winding is placed on the rotor. When well up in speed, a centrifugal device, working as a sleeve around the shaft, throws a short-circuiting ring against the commutator bars from the inside. At the same time, it may push the brushes away from the commutator, which for this type of machine is radial rather than cylindrical. The motor, therefore, starts as a repulsion motor and runs as a single-phase induction motor on the same winding.

These motors have a starting torque several times rated torque, take a relatively low starting current, and have good speed regulation. They are built in sizes about $\frac{1}{10}$ hp to 15 hp and are limited in size only to that permitted on the power system. Applications include pumps, compressors, milling machines, hoists, refrigerators, oil burners, stokers, conveyors, machine tools, washers, floor polishers, and coffee, meat, and food grinders.

12.8. Series Motors. The a-c series motor, Figure 12.5, is similar in construction to the d-c series motor, and it has similar characteristics. It is a commutator machine with the field winding in series with the rotor circuit. Depending upon the relative connection of the field and armature, the motor will run in either direction. The number of turns in each field coil of the a-c machine is less than for the d-c machine so as to decrease the inductance, and the flux density is reduced. To provide the same torque the armature current is greater, and this increases the cross-magnetizing field which leads to a considerable distortion of the air-gap flux and very poor commutation. Compensating windings, however, which carry armature current and are placed in the pole faces, neutralize armature reaction. Better power factor is obtained in addition to better commutation. Commutation also is improved with resistance between the armature coils and the commutator. A variation in design is to short-circuit the compensating windings upon themselves, in which case compensation is inductive rather than conductive.

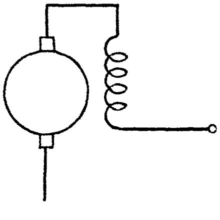


FIG. 12.5. The series motor.

The a-c series motor has a large starting torque, as does the d-c series machine, and, similarly, it has a large speed regulation. The power factor is good. The motor can run on direct current, and with a higher efficiency, as well as it can run on alternating current. A few minor changes are made in the circuits of the larger motors when they are to be connected to d-c lines, and the voltage is reduced. Very small machines are universal motors, that is, they will run on direct or alternating current of the same voltage and with no changes of any sort in them. In the smaller sizes, the a-c series motor finds application in sewing machines, vacuum cleaners, food mixers, fans, etc. (see Universal Motors).

Large series motors are used in main line and interurban railway electrifications. Here they are found with ratings of several hundred horsepower and possibly over a thousand horsepower. The motors are designed for voltages between 170 and 230 volts, with 300 volts maximum. They take their power from the secondaries of transformers, the primaries of which are connected to 11,000-volt 25-cycle lines, standard for this country. The power factor of the series motor for railway service is about 95 per cent at the continuous rating of the motor, and the efficiency is in the neighborhood of 88 per cent.

The useful charaacteristics of any series motor are high starting torque and high running speed. The speed regulation on constant voltage, however, is high. Speed regulation or variable-speed operation is obtained with railway motors through a change of voltage by means of taps on the transformer secondaries. There is far less loss than with rheostatic control used for d-c series motors in railway service.

12.9. Universal Motors. Universal motors, as the name implies, are machines that will run about equally as well on direct or alternating current. They are small series motors, built in sizes about $\frac{1}{64}$ hp to approximately $\frac{1}{2}$ hp. They are designed for various voltages, and for the frequency range from direct current to 60 cycles. The most common speeds are in the range of 3,500 to 8,000 rpm, although in some cases the speed is very much higher. Starting torque is high, and the power factor under load is good. The efficiency is low.

Of very great value in their general use is the fact that they are small, light, portable, and powerful for their size because of the high speeds at which they run. Speed control is obtained easily by varying the armature voltage by way of series resistance in the circuit. The governor-controlled motor has a centrifugal device which works extremely rapidly in closing or opening contacts across a series resistance. A condenser in parallel with the resistance absorbs energy as the contacts open. This type of motor is found in some food mixers, and it holds constant speed regardless of variation in torque. The universal motor has a great many applications. Common ones are in vacuum cleaners, portable drills, small ventilating fans, sewing and dish-washing machines, food and drink mixers, floor polishers, polishing and buffing machines, cameras, moving picture projectors, oil burners, grinders, dental and surgical equipment, laboratory equipment, adding and calculating machines, metal and woodworking machines, and novelties and games.

12.10. Shaded-pole Motors. The shaded-pole motor has projecting poles on the stator, and its field structure is similar to that of the d-c machine. The poles are split, and a short-circuiting coil or heavy collar, called the shading coil, surrounds one part of the pole, Figure 12.6. As the main flux pulsates through the pole body, a current is induced in the shading coil which acts as the secondary of a short-circuited transformer. The flux Φ_2 thereby is prevented from reaching its maximum value at the same time as the flux Φ_1. The result is a shift of flux across the pole face from the main section to the shaded area. Consequently, a rotating field, although very poor, is produced and the motor runs.

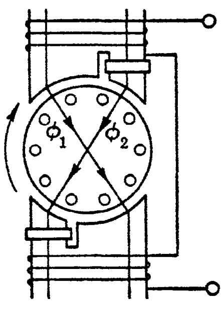


Fig. 12.6. The shaded-pole motor.

Shaded-pole motors have limited application because of their weak starting torque. They are found principally as drivers for small desk fans, for timing motors, in domestic furnace control, and in the light-load adjustment of watthour meters. They are built only in very small sizes, $\frac{1}{250}$ hp to about $\frac{1}{30}$ hp, and usually run on 110 or 220 volts, 50 or 60 cycles.

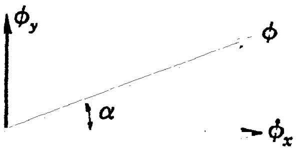
12.11. Synchronous Motors. There are a few types of motors which run at synchronous speed. They are used primarily for timing devices and where accurate and constant average speed is desired. Specific applications are in clocks, timers, sound cameras, motion picture projectors, recording instruments, control of heat instruments, relays, X-ray equipment, governor control, and stoker drives. They are a source of low-speed torque or power delivered at constant speed. Some may be started and run in either direction. In stoker drives, a spring arrangement will reverse the motor if it jams, and start it forward again at the end of its back travel.

Without reduction gears, the speeds may be as low as 75 rpm on 60 cycles and, with gears, as low as 1 rpm. Ratings are given in ounce-inches of torque at a given low speed or in terms of fractional horsepower, for example, $\frac{1}{12}$ hp at 1,800 rpm. Voltages are 110 and 220 volts, and frequencies are standard, from 25 to 60 cycles.

Hysteresis, or reluctance, torque provides the means of keeping these small motors in synchronism. The rotor construction is of salient-pole character so that the rotor iron can become magnetized with definite polarity. They are started as induction motors, using shading coils or a split-phase circuit employing either resistance or capacitance. A few are given a quick start by hand.

Problems

1. The stator coils of a 2-pole capacitor motor produce x and y component space fields as shown. In time, $\phi_x = A \cos \omega t$ and $\phi_y = A \cos (\omega t - 90°)$. Plot the locus of the resultant field, ϕ, for one complete cycle. Derive an expression for the rotating field, similar to that of equation 8.1.



2. The stator coils of a 2-pole, split-phase motor produce the following component fields $\phi_x = A \cos \omega t$, $\phi_y = (A/2) \cos (\omega t - 60°)$. Repeat problem 1 for the resultant field.

3. A 5-hp, 230-volt, 1,720-rpm, 60-cycle, single-phase capacitor motor has an efficiency of 83% and a pf of unity at full load. The resistance of each leg of the split-phase winding is 0.9 ohm. It is to be assumed that at rated load the leg currents are equal and the power factor of each leg is 0.71. Draw the power-flow diagram showing, input, stator copper loss, rotor loss, fixed losses, and output

4. A locked-rotor test of the motor of problem 3 made on 230 volts shows 85 amp and 36 lb-ft. Express both of these values in per cent of rated values.

5. A Y-connected, 3-phase, 10-hp, 440-volt, induction motor is delivering 7 hp at an efficiency of 84% and 0.86 pf. Stator copper losses for this load amount to 3.4%. One of the 3-phase lines is now opened and the motor continues to deliver 7 hp to the shaft load. Making reasonable approximations, calculate the new stator copper loss If the total stator copper loss is not to exceed that for 3-phase operation, what should be the approximate rating of this motor for single-phase operation?

6. At no load, a 3-phase, 220-volt, 10-hp, squirrel-cage motor draws 9.8 amp and 670 watts. Give the approximate value of the line current if the fuse in one of the lines is removed.

7. A manufacturer gives the following data for a $\frac{1}{6}$-hp, 115-volt, 60-cycle, 2-pole, series (of the universal type) motor: 2.03 amp, 0.9 pf, torque of 2.8 oz-ft, resistance of armature circuit of 5.2 ohms, resistance of field of 7.6 ohms. Calculate the efficiency, rpm, and the sum of windage, friction, and iron losses. The speed of this motor is approximately inversely proportional to the line current. Calculate the rpm for 0.64 amp.

CHAPTER XIII

DIRECT-CURRENT MACHINES

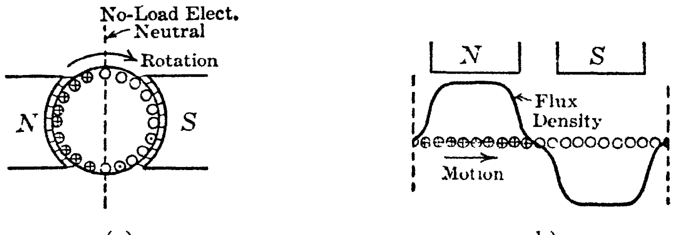
13.1. Introduction. The d-c machine already has been discussed to good extent. General features in its mechanical construction have been noted, the manner of generating voltage and of producing torque has been explained, the function of the commutator was described, and the armature windings were taken up specifically. We come now to the operating characteristics of this type of machine, and the changes, primarily in its electrical circuits, whereby the terminal voltage of the generator may be modified and control obtained over the torque and the speed of the motor.

In comparison with the high ratings that a-c machines have, the d-c machine is moderate in size. At one extreme, both generators and motors are built with fractional horsepower ratings. Generators reach capacities of several thousand kilowatts, and motors may have a rating of many thousand horsepower, for example, 8,000-hp motors for steel mills. Voltages range from a few volts, as typified by those for automobile generators and motors, through an extremely great number of standard and odd voltages to a few thousand volts. One of the most fundamental difficulties in building d-c machines in very large sizes or for relatively high voltages as those used with a-c machines lies in the problem of commutation, a problem more mechanical than electrical. Again, the power output of the d-c machine is represented by its armature, which is the rotating element. Like the synchronous machine, it is far easier to build a machine in which the field structure rather than the armature revolves, but this cannot be done, of course, with the d-c machine because rectification is obtained by rotation.

Since d-c machines do not have the reactive effects of a-c machines to be considered in design, they have a higher ratio of power to volume for a given sp ed. They are fully as reliable as a-c machines and have the distinct advantage of flexibility which the a-c machine in general has never had. Voltage, speed, and torque are readily controlled, and the d-c machine has a good efficiency. Over the last few years, and for machines in the smaller sizes, the curve of horsepower of d-c motors sold, as a function of time, has had a rate of increase greater than that of a-c motors. A limitation on the use of d-c motors in many localities lies in the fact that d-c power is not available.

13.2. Armature Reaction. Armature reaction, which is the effect of the armature mmf upon the flux distribution of the machine, is a fundamental problem in d-c generators and motors. It has a pronounced influence on commutation, which is perhaps the most serious limitation on the size and voltage rating of d-c machines. Armature reaction is treated here mainly as it applies to the generator, but the student should have no difficulty in carrying the discussion over to the d-c motor.

The ampere-turns of all windings on the field poles act on the magnetic circuit of the machine along the center line of the poles, that is, in the direct axis, the same as in the synchronous machine. With no armature current flowing, the field mmf gives rise to a uniform flux distribution under the pole faces, symmetrical with the direct axis, and with a point of zero flux density approximately midway between poles. Figure 13.1. This point of zero flux density is called the electrical neutral and, in general, will not be exactly at the mechanical neutral position midway between poles.



(a) (b)

FIG. 13.1. Distribution of air-gap flux of a generator at no load.

With no field excitation, the armature winding, when carrying current, produces its own flux distribution, Figure 13.2. The armature mmf always acts along the plane at 90 electrical degrees with the axis of the brushes which are placed approximately under the pole centers so as to short-circuit each coil as it passes through the electrical-neutral plane. The armature mmf, then, exerts its effect on the quadrature axis midway between poles, and all or the greater part of this mmf is said to be cross-magnetizing. The flux distribution is symmetrical about the quadrature axis only when the brushes are directly under the pole centers so that the coils they feed lie in the mechanical-neutral plane.

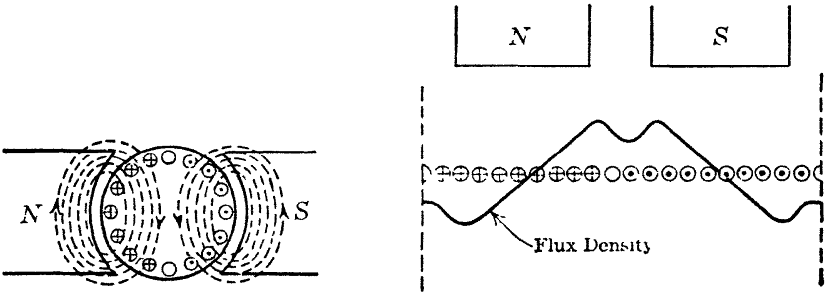


FIG. 13.2. Flux distribution due to armature current.

Under normal operation, both field and armature carry current, the axis of their resultant mmf being somewhere between the direct and quadrature axes. The armature mmf, therefore, has a distorting influence upon the main-field flux, Figure 13.3, which is shifted from the center of the pole toward one of the pole tips. The shift is forward in the direction of rotation for a generator, and backward against rotation for a motor. The electrical neutral hence moves along the armature surface as the armature mmf is increased, that is, with load current.

The magnitude of flux shift depends upon saturation in the pole tips as well as upon the value of the armature current. If saturation does not exist, the increase of flux in one pole tip is offset by a corresponding decrease in the other. With saturation, however, there is a net decrease in total flux, resulting in a decrease in terminal voltage of the generator and an increase in speed of the motor. The distorting effect of armature reaction is used in the principle of operation of the third-brush constant-current automobile generator and in speed regulation for some small motors.

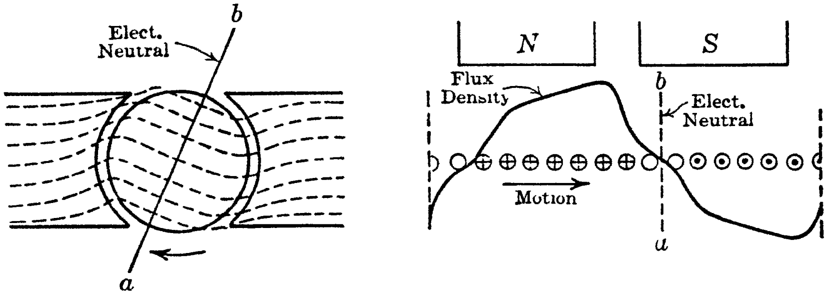


FIG. 13.3. Distribution of air-gap flux of a generator under load.

13.3. Commutation. When an armature coil moves from one pole to another, the current in it is reversed from a maximum in one direction to a maximum in the opposite direction. This reversal of current is called commutation, and it takes place during the time the 2 commutator bars to which the coil ends are fastened pass under, and are short-circuited by, a brush. Figure 13.4 shows a coil undergoing commutation. In position (a), the current through the coil is from terminal 1 to terminal 2. At (b), the coil is short-circuited, and at mid-position should have no current in it. At (c), commutation has been completed and the current in the coil has been reversed, as shown by the arrows. The time to complete commutation may be as low as 0.0005 sec in a commercial machine.

If the coil sides cut flux while they are short-circuited, the induced voltage in them may cause a heavy circulating current to flow in that coil. Consequently, the coil, bar, and brush will overheat, and, further, severe sparking will follow as this current is interrupted by one of the coil ends leaving the brush. It is necessary, therefore, that coils being commutated lie in planes of approximately zero flux.

Quite apart from the fact that a coil may be cutting no flux during commutation, in all coils undergoing current reversal there is an opposing self-induced voltage appearing solely because the current is changing. The magnitude of this reactance voltage depends upon the inductance of the coil, the load current being carried, and the time of commutation. For successful commutation the reactance voltage must be nullified. Consequently, provision must be made in some way to stabilize the electrical neutral of the machine and to permit at the same time the generation of a voltage equal and opposite the reactance voltage of commutation, or else to provide for shift of the brushes so that they can be moved to keep in position with the electrical neutral—perhaps slightly ahead. Modern machines employ the first idea. Many old-type machines have a brush-shifting mechanism so that the brushes can be moved with load or at least set at that point which gives the best average commutation for the load range of the machine.

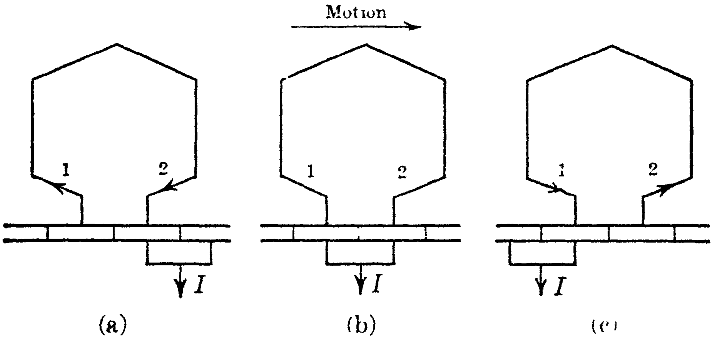


FIG. 13 4. An armature coil undergoing commutation.

13.4. Commutating Poles. While several schemes have been devised in the past to overcome the distorting effects of armature reaction, the most practical is that of using commutating poles. Commutating poles, or interpoles as they also are called, are narrow laminated auxiliary poles placed midway between the main poles and in the plane of commutation. They carry windings of a few turns and large cross section, are in series with the armature, and are wound to oppose and nullify the armature mmf in this region. The magnitude of the commutating-pole mmf is directly proportional to the load current, and the distortion of armature reaction thus is neutralized in the commutating plane for all values of load.

Further, commutation is assisted in that the commutating poles also provide a flux for an induced voltage equal and opposite the reactance voltage of commutation, as shown in Figure 13.5. Commutating poles are used of necessity on machines having high voltages between bars so as to prevent sparking that might cause flashover, and they also lead to a reduction in iron losses in the armature teeth. As indicated in Figure 13.5, a commutating pole of S polarity lies beyond a main pole of N polarity for either direction of rotation of the generator. With motors, an N commutating pole lies ahead of an N main pole for either direction of rotation. Changing from generator to motor action, the polarity of the commutating pole automatically changes, thereby giving this required result without a change in connections.

Commutating poles automatically hold the electrical neutral at the same point for all conditions of load. Their use is equivalent to shifting the brushes with load, could such a measure be accomplished. Commutation in commutating-pole machines is not affected by a change in the direction of rotation, by a change from generator to motor operation, or by heavy loads carried at high speed. Sudden load variations of large magnitude can be handled successfully, and a properly designed machine can carry 200 per cent or more full load without harmful sparking.

Because of their mechanical position, commutating poles increase the effect of armature reaction on either side of the commutating region and may have a weakening effect on the main field flux because of leakage. Under load, the tendency then is towards a decrease in induced voltage of the generator. With motors, the speed may not drop so much under load—in fact, it may increase, particularly if the field current is low or if the load is much above normal. Instability may result at fairly high overloads on motors.

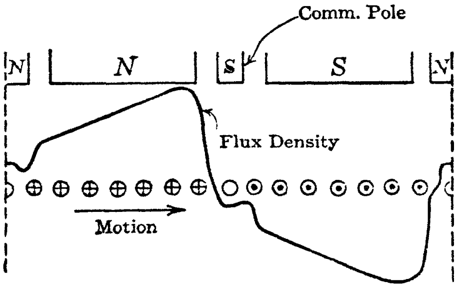


FIG. 13.5. Distribution of flux in a commutating-pole generator.

From Figures 13.2 and 13.5 it can be reasoned that one commutating pole of double strength will perform the service of two. Many machines, therefore, are built with only half the number of commutating poles as main poles. Any commutating pole acts on coil sides leading to the 2 adjacent paths through the armature, see Figures 9.6 and 9.7. Consequently, only one commutating pole is necessary for each pair of paths. A P-pole lap-wound machine may have P or $P/2$ commutating poles. A wave-wound machine requires only one, although P or $P/2$ are provided. In practice, commutation is found to be better if machines above 10 kw are provided with as many commutating poles as main poles.

13.5. Effect of Brush Shift. The distorting effect of armature reaction depends also upon brush position. If the brushes are moved so as to commutate coils when at some small angle, α, from the quadrature axis, the armature reaction mmf is said to have 2 components. The larger component is cross-magnetizing, as before, although, as pointed out, some demagnetization occurs because of saturation. The second component directly aids or opposes the main field mmf and is said to be magnetizing or demagnetizing, depending upon the direction of brush shift. In a generator, for example, the armature ampere-turns within the angle 2α are demagnetizing if the brushes are given a forward lead; they are magnetizing if the brushes are given an angle of lag. Note that there is a direct similarity to the armature reaction effects in synchronous machines.

In non-commutating-pole generators, a forward movement of the brushes decreases the total flux and, consequently, the generated voltage. A backward shift achieves opposite results and may lead to a terminal voltage under load higher than at no load. In the motor, a forward shift, with rotation, increases the total flux and, hence, leads to a decrease in speed. Moving the brushes backward weakens the total flux and causes an increased speed under load. Commutating poles exaggerate these effects of brush shift, and a very slight movement of only a small fraction of an inch may give pronounced results.

Practically, however, the amount of brush shift which can be tolerated in a d-c machine is extremely limited because of commutation and generally is not feasible. It is intended in commutating-pole machines that the brushes be set for the no-load electrical neutral and remain fixed for all conditions of operation. Brushes cannot be moved backward in the generator or forward in the motor without serious or destructive sparking. For this reason, any brush shift for improving commutation will be in the forward direction for a generator, hence always leading to demagnetization by the armature mmf. Brush shifting is not applicable to reversible-speed machines, nor can it be used satisfactorily with variable-speed motors controlled by the field rheostat since the ratio of field to armature ampere-turns is different for each basic speed.

13.6. Compensating Windings. Although the windings on the commutating poles neutralize the effect of armature reaction in the interpolar regions, they do not prevent distortion of the flux in the air gaps over the pole faces. This distortion may lead to voltages between commutator bars sufficient to initiate flashover between brush arms. Consequently, many machines are equipped with compensating windings which, with weaker interpole windings that now may be required, neutralize armature reaction over the entire armature periphery. These windings are placed in slots in the pole faces; they carry the armature current just as do the commutating windings and, hence, at all times have an mmf equal and opposite that of the armature mmf. Compensating windings are used on some a-c motors as well, for the same purpose of counterbalancing armature reaction.

13.7. Field Excitation. The general manner of classifying d-c machines is according to the method of field excitation, the 3 major types being shunt-, series-, and compound-wound field circuits. In shunt-wound machines the field circuit is placed in parallel with the armature circuit. The coils have a great many turns of small wire and carry a current of magnitude 0.2 to 5 per cent of the armature current. In all but a very few cases the field spools are connected in series, and in all instances they provide consecutive poles with opposite magnetic polarity.

It is unusual with motors for the field circuit to receive its power from a source other than that for the armature, but with generators the field excitation may be obtained from the armature or from a separate source. In the first case, the generator is said to be self-excited; in the second instance, it is separately excited and may or may not have a field-circuit voltage of the same magnitude as the armature voltage.

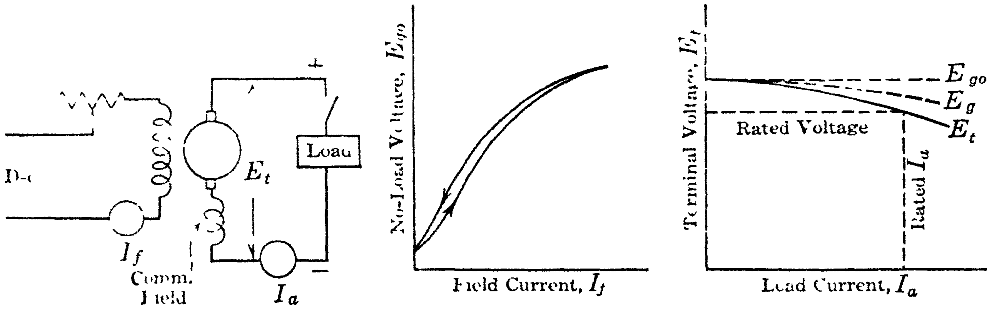
Series-wound machines have field coils composed of a few turns of heavy wire, are placed in series with the armature, and hence carry and are excited by the armature or load current. Series generators are infrequently met, but series motors have a wide field of application.

Compound-wound machines have both shunt and series windings on all field poles, the series coils being placed over the shunt coils. The mmf's of the 2 windings may aid or oppose each other, as desired. Whether the shunt field circuit is placed directly across the armature (short shunt) or includes the series coils as well (long shunt) is immaterial, practically, in terminal or operating effects.

If the ampere-turns of the series field are very low with respect to those of the shunt field, the machine may be called a stabilized-shunt generator or motor, although it actually is a compound machine. By far the great majority of d-c machines are compound wound, even though there may be only 2 or 3 turns in each series-field coil.

Control of the shunt-field mmf is obtained through the field rheostat which sets the no-load voltage of the generator or the no-load speed of the motor. The series-field mmf can be changed for a given load current only by shunting that circuit with a very low resistance called a diverter. It is not usual, though, to vary the series-field current except as the load current changes.

13.8. The Separately Excited Generator. Figure 13.6a shows the schematic diagram of the separately excited generator, and Figure 13.6b the no-load satura-



(a) Schematic diagram (b) Saturation curve (c) External characteristic

FIG. 13.6. No-load saturation curve and external characteristic of the separately excited generator.

tion, or magnetization, curve for the machine. This open-circuit characteristic is taken at constant (rated) speed, or at least is plotted for normal speed from corrected readings. Since the generated voltage is directly proportional to the flux, and the field current is directly proportional to the magnetizing force, the saturation curve is but a part of the complete *B-H*, or hysteresis, loop for the generator. The shape of the curve is determined solely by the characteristics of the magnetic circuit. The voltage does not start from zero, except for a new machine, because of a small amount of residual magnetism in the field structure

In taking such a curve on test, the field current must be changed progressively in one direction only, else minor hysteresis loops will appear. At no load, the generated and terminal voltages are one and the same.

When the generator is loaded, the terminal voltage decreases as a result of armature reaction and also because of the armature resistance drop I_aR_a. The external characteristic, Figure 13.6c, shows the manner of variation of terminal or load voltage with respect to the armature or load current. The voltage relation is

$$\begin{aligned} E_t &= E_g - I_aR_a \\ &= E_{g0} - \text{drop due to armature reaction} - I_aR_a \qquad (13.1) \end{aligned}$$

where E_{g0} = generated voltage at no load
E_g = generated voltage under load. Armature reaction decreases the net air-gap flux and hence the generated voltage. The field current is held constant.
R_a = armature-circuit resistance between brushes

The armature-circuit resistance drop at full load is small in comparison with the nameplate voltage rating of the machine and usually is 2 to 5 per cent of it. The circuit resistance includes that of the armature winding, the resistance of the commutating winding which always is considered part of the armature circuit, and the brush-contact resistance.

As has been shown (equation 9.5), the no-load generated voltage is

$$E_{g0} = \frac{\Phi PNZ}{60p'} \text{ volts} \qquad (13.2)$$

this equation holding only if the position of the brushes is such that the voltages induced in all conductors connected in series between the brushes are directly additive; that is, it is the maximum possible voltage.

Voltage regulation for all d-c generators is defined just as it was for the alternator, that is,

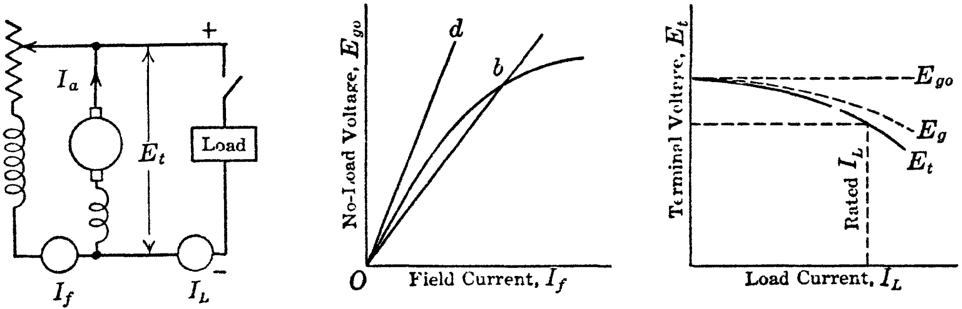
$$\frac{E_{\text{no-load}} - E_{\text{full-load}}}{E_{\text{full-load}}} \times 100 = \text{per cent voltage regulation} \qquad (13.3)$$

A voltage regulation test usually starts from full-load conditions, the load then being decreased gradually to zero. It must be pointed out that any decrease in speed of the prime mover under load will be reflected directly in the voltage regulation of the generator. Therefore, voltage regulation for a particular case may be so defined as to include also the speed regulation of the driving machine.

13.9. The Self-excited Generator. Figure 13.7a shows the schematic diagram of the self-excited generator, and Figure 13.7b the saturation curve for the machine together with field resistance lines. The saturation curve is the same whether it is taken with the field circuit supplied by the armature or by a separate power source. The no-load voltage is determined by the total resistance R_f of the field circuit and by the magnetic circuit of the machine as shown by the saturation curve. The no-load field current I_f is

$$I_f = \frac{E_{g0}}{R_f} \tag{13.4}$$

The field current produces a generated voltage that is also the voltage drop $I_f R_f$ across the field circuit. Therefore, the stable operating point can be found by drawing a line *ob* as indicated, the slope of the line being equal to the resistance of the field circuit. The point of intersection of this field resistance line with the no-load saturation curve fixes the no-load voltage and field current of the generator. The critical value of field resistance is determined by the tangent to the saturation curve, a resistance higher than this preventing the machine from building up its voltage.



(a) Schematic diagram (b) Saturation curve (c) External characteristic

FIG. 13 7. No-load saturation curve and external characteristic of the self-excited generator.

The mechanism of voltage build-up in the self-excited generator may be explained in the following manner. A small voltage is generated because of residual flux. When the field circuit is closed, this small voltage produces a small field current which increases the flux, which in turn increases the generated voltage. The cumulative action continues until the operating point on the saturation curve is reached. Saturation prevents the process from continuing indefinitely, and the highest no-load voltage is obtained when the resistance in the field rheostat is zero.

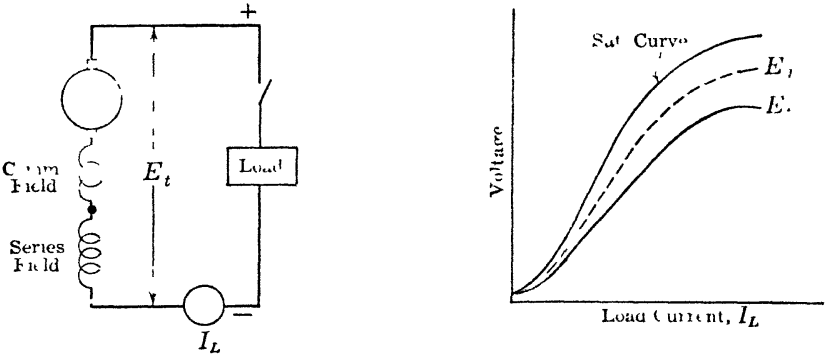
From the consideration of the process of voltage build-up in the self-excited generator, it is evident that the voltage will not rise as it normally should for any one of the following reasons:

1. No residual flux. A separate d-c source, perhaps a battery, then must be used to magnetize the field structure. Polarity also can be reversed by this method.
2. Field circuit open.
3. Reversed connections between armature and field circuit. The field circuit must be so connected across the armature that the field current produced by residual voltage increases the air-gap flux.

4. Field circuit resistance too high. In such a case, the field resistance line (*od* in Figure 13.7b) intersects the saturation curve at a very low value of voltage. Too low a speed of the prime mover or a short circuit on the armature terminals also will prevent voltage build-up.

The external characteristic of the self-excited generator is shown in Figure 13.7c, and, for the same machine, E_{g0} now must be greater than with separate excitation. The field circuit of the self-excited generator receives its power from the armature. Under load, as the terminal voltage drops because of armature reaction and armature resistance, the field current, therefore, also decreases. The terminal voltage is thereby further decreased, although the effect is not cumulative. It follows that the terminal voltage of the self-excited generator falls more rapidly with load than does the voltage of the separately excited machine.

13.10. The Series Generator. The series generator, Figure 13.8, has only the one field circuit which is in series with the armature. The saturation curve is obtained by separately exciting that winding. The external characteristic of the machine has the same general shape as the saturation curve and falls below it. The magnitudes of the generated and terminal voltages depend upon the load resistance which has the same effects as does the field resistance for the self-excited generator. The no-load voltage of the series machine is always the



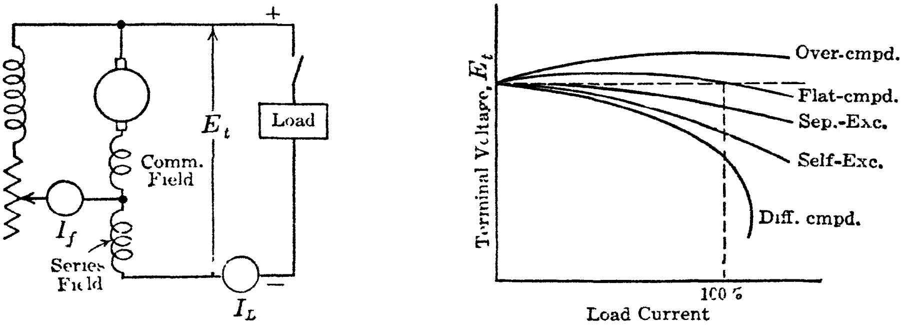
(a) Schematic diagram (b) Voltage characteristics

FIG. 13.8. Circuit diagram and voltage curves of the series generator.

residual voltage, and the load resistance must be below the critical value before the generator will build up in voltage. The series generator has a very limited application and is seldom met in practice.

13.11. The Compound Generator. The reduction in terminal voltage of the separately and self-excited generators can be compensated by placing an additional field winding, which carries the load current, on the main field structure, Figure 13.9a. This series field winding furnishes an mmf which is proportional to the load current and can add directly to that of the shunt field. The machine is said to be a compound generator.

If the number of turns of the series field is chosen so that the voltage drops due to armature resistance and reaction are exactly compensated at full load, the machine will have the same terminal voltage then as at no load. The generator hence is said to be flat-compounded. If the full-load voltage is greater than at no load, the generator is over-compounded, and if the full-load voltage is less than at no load, the generator is under-compounded. In all of these cases, the generator is cumulatively compounded because the mmf's of the shunt and series fields are in the same direction.



(a) Schematic diagram (b) External characteristics

FIG. 13.9. Schematic diagram of the compound generator and comparison of its external characteristics.

If the series field is so connected that its mmf opposes that of the shunt field, the generator is differentially compounded. The external characteristic may show a rapid decrease in terminal voltage with load current.

Figure 13.9b shows a set of typical external characteristic curves for d-c generators, the series machine omitted. For the sake of comparison, all curves are started from the same no-load voltage. It is from knowledge basic to these curves that the several types of d-c generators are found applicable to certain load requirements.

The amount of compounding is determined primarily by the relative strength of the series-field winding, and this may be varied by providing a resistance, called a diverter, in parallel with the series-field coils. If the resistance is variable, the external characteristics of the generator can be controlled and can be made to fall anywhere within the range from full over-compounded operation to full differentially compounded operation. In practice, the diverter is a fixed resistor such that the generator will produce a given external characteristic. Note that with over-compounding, the increase in terminal voltage at full load can be adjusted to offset the line drop if the load is at some distance from the generator.

13.12. Effects of Saturation and Speed on External Characteristics. The saturation curves of all generators are affected alike by a change in speed, and if this characteristic is known for one speed of a machine it can be calculated for any other speed. For the same shunt-field current, the ordinates (E_g) are directly proportional to the speeds.

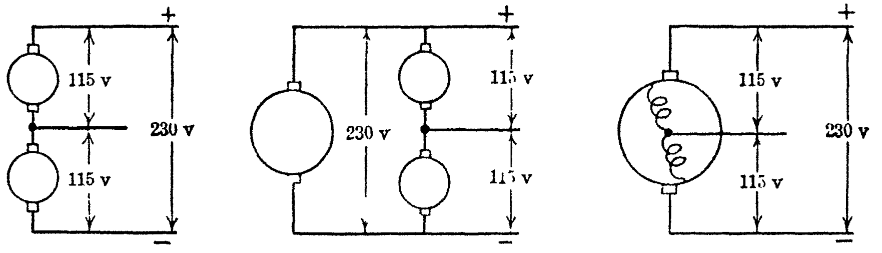
The external characteristics of all generators are affected by the initial degree of saturation. For the same speed, the lower the initial voltage of the separately and self-excited generators, the less will be the required flux of the shunt field. Consequently, the less the saturation, the more will armature reaction influence the external characteristics. These curves then fall more rapidly the lower the value of E_{g0}.

On the other hand, the less the degree of saturation, the greater will be the effect of the series field. Hence, the lower the value of E_{g0}, the greater will be the compounding action. In all cases, armature reaction and the series field both have more pronounced effects at lower saturation than at higher initial saturation.

For the same value of E_{g0}, a lower speed will require a greater shunt-field current, with a corresponding increase in saturation. Armature reaction then produces less distortion, and the external characteristics will not drop so fast for the separately and self-excited generators. Also, compounding action will be less, because the series field is not so effective. For the same value of E_{g0}, the higher the speed, the greater the influence of armature reaction and the series field.

13.13. Three-wire Systems. D-c distribution systems frequently provide 3-wire service, for example, 115 230 volts, in like manner to 3-wire single-phase circuits, and for the same reasons. There is a saving in apparatus and copper by running motors and power equipment from the 230-volt outer lines, and by operating lamps and small appliances from the 115-volt circuits. Electrical power supplied by 3-wire generators on board ship is an outstanding application.

Several methods are in general use for 3-wire service. Two 115-volt generators, for example, may have their armatures electrically in series, the neutral lead being carried from their common junction, Figure 13.10a. This is an early form for such a supply and is still common in practice. Another early form, now obsolete, employed a single generator having 2 commutators in series, the midtap between them forming the neutral junction. The lower-voltage loads, in all cases of 3-wire service, are so distributed between the outer lines and the neutral that they balance each other as nearly as possible. The resultant current in



(a) Two generators (b) Balancer set (c) Three-wire generator

FIG. 13.10. Schematic diagrams for 3-wire service. Field circuits omitted.

the neutral wire is generally expected to be no greater than 25 per cent of the rated current for the outer lines, thereby requiring less copper for that line.

Again, a 230-volt generator, for example, Figure 13.10b, may have across its armature a balancer-set composed of two 115-volt machines connected mechanically and having their armatures in series electrically. The balancer-set is not driven mechanically. The third line is carried, as before, from the common junction of the 2 small machines. If the loads between outer lines and neutral are equal, the balancer-set draws only its no-load current. When the loads are unequal, the machine on the lightly loaded side tends to run as a motor, driving the other as a generator. The voltages on the 2 sides of the system thereby are equalized to a great extent. Balancer-sets are frequently met in practice.

The 3-wire generator, Figure 13.10c, avoids the use of a balancer-set. In the simplest form, a high-reactance coil, wound on an iron core, is connected to taps 180 degrees apart on the generator armature. The voltage across the coil is alternating, and the midpoint of the coil becomes the neutral connection for the third lead. This balance coil may be within the machine, perhaps wound in the same slots as the armature winding, in which case the third lead is brought out through a slip ring. In the majority of instances, the coil, also called a compensator, is external to the generator, hence requiring 2 slip rings. Taps to the armature at 90-degree points call for 4 slip rings. The 3-wire generator is very common, and there are many variations in the scheme of obtaining the neutral point.

The series-field coils of 3-wire compound-wound generators are split, all of the N poles, for example, being placed on one side of the armature, and the S poles on the other. In this way, both sides of the system receive the same degree of compounding regardless of current unbalance. Voltage unbalance, however, is not influenced by splitting the fields. Commutating windings on both shunt- and compound-wound 3-wire generators are split in the same way. It is simpler to connect the shunt field to include the armature and commutating fields only. The voltage difference between the 2 sides of the system will not exceed 3 per cent of the voltage between the outer lines.

13.14. Parallel Operation of D-c Generators. D-c generators operating in parallel permit good efficiency to be maintained over wide variations in total load. The machines on the line can be run at or near their rated capacities, the number of generators being changed as the load varies. Again, whereas a present load may demand a given power output, future expansion may call for additional kilowatts that can be obtained through parallel operation. Continuity of service may require parallel operation in case a generator must be taken from the lines. In all cases, there is greater flexibility in controlling the power supply.

The requirements for parallel operation are few and are similar to those for alternators. The speed-load characteristics of the prime movers should droop. The polarity and voltage of the incoming machine must be the same as those of the system. For automatically proper load division among generators in parallel, the external characteristics of all machines should show the same change in voltage for the same percentage of their rated loads. Since, of course, an increase in field current leads to an increase in generated voltage and hence an increase in output, load can be shifted from one generator to another by field control of either or both of them. This may be necessary for generators having dissimilar characteristics, but it is not a satisfactory procedure. It can be accomplished only for a steady value of load and not for fluctuating loads.

All generators having drooping voltage curves will normally operate stably in parallel. Any increase in load is shared according to their external characteristics, Figure 13.11 showing the manner of load division between two generators, A and B. Machine A has the higher voltage curve, hence will always carry more load than B. If, however, I_A and I_B are their respective full-load currents, the generators are dividing the total load properly.

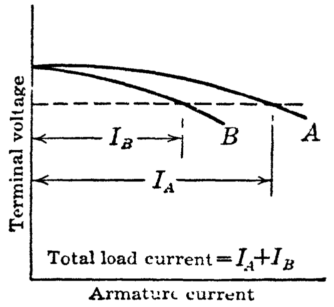


Fig. 13.11. Load division between two generators having drooping voltage characteristics.

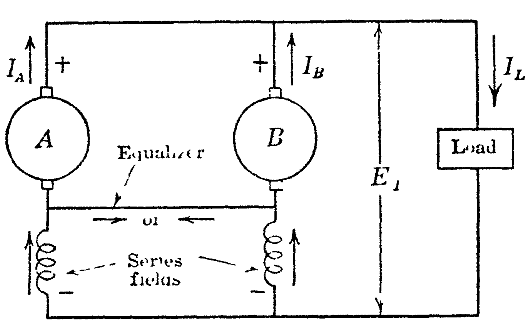


Fig. 13.12. Schematic diagram for parallel operation of compound-wound generators. Shunt-field circuits are omitted.

With drooping external characteristics there is no tendency for any generator to increase its own load by taking it from the others, that is, to exhibit a cumulative action. An increase in load on any one always lowers its internal voltage and automatically shifts load to the others as well.

Generators having rising voltage characteristics, for example, over-compounded machines, are inherently unstable when paralleled. In general, the generated voltage of that machine having the greatest compounding will increase with load, thereby causing it to take more than its share, drawing it from the others. The action is cumulative, and very quickly that generator may force current in the opposite direction through the other machines, tending to drive them as differential motors. To overcome this instability, a lead, called an equalizer, is connected between the series fields of compound generators, Figure 13.12. The equalizer is necessary only when the terminal voltage of a compound generator rises above the no-load value. A generator with such a terminal voltage rise cannot be paralleled successfully with a shunt generator over the complete load range of both.

The equalizer parallels the series fields of the generators so that a change in load on any machine also provides a corresponding change in series-field excitation for all the others, and each will share in the load variation. For proper load division, the series-field currents of the generators should be proportional to their respective kilowatt ratings; hence, the generators should have resistances of series fields and connecting leads to the equalizer bus inversely proportional to the generator ratings. Further, the resistance of the equalizer bus should be as low as possible, since too large a value will not permit proper proportioning of the series-field currents. Between any 2 compound-wound machines, the equalizer connection circuit should have a resistance not over 20 per cent of the resistance of the series-field circuit of the smaller machine.

Compound-wound 3-wire generators require 2 equalizer busses, one on either side of the armature. The commutating and stabilizing, or series-field windings, are split, and ammeters are so placed as to indicate armature currents. If a lead to either equalizer bus is opened, a circuit breaker removes that generator from the line.

To obtain correct load division among compound-wound generators, the governors of the prime movers should be adjusted first so as to give these driving machines the same speed-load characteristics. After that is done, the generators may then be adjusted for the same voltage regulation. The degree of compounding of a machine may be altered by changing the basic speed of the prime mover, by shifting brushes if commutation permits, or by inserting a small value of resistance in the connecting leads to the main bus. Use of a diverter on the series field is not feasible because it also shunts the series fields of all machines. In general, the characteristic of a generator is lowered by any adjustment for compounding.

Different saturation curves among generators may give the same voltage regulation at full load but improper load division at partial loads or overloads. Successful parallel operation is considered attained if the load on any generator does not differ more than ±15 per cent of its rated kilowatt load from its proportionate share, based on the generator ratings, of the combined load, for any change in the combined load between 20 and 100 per cent of the sum of the rated loads of all generators.

13.15. Application of D-c Generators. Figure 13.13 shows a set of typical external characteristics for d-c generators, the series machine omitted. For the sake of comparison, all curves are started from the same no-load voltage. It is from knowledge basic to these curves that the several types of d-c generator are found applicable to certain load requirements. Flexibility of speed and torque control of d-c motors has always given them a strong position in industrial applications, and their value is increasing in this respect. The d-c generator supplies the power for them, and it can do so with smooth voltage control. It is especially adaptable to reversible drives.

The separately excited generator has its primary use in requirements for voltage control, where multiple circuits are involved, and for Ward-Leonard control of the speed of motors (see later section in this chapter). Because the field circuit is independent of the armature, the machine can be designed for very low or relatively high voltages. In electroplating work, the lap-wound machine with many poles may carry as high as 20,000 amp at 6 or 12 volts. In supplying power for radio apparatus, the separately excited generator often is designed for 1,000 volts or more. This type of generator is, therefore, most advantageous for extremes in voltage or where an independently controllable armature voltage is desired. This type of generator also is used for propulsion on board ship.

The shunt generator is used as an exciter for both synchronous and d-c motors and generators. It is often controlled in voltage by an automatic voltage regulator. Because of its drooping characteristic, the shunt machine is well adapted to charging storage batteries. It also is a supply commonly used for electric furnaces. Shunt-wound generators with automatic voltage regulators are finding a more increasing application in parallel operation, where constant voltage is required, than compound-wound generators with equalizers. This is especially true in steel mills.

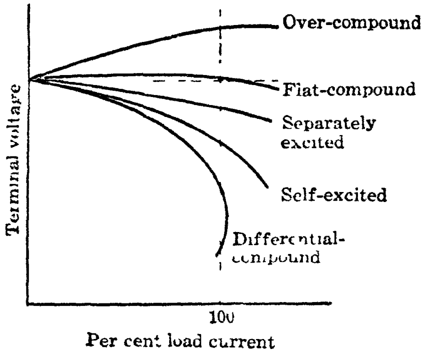


FIG. 13.13. Voltage characteristics of various d-c generators

Most d-c generators are compound-wound, even though the series field may have only 2 or 3 turns simply to stabilize the voltage. The cumulatively compounded generator is well suited to multiple systems and to long feeder systems, the rise in terminal voltage offsetting the line drop which increases with the load current. Loads may be either lighting or industrial, or both. Office buildings often take power for lighting and for motors which drive elevators, fans, etc., from a d-c system.

Flat-compounded generators are used where essentially constant voltage is desired and where the generator and its load are near each other. They have had application in street railway systems for many years, although a great amount of power for this type of service has been supplied by rotary converters. The flat-compounded generator also is used in some arc-welding sets.

The differentially compounded generator is particularly applicable in arc welding and in driving such machines as the crowding motor of shovel sets. The voltage can be made to drop abruptly at a given load current, for example, when the arc is struck or when the crowding motor stalls.

Series generators are seldom found. Initially, they were employed in series street-lighting systems but have practically lost this field. Their principal use today is as series boosters on long feeder circuits. Here they simply take the place of an extremely strong cumulative series field of a compound generator.

13.16. D-c Motors. As has been pointed out, there is no fundamental difference in the mechanical construction of the d-c generator and motor, and in the preceding discussions they have been treated as the same machine. The analysis has been general and applies equally to both types. The major electrical difference between the two lies in the direction of flow of armature current. For the same polarity of the field poles and the same direction of rotation, the flow of armature current in the motor is opposite the flow for generator action. The mmf of armature reaction is therefore, in the opposite direction, and, as a result, the electrical neutral in the non-commutating-pole motor moves back against the direction of rotation. Commutating poles hold the electrical neutral always at the same position. It is not necessary, for the motor, to interchange leads to the commutating windings when passing from generator to motor action. The flow of current in the opposite direction automatically provides the required change in polarity of the commutating poles.

D-c motors are classified according to the manner of field excitation, that is, shunt, series, and compound, just as are d-c generators. Since both torque and speed are dependent upon the field flux, the several types of motor will have operating characteristics that differ and, therefore, determine the suitability of a particular type for a given duty. It is to be recognized that applications of d-c motors lie in the speed and torque requirements of the loads they are to drive.

As with generators, the nameplates of motors carry information on their ratings. The data include the horsepower (shaft output), kind of rating (continuous, intermittent, etc.), voltage, line current, speed, temperature rise, and manner of field excitation. Unless specifically stated otherwise, the several items in the complete rating are for full-load continuous operation. Motors are designed so that they will give satisfactory performance at rated horsepower within ±10 per cent of rated voltage but with characteristics not quite as good as at normal voltage.

From experimental evidence, the magnitude of the generated voltage (E_g) in the armature of a d-c machine is dependent upon the magnetic field strength (expressed in terms of Φ lines of flux per pole), and the speed of rotation (N) in rpm. Hence

$$E_g = K_n\Phi N \text{ volts} \tag{13.5}$$

The constant K_n takes into account the manner of construction of the winding, the number of poles on the machine, and conversion factors. It is fixed for a particular machine once that machine is built.

When the machine is running as a motor, the terminal voltage is greater than the generated voltage by the armature-circuit resistance drop. Thus,

$$\begin{aligned} E_t &= E_g + I_aR_a \text{ volts (motor)} \\ &= K_n\Phi N + I_aR_a \end{aligned} \tag{13.6}$$

whence the speed of the motor can be expressed as

$$N = \frac{E_t - I_aR_a}{K_n\Phi} \text{ rpm} \tag{13.7}$$

Judgment must be used, however, in interpreting this last expression. In calculating motor speeds, it is the best practice to obtain them from ratios of generated voltages, taking into account also any percentage change in flux caused by the armature current.

Because the IR drop of the armature circuit is so small in comparison with rated terminal voltage, the speed of the d-c motor is nearly inversely proportional to the flux. Practically, the flux is controlled by changing the field current through settings of the field rheostat for shunt and compound motors. In series motors, the speed changes as the load resistance is altered since the armature current also is the field current. All d-c motors are, of course, subject to speed control by variation of the voltage applied to the armature.

The speed regulation of all motors is defined as follows:

$$\frac{\text{no-load speed} - \text{full-load speed}}{\text{full-load speed}} \times 100 = \text{per cent speed regulation} \qquad (13.8)$$

A test for speed regulation is taken by adjusting the motor for full-load conditions and then dropping load to zero. This is not applicable for the series motor.

Both generator and motor develop torque in their armatures by virtue of the current-carrying conductors in the magnetic field of the machine. The torque of the generator opposes the external torque of the prime mover and is a countertorque. That of the motor, since it is the driving torque, opposes the load torque placed on the shaft. For the same field strength and the same value of armature current, the torque in a machine will be the same whether it is run as a generator or as a motor.

Again, from experimental evidence, the torque (T) developed in an armature winding is given by the relation

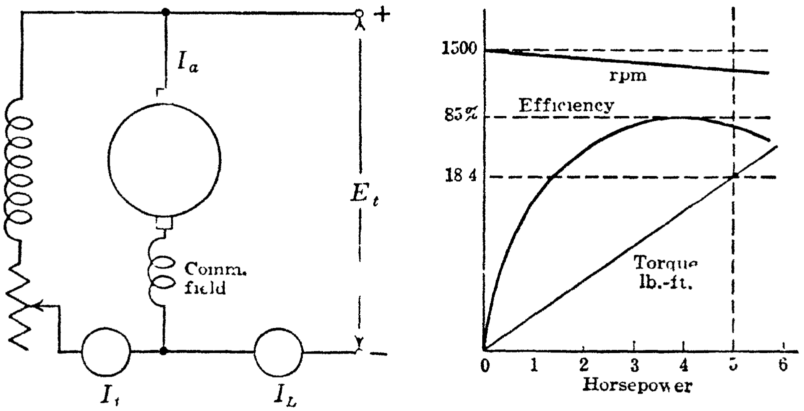
$$T = K_t \Phi I_a \text{ pound-feet} \qquad (13.9)$$

where K_t also is a constant of the particular machine. This developed torque includes components for windage, friction, and iron losses in the motor as well as that needed for the load.

13.17. The Shunt Motor. The circuit connections for the shunt motor, Figure 13.14a, are the same as those for the shunt generator. The field circuit is independent of the armature and draws a small current which in magnitude is in the order of 1 to 5 per cent of the line current. For a given field rheostat setting, the field current is constant. Except for effects of armature reaction, the total flux remains the same, and the shunt motor is called a constant-flux machine.

With constant flux, changes in speed are directly proportional to the armature current, the speed falling as the armature current increases. The change in speed from no load to full load is generally 5 to 12 per cent and the shunt motor is classed as a constant-speed motor. If armature reaction is pronounced, the motor may have a flat or a rising speed characteristic (addition of a weak series field, as in the stabilized shunt motor, prevents speed instability). Speed is controlled usually by means of the field rheostat but can be made to vary automatically by adding a fixed resistance to the armature circuit, if a fairly large speed regulation is desired. Speed regulation and reversible operation can be achieved by varying the applied voltage, the field circuit then being connected to a separate supply. Rotation is reversed by interchanging either the leads to the field circuit or the leads to the armature circuit, but not both sets of leads.

For constant flux, the developed torque of the shunt motor is directly proportional to the armature current. The torque-current curve for the average shunt motor is linear from zero to about full-load current, where it begins to fall off as the result of saturation caused by armature reaction.



(a) Schematic diagram (b) Performance curves

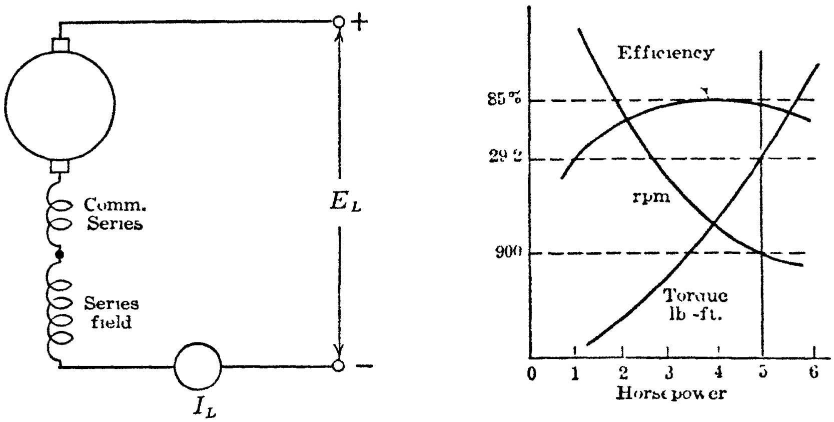
FIG. 13.14 Circuit diagram for shunt motor and typical performance characteristics for a 5-hp machine.

The efficiency of the shunt motor is good at high speeds but poor at low speeds for the same machine because of the decrease in output. If speed is controlled by series resistance in the armature circuit, the efficiency is still lower. Typical performance characteristics of a shunt motor are shown in Figure 13.14b.

The process whereby a shunt motor takes load is no different from that in any motor. As torque is applied at the shaft, the motor slows down, and the counter-emf drops with the speed. More current, therefore, can flow in the armature, and a greater torque is developed so as to equal that demanded by the load. A decrease in load leads to opposite effects, the motor running at a higher speed and developing less torque. At all times a motor so adjusts itself automatically in speed and generated voltage that the developed torque is just sufficient to equalize the total shaft torque of the load and certain losses in the rotating member. The synchronous motor, as we have seen, takes load in a similar way by slowing down momentarily to a new load angle; this results in a difference between the terminal voltage and that generated by the main field and a consequent change in armature current.

Similar to the above discussion on the shunt motor, the machine may be analyzed with respect to changes in speed corresponding to changes in flux. A decrease in flux, for example, leads to a drop in counter-emf and an increase in armature current. The percentage decrease in flux is less than the percentage increase in current, hence the torque actually increases. Torque in excess of that required by the load accelerates the armature to that value of speed at which the new counter-emf automatically regulates the current flow. This analysis is helpful in understanding the reason why a shunt motor will run at an excessive speed if the field circuit is opened, and why the field rheostat is set at minimum resistance at starting. In practice, the field circuits of shunt motors are not provided with field switches because of the danger of opening them inadvertently.

13.18. The Series Motor. The field coils of the series motor, Figure 13.15a, are in series with the armature, and, because they carry the line current, they have only a few turns of heavy wire. The air-gap flux is produced solely by the armature current and is directly proportional to it if saturation is absent. The series motor, therefore, is a machine with a highly variable flux.



(a) Schematic diagram (b) Performance curves

FIG. 13.15. Circuit diagram and performance characteristics for a 5-hp series motor.

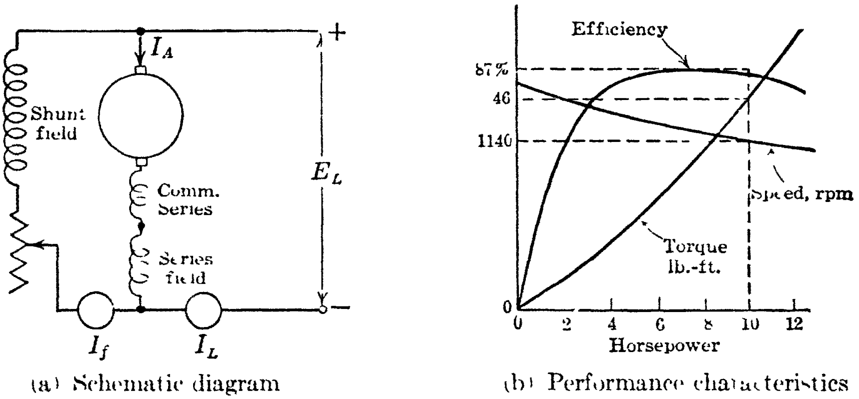
Since the flux is proportional to the armature current, the torque of a series motor is proportional to the square of the armature current. The starting torque of the motor hence can be very large for a relatively low starting current. An armature current of 150 per cent of normal value will produce 225 per cent of normal torque if saturation is absent, but because of saturation the torque usually is somewhat less, perhaps nearer 200 per cent.

The speed of the series motor is inversely proportional to the load, since the flux increases directly with the current. The speed regulation hence is very high with changes in load, and at light loads the motor runs at a high speed

For this reason, series motors in practice are always connected mechanically to their loads which act as brakes to limit the speed.

There are no constant shunt-field losses in the series motor, and its efficiency, therefore, is generally good, regardless of speed. Typical performance characteristics of a series motor are shown in Figure 13.15b.

13.19. The Compound Motor. The compound motor, Figure 13.16a, has both a shunt and a series field, and in the cumulatively compounded machine their mmf's are additive. This is the usual type, the differentially compounded motor being seldom found in practice. The characteristics of the compound motor partake of those of the shunt and series motors, and it may be considered a compromise between them. For 150 per cent of normal current at starting, the compound motor produces a torque about 180 per cent of normal torque. Because of the shunt field, the motor has a definite no-load speed which is not excessive as it is with the series motor.



(a) Schematic diagram (b) Performance characteristics

FIG. 13.16. Circuit diagram and performance curves of a 10-hp compound-wound motor.

Desired torque and speed characteristics of the compound motor are obtained through design by the proper choice of the relative number of ampere turns of the shunt and series coils. In practice, a diverter across the series field can be adjusted so that the motor will approach the shunt machine in its performance. In some instances, the series-field coils on alternate poles are short-circuited in order to alter the characteristics of the motor. A few compound machines use the series fields only for starting service, the motors running as shunt machines when on the line. The series fields are not generally very strong, and, while the compound motor can function as a shunt machine, it cannot be run as a series motor.

Speed control is effected by the shunt-field rheostat, by a variable resistance in the armature circuit, or by varying the voltage impressed on the armature. In this last instance, the shunt field is excited from a separate constant-voltage source. Reversal of rotation of the compound motor is usually accomplished by interchanging only the connections to the armature, thereby leaving the two fields in cumulative arrangement. Where regular reversing service is required.

the necessary features for that type of duty are built into the controller for the motor. This is true for any motor.

13.20. Losses: Efficiency. As with all machines, efficiency is the ratio of the useful power output to the total power input.

$$\text{per cent efficiency} = \frac{\text{output}}{\text{input}} \times 100$$

$$= \frac{\text{output}}{\text{output} + \text{losses}} \times 100$$

$$= \frac{\text{input} - \text{losses}}{\text{input}} \times 100 \qquad (13.10)$$

In general, efficiency is obtained by calculation of the separate losses, although in some instances an input-output test may be taken. When this is done, a dynamometer is used to measure the mechanical input or output, and calibrated electrical instruments to measure the electrical output or input. With small motors, a Prony brake may take the place of the dynamometer.

The types of losses in the d-c machine are copper, brush-contact, core, stray-load, and mechanical. Copper losses exist in the windings of the armature; in the shunt, series, and commutating fields; and in the compensating windings. In practice, the loss in the shunt-field rheostat is charged against the plant and not against the machine unless otherwise agreed upon. This is true also of exciter losses.

The brush-contact loss is calculated from an assumed total brush-contact drop of 2 volts for the machine It varies with the kind of brushes and with their connections. The total *IR* drop of the armature circuit includes the armature; the series, commutating, and compensating windings; and the brush-contact drop.

The core loss is a loss in the iron due to hysteresis and exists primarily in the armature core. This loss is obtained at no-load, at rated speed, and at rated generated voltage, and can be measured fairly accurately by test. Under load, flux distortion of armature reaction increases the core loss and also gives rise to eddy currents in both the iron and the copper of the machine. This added loss is called the stray-load loss and is taken arbitrarily as 1 per cent of the machine output. It is very difficult to calculate.

Mechanical losses are caused by windage and by bearing and brush friction. They are calculated only by empirical formulas but can be measured closely by test. The mechanical losses, together with the shunt-field and core losses, form the total no-load loss of the d-c machine, a loss corresponding closely to the so-called fixed loss of the induction motor. The no-load armature input to the d-c motor is often called the "stray-power loss," an unfortunate designation.

The power developed in the armature of the d-c machine is the product of the generated voltage and the armature current. In the generator, it includes the copper losses of the armature circuit, and of the field if self-excited, as well as the power taken by the load. All of these form the power which has been converted from mechanical to electrical. The core and mechanical losses may be considered as a separate part of the output of the driving machine. In the motor, the developed power includes the iron and mechanical losses, as well as the shaft output, and is that power which has been converted from electrical to mechanical. The copper losses may be considered as a separate part of the electrical power supplied the motor.

There are several ways of testing machines for over-all efficiency. Common methods are by Prony brake for small motors, and by a mechanically coupled dynamometer or calibrated machine for the motor or the generator. In the Prony-brake test, voltage and current input to the motor are measured, together with the speed and the torque at the brake arm. The first two items give the watts input, and the last two lead to the horsepower output from the horsepower equation.

The dynamometer is a machine with the stator also mounted on bearings and connected to a torque arm. Depending upon its function as a generator or a motor, speed and torque readings for it give horsepower input or output. In any case, it measures the mechanical power transferred through the shaft. The dynamometer and the machine to be tested may be connected for pump-back, or load-back, operation, that is, both tied electrically to the same lines. A-c and d-c machines can be tested conveniently in this way, only the energy for losses being unclaimed, although the method is not too accurate for efficiency measurements, and it requires machines of the same ratings. Over-all readings in kilowatts can thus be obtained, as well as the power transferred between the two machines. A dynamometer test, particularly if the dynamometer is small in comparison with the machine to be tested, will give good results on the separation of losses in the tested machine.

The calibrated machine simply takes the place of the dynamometer, but, being bolted to a base, readings on torque are not available. From previous tests on it, preferably by a dynamometer, the efficiency of the machine is known for various values of output or input. For example, a machine running as a calibrated generator under a motor test may have a calibration chart for it showing torque in the connecting shaft as a function of its armature current or electrical output. Any such curve would be taken at a given field current, held constant.

A running-light (no-load) test on a motor, at rated speed and with an impressed armature voltage approximating the counter-emf under full load, will give the shunt-field loss and the core and mechanical losses assigned to the armature. These are measured in the separate circuits. The armature copper loss at no load is very small and sometimes is neglected. The resistance of the armature circuit is obtained by a separate test. For an assumed input current to the motor, the losses then are available, and hence the output can be calculated. Such a form of calculation will give the performance characteristics of a motor from a knowledge of no-load information, effects of saturation being neglected.

13.21. Example. The nameplate rating of a shunt motor is 150 hp, 600 volts, 205 amp, 1,700 rpm. The resistance of the shunt field circuit is 240 ohms, and the total armature circuit resistance is 0.15 ohm. The motor has a commutating winding, and armature reaction is neglected.

(a) Calculate the full-load efficiency of the motor and the speed regulation.
(b) Calculate the efficiency and the delivered torque when the motor draws half-load current from the line.
(c) If the motor is to deliver full-load torque at 1,200 rpm, what value of resistance must be added to the armature circuit?

Solution:

(a) At full load

$$\text{motor input} = \frac{600 \times 205}{1{,}000} = 123.0 \text{ kw}$$

$$\text{motor output} = 150 \times 0.746 = 111.9 \text{ kw}$$

$$\text{motor losses} = 11.1 \text{ kw}$$

$$\text{efficiency} = \frac{111.9}{123.0} \times 100 = 91.0\%$$

The losses are

$$\text{shunt field} = \frac{600 \times 2.5}{1{,}000} = 1.50 \text{ kw}$$

$$\text{arm. copper} = \frac{(205 - 2.5)^2 \times 0.15}{1{,}000} = 6.15 \text{ kw}$$

$$\text{stray power} = \text{remainder} = 3.45 \text{ kw}$$

$$\text{total losses} = 11.10 \text{ kw}$$

The no-load current to the motor will be that required for the field and the stray-power losses, that is,

$$\text{no-load current} = 2.5 + \frac{3{,}450}{600} = 8.25 \text{ amp}$$

The generated voltages are

$$E_g \text{ full-load} = 600 - (205 - 2.5) \times 0.15 = 569.6 \text{ volts}$$

$$E_g \text{ no-load} = 600 - (8.25 - 2.5) \times 0.15 = 599.14 \text{ volts}$$

whence the no-load speed is

$$1{,}700 \times \frac{599.1}{569.6} = 1{,}790 \text{ rpm}$$

and

$$\text{speed regulation} = \frac{1{,}790 - 1{,}700}{1{,}700} \times 100 = 5.3\%$$

The delivered torque at full load is 463 lb-ft, obtained from the horsepower equation. The developed torque is 478 lb-ft and includes the stray-power loss.

(b) At half-load current, the motor draws 102.5 amp, of which 100 amp flow through the armature circuit. The field and stray-power losses are assumed to remain constant, but the new armature copper loss is 1.5 kw. The total losses are 6.45 kw, the motor input is 61.5 kw, the output is 55.05 kw, and the motor efficiency is 89.6%. The generated voltage for the new condition is 585 volts. the speed is 1,746 rpm, and the delivered torque is 222 lb-ft.

(c) With constant flux, full-load torque requires full-load armature current of 202.5 amp. Hence,

$$E_g \text{ at } 1{,}200 \text{ rpm} = \frac{1{,}200}{1{,}700} \times 569.6 = 402 \text{ volts}$$

and the resistance to be added in the armature circuit is

$$\frac{600 - 402}{202.5} - 0.15 = 0.83 \text{ ohm}$$

13.22. Starting D-c Motors. There is no back voltage generated in the armature of a motor at standstill, and without a protective resistance the armature current is limited only by the very small resistance of the armature circuit. On full voltage, the motor of the example above would draw about 20 times full-load current at starting. The motor might be permanently damaged because of the high current or the very high accelerating torque. Circuit breakers, of course, protect motors from such high currents.

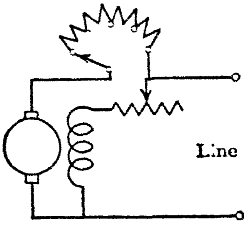


Fig. 13.17. Motor-starting resistance.

To limit the armature current to a reasonable value—perhaps 150 per cent of rated current—when starting from a full-voltage supply, a resistor with taps is placed in the armature circuit, Figure 13.17. As the motor increases in speed and develops a back voltage, the series resistance is gradually reduced to zero. The shunt-field circuit always is connected to the line side of the resistance so that full voltage is applied to the field at starting. Further, the field rheostat is set at minimum resistance so that the no-load speed will be a minimum when the motor is on the line.

All d-c motors are started with a starting box, a controller, a water rheostat, or some similar device to limit the current when full voltage is impressed on the armature circuit. Large motors may be started from a low-voltage source, the voltage being increased gradually as the motor increases in speed. Both large and small motors may be started by automatic contactors or push-button control. At predetermined values of counter-emf, the contactors short-circuit parts of the series resistance until it is all cut out of the circuit.

Figure 13.18a shows the connections of a commercial type of starting box, known as a 4-point box, with a no-voltage release. The lever is gradually moved over to its final position as the motor picks up in speed. In the running position all the resistance has been cut out, and the lever is held by the electromagnet *M*. If the magnet is de-energized, a spring pulls the lever back to the starting position. When, for any reason, the electric power is interrupted, the lever returns to its starting position and the motor must be restarted when voltage reappears on the line. The box has the disadvantage that the field circuit can open without the holding coil de-energizing and taking the motor from the line.

The connections for a 3-point box are shown in Figure 13.18b. The field current flows through the holding coil of the box, hence the motor will stop automatically if the field circuit opens. A disadvantage is that a very weak field current may permit the same thing. The 4-point box is more satisfactory when the field current must be varied over a wide range.

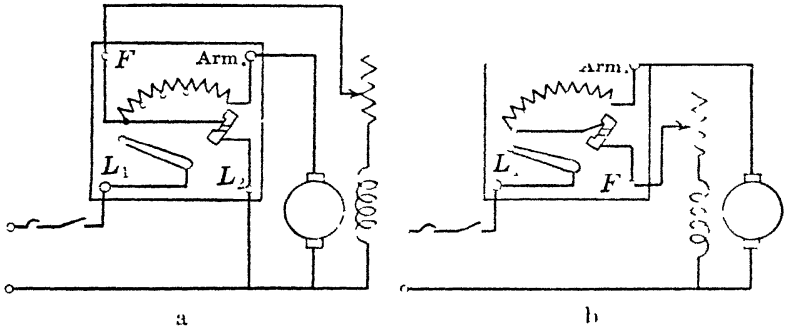


FIG. 13.18. Circuit connections for (a) four-point and (b) three-point starting boxes.

13.23. Speed Control of D-c Motors. Shunt and cumulatively compounded motors have definite no-load speeds, with positive speed regulation as the motors are loaded. Series motors drop rapidly in speed with an increase in load. Differentially compounded motors also have a definite no-load speed with, however, a flat or rising speed characteristic, generally reaching instability a little above full load. Shunt motors may show the same effect if armature reaction is strong in decreasing the flux, and occasionally shunt motors are so designed on this principle that they have flat speed characteristics. Stabilizing windings on shunt motors are for the purpose of preventing a rise in speed or of obtaining a slight drop in speed with increase in load. On the other hand, the series-field coils on alternate poles of compound motors may be short-circuited so as to obtain a lower speed regulation than normal.

Where reasonable changes in speed are desired, shunt and compound motors usually are controlled by field rheostat. This method is the most satisfactory and efficient manner. A few shunt motors, called adjustable-speed motors, are set in speed by field control and may have no-load speeds in the ratio of five or six to one. The speed regulation, from no load to full load, is about the same regardless of the particular no-load speed. Series motors can be controlled by varying a resistance in parallel with the series field or by short-circuiting a few of the turns. These schemes, though, are not common in industrial applications.

Large changes in speed may be accomplished for all motors by inserting a resistance in the armature circuit or by varying the voltage applied to the armature. In the first method, speed variation is automatic with load, but it has disadvantages. The efficiency of the motor is much lower because of the copper losses in the external resistance. Again, the speed fluctuates too much with changes in load and may be highly undesirable. Speed regulation by armature resistance usually is under the direct control of an operator, particularly in the case of series motors.

In all d-c motors, the speed is directly proportional to the generated voltage and inversely proportional to the flux. The torque required determines the armature current which must flow and, with it, the corresponding flux it produces if the motor has a series field. This information will permit calculation, at least approximately, of the external resistance to be added to the armature circuit when a given torque is to be obtained at a given speed.

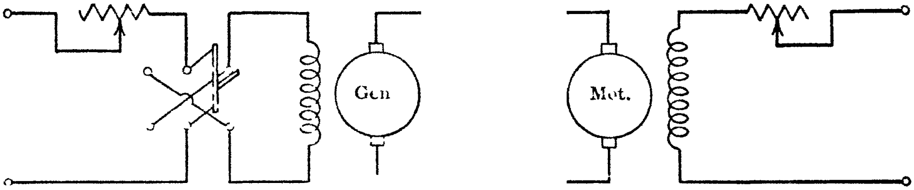


FIG. 13.19 Ward Leonard system of speed control.

Armature voltage control of speed usually is accomplished by means of the Ward Leonard system, Figure 13.19. A constant-speed generator, with variable field control, supplies the power taken by the motor armature. The fields of both machines are separately excited from a constant-voltage source. The generator voltage can be changed uniformly from full value in one direction to full value in the other, thus carrying the motor through the corresponding speed range. The control is smooth, and the efficiency of the method is much better than that in which armature resistance is used. The disadvantage is the cost of a separate generator for each motor so controlled, the losses in the generator, and the added floor space required.

Ward Leonard control is particularly applicable to reversing drives in steel mills. It also is employed in conjunction with electric drives for mine hoists, ship propulsion, locomotives, shovel sets, passenger and freight elevators, and large machine tools. Its application in industry is very great. Many large motors are started in this manner, especially in those cases demanding frequent starting. There are also a few equivalent methods of armature voltage control where the motors are not reversible in speed, but where large changes in speed are desired. Field control also can be used, of course, with any of these systems.

A few small shunt and compound motors are regulated to hold constant speed. A centrifugal device on the shaft of the machine opens and closes contacts across a resistor in the shunt-field circuit. The manner of operation is apparent.

13.24. Electric Braking. To stop a motor and its connected load quickly or to control its retardation, dynamic braking may be employed. The motor becomes a generator, converting the stored energy of rotation into electrical energy and thence to heat in a resistance. The main field of shunt and compound motors is left connected to the lines and usually kept at full strength. The armature is disconnected from the lines and across it is placed an external resistance that can be varied and that absorbs the energy. Series motors usually are reconnected through the controller so as to run as shunt machines. The lower the value of the external resistance, the greater the braking effect. Often this resistance is that in the starting rheostat or control circuit. Dynamic braking is applicable to elevators, hoists, cranes, derricks, winches, coal and ore unloaders, newspaper presses, steel-mill drives, etc.—in general, where loads are overhauling.

If the motor is connected so that it returns energy to the power system rather than dissipates it as heat in a resistance, the process is called regenerative braking. It is applicable to all motors, but series machines require a modification of the field circuit. Regenerative braking is used in some mine-hoist installations and in several large electrified-railway systems in the United States. Both dynamic and regenerative braking apply to a-c motors as well as to d-c.

A motor may be stopped very quickly by a scheme called plugging, commonly used with series motors. When the stop button is pushed, the controller automatically reverses the line voltage applied to the armature and inserts a current-limiting resistor in the armature circuit. The method is used with many reversing motors in order to bring them to full speed rapidly in the opposite direction. Plugging also is applicable to a-c drives.

13.25. D-c Motor Control. Control systems place a motor on the line, protect it during operation, and remove it from the line. In many instances, reversible operation also is provided. Control units are almost always magnetic and may be full- or semi-automatic. Manual operation of motors is infrequent.

All control systems embody the same general ideas and circuits, but the variations are extremely great in number. This is true even for a given manufacturer. Every type of motor and every form of application, in general, has a control circuit different from the next one. Such circuits never have been standard, and only general remarks can be made about them. The action of any particular system should be discussed from the actual device or from the connection diagram supplied by the manufacturer.

13.26. Applications of D-c Motors. Although a-c machines, particularly induction motors, are far more common in industrial applications than d-c machines, nevertheless the d-c motor is found in a great many fields. There are some services for which the d-c motor is especially well suited. The field of traction is an outstanding example. The d-c motor has been selected for many years for extremely hard service in steel mills. In making paper, the high-speed and concurrent operation of machines over wide speed ranges, together with close speed regulation, has greatly favored use of the d-c drive. Textile mills are another example.

The maximum torque, either starting or running, of the d-c machine is limited in general only by commutation. It may far exceed the torque of a-c motors of the same size, and starting currents are much less. Further, the efficient speed adjustment of the d-c machine cannot be realized with a-c motors, nor is there the same flexibility. The d-c motor is highly flexible, produces an excellent starting torque, gives constant-speed performance, and has a good efficiency.

The d-c machine has admitted disadvantages, primarily because of the commutator. Its first cost and maintenance are higher than that of the squirrel-cage motor. Dust, fumes, moisture, etc., must be kept from the commutator, thus necessitating at times enclosed motors with lower ratings for their sizes than would otherwise be required. Induction motors can often withstand such surrounding conditions with no or little enclosure.

On the other hand, the advantages of the torque and speed characteristics of the d-c motor often outweigh the disadvantages. Although the motor necessitates, of course, a d-c source of energy, there are few instances in industry in which this problem is of any concern. Motor-generator sets, synchronous converters, or mercury-arc rectifiers readily can convert a-c power delivered to d-c power and may not, in general, present a serious economic problem.

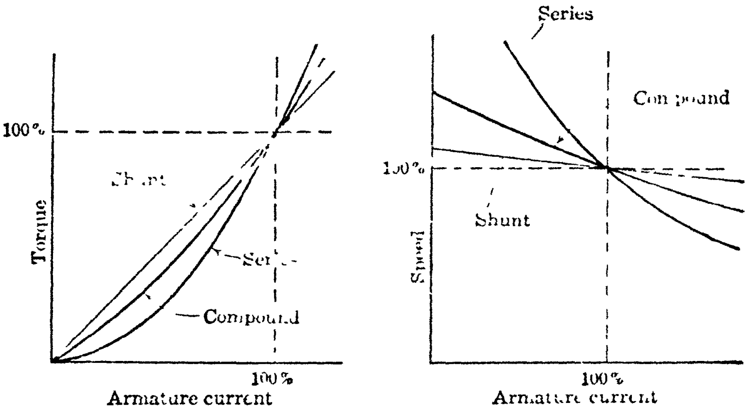
There is great overlapping in the application of the three types of d-c motor. Especially is this true with the shunt and compound machines, and for some services of the series motor. Again, overlapping occurs with shunt, compound, and induction motors. It simply must be realized that, in the end, special conditions affecting the particular job will determine that type of drive which best fits. These conditions may not always be of a purely technical nature.

Selection of a d-c motor for a given duty will depend upon the load requirements to be met, namely, torque and speed. For comparison of the characteristics of the types of d-c motor, general torque-current and speed-current curves are shown in Figures 13.20a and 13.20b, respectively. The reference point for the curves is rated current and either rated torque or rated speed. The curves thus become representative of all sizes of d-c motors.

The shunt motor is classed as a constant-speed machine, and it is the speed characteristic that almost always determines its use. It has a close speed regulation over a wide range of load and can be controlled readily in speed from zero to 25 per cent overspeed. The adjustable-speed shunt motor can have no-load speeds which differ greatly from each other. The shunt motor is well adapted to reversing drives. It is classified for medium starting duty and is used where such conditions are not severe.

A list of applications of shunt motors is long. A few of the services, with no emphasis on the order, are with blowers, fans, compressors, pumps, conveyors, elevators, skip hoists, lineshaft drives, paper-mill drives, textile-machine drives, rubber mills, steel mill main rolls and auxiliaries, material handling, power shovels, machine and woodworking tools, printing presses, motor-generator sets, and ship propulsion.

The compound motor is suitable for services in which frequent or heavy starting duty is required and in which the speed under load need not be as constant as with the shunt motor. The starting torque may reach 450 per cent of rated torque, depending upon the strength of the series field. In some instances, the series field is used only for starting. Speed regulation varies between 10 and 30 per cent, again dependent upon the series field. Speed is controlled by the same means used for the shunt machine. The speed characteristic of the compound motor makes the machine particularly applicable to drives having high torque peaks. With a flywheel or considerable flywheel effect in the set, the motor can take sudden loads with little line disturbance.



(a) Torque curves (b) Speed curves

FIG. 13.20. Torque and speed for shunt, series, and compound motors.

The compound motor will satisfy almost all applications of the shunt motor. It is used for steel mill drives, and for shovels, cranes, skip hoists, elevators, pumps, fans, compressors, punches, shears, bending rolls, machine tools, printing presses, crushers, etc.

The differentially compounded motor appears infrequently and only in small sizes. It is limited to special services in which a flat speed characteristic is desired.

The series motor is designed for very heavy and frequent starting duty, for loads having high inertia, and for speed which increases rapidly as the load decreases. The motor develops a powerful starting torque which is limited by commutation, heating, line capacity, and permissible bearing loads. Its maximum starting torque is about 500 per cent of full-load torque. The starting current is relatively low, but strong effects of saturation on overcurrent will prevent the torque from being proportional to the square of the current. Because of its strong torque, the series motor accelerates its load better than other types of motor.

The speed of the series motor is very low at heavy loads, a highly desirable feature of the machine. Speed variation is automatic with the load, and at light loads the speed is high. The series motor always must be connected to its load so that it cannot race. Speed control usually is effected by armature resistance if the duty is intermittent. Otherwise, speed is varied by armature voltage control. In railway service the speed may be controlled by placing motors in series, parallel, or series-parallel, depending upon the number of motors.

The most pronounced application of the series motor lies in traction service for main line, interurban, street, and mine railways, and in trolley busses. It is used also in all types of cranes and hoists, and in such services as with elevators, conveyors, bridges, gates, car dumpers and retarders, blowers, and fans.

13.27. Synchronous Converters. The synchronous converter is a rotating machine which converts a-c power, at frequencies usually between 25 and 60 cycles, to d-c power. As an inverted rotary converter, it also can convert d-c power to a-c power, but its use for such service is infrequent. Except for a few details, the converter structurally is a d-c generator with taps on the armature brought out through slip-rings at the end opposite the commutator. Figure 13.21 shows the schematic diagram for a 2-pole 3-phase machine. The number of slip-rings depends upon the number of phases, and the number of taps depends upon both the number of phases and the number of poles. The machine is essentially a revolving-armature synchronous motor and a d-c generator combined into one and hence is considerably more efficient than a motor-generator set. It has the general performance of the synchronous motor and the external characteristics of the shunt or compound generator, depending upon the design of the field circuits.

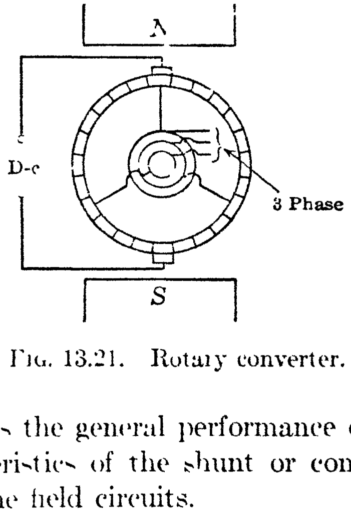


FIG. 13.21. Rotary converter.

The currents in the armature of the synchronous converter may be thought to consist of two components. One is the sine-wave synchronous-motor input current, and the other is the block-wave d-c generator output current. Since the first is in opposition to the generated voltage and the second is in the same direction, the two components are plotted graphically on opposite sides of the axis. For unity power factor, the two curves cross the axis at the same points, and the resultant current has an odd wave shape with pronounced peaks adjacent to the points at which the component curves cross each other. Converters are designed mainly for 1.0 pf, occasionally for 0.95 pf, and seldom for a power factor less than 0.9. Heating, which limits the rating of the machine, increases very rapidly as the power factor moves off unity. This can be shown readily by sketching component current waves for a power factor other than unity. Heating is quite different from that in either the synchronous or the d-c machine. It is sufficiently less so that the 6-phase converter, for example, has a rating about double the rating it would have if run only as a d-c generator.

Although the armature reaction effects of the component currents do not neutralize each other point by point along the armature surface, the over-all armature reaction is practically balanced out. The commutating field can be weaker and is about 37 per cent as strong as it is in the corresponding d-c generator. No mechanical energy transfer takes place in the machine because the torque is consumed in the same armature conductors in which it is developed. Consequently, the bearings are built light, mainly to support the weight of the machine. They are not strong enough to carry the reactions of shaft torque should it be desired to operate the converter as a synchronous motor. The torque angle of the converter never is greater than that which represents its mechanical losses.

Because the same winding on which the a-c voltage is impressed also produces the d-c terminal voltage, there is a fixed ratio between the two voltages. The brushes, in normal position, always pick off the maximum a-c voltage in the coils, hence the d-c voltage is $\sqrt{2}$ times the effective value of the diametrical, or single-phase, a-c voltage. The following table gives the theoretical ratios of the a-c voltage between slip-rings to the d-c voltage.

Phases	*No. of Rings*	E_{ac}/E_{dc}
1	2	0.707
3	3	0.612
quarter	4	0.500
6	6	0.354
12	12	0.183

Because of the fixed voltage ratio, it almost always is necessary to furnish *transformers with converters*, the turn ratios of the transformers providing for the proper slip-ring voltage. Balancing cost, rating, and efficiency, the 6-phase converter is the most common. Power for it is taken from 3-phase lines, the diametrical connection of the transformer secondaries being practically the only type of connection ever used. The diametrical voltage conforms to that of the single-phase converter. The line currents on the a-c side are calculated from a knowledge of the output, the efficiency, and the power factor.

Voltage drop in the supply system and within the converter decreases the output voltage, and several ways are used in practice for compensating the voltage drop. The simplest is the use of induction regulators or tap-changing transformers. A second method employs an auxiliary machine, called a booster, on the same shaft and placed between the supply and the converter armature. Fundamentally, the booster is an a-c generator in series with the line and can be controlled for a 15 per cent voltage boost or buck. Again, field control, either manual or automatic, can be used for voltage regulation. An increase in excitation will not change the generated voltage directly, but it makes the power factor less lagging or more leading, as the case may be. By way of inductive reactance in the lines, in the transformers (sometimes purposely provided), or in reactance coils, a power factor more leading raises the slip-ring voltage. The excitation of the compound-wound converter is usually set to give unity power factor at 75 per cent of full load. Compounding never is greater than that for a flat voltage characteristic.

Synchronous converters may be started and synchronized like synchronous motors, on squirrel-cage windings in the pole faces. They also may be started as d-c motors from the d-c side if power is available, or they may be brought up to speed by a separate motor, but this last method is practically obsolete. The machines are sensitive to sudden changes in load, particularly large loads quickly dropped, and hunting may lead to a serious flashover between brush arms. Most converters are equipped with flash barriers to assist in quenching any initial spark between commutator bars. The inverted rotary converter is not too stable. A lagging power factor weakens the field, the machine increases in speed (it now runs as a d-c motor), the frequency increases, the inductive reactance increases, the power factor becomes still worse, and the action is cumulative. Such operation requires a speed-limit device to take the converter from the line at about 25 per cent overspeed.

Synchronous converters are used primarily as a d-c supply for electric railways, in industrial and chemical plants, in mining service, and for a few large lighting loads. Power for electric railways is almost always produced as alternating current and transmitted in that form to substations where it is stepped down through the converter transformers. Standard railway voltages are 600, 1,200, 1,500, and 3,000 volts. Because of limiting voltages between commutator bars (about 15 volts average) and limiting peripheral speeds of the commutators, 60-cycle converters are designed for a maximum of 750 volts d-c, and 25-cycle converters for a maximum of 1,500 volts d-c. The armatures of two 600-volt, 60-cycle converters are connected in series for 1,200 volts, and, similarly, two 25-cycle machines are required for 3,000 volts d-c. The maximum d-c voltage is thus dependent somewhat upon the frequency of the a-c supply.

In chemical plants converter voltages are lower, with consequently higher currents. The standard low voltage for converters is about 250 volts. Direct current is used in chemical plants for electrolytic processes, and in industrial plants as a general supply for motors and lighting. In mine service, d-c power may be required for power shovels and for cutting, loading, conveying, hoisting, and pumping. Converters for general power and lighting loads very often supply 3-wire systems. The neutral for the third wire is obtained at the transformers, for example, the zigzag connection previously discussed. Synchronous converters are designed for standard voltages and frequencies and in standard sizes from 100 to 4,000 kw. They are especially suitable as a d-c supply where small adjustments of voltage are not required, although with a synchronous booster or other auxiliary apparatus they can give close voltage regulation.

Within the last 15 years, the large steel-tank mercury-arc rectifier has greatly displaced the synchronous converter. Rectifiers may reach ratings of a few thousand kilowatts, and voltages range from 250 to 3,000 volts. They are built with at least 6 or 12 electrodes so as to give a smoother d-c output voltage, the phase transformation from a 3-phase power source being accomplished by transformer connections. Because of a constant arc drop of about 20 volts, the efficiency is better the higher the d-c output voltage and the greater the kilowatt rating. Efficiencies, including auxiliary apparatus, are in the region of 94 to 96 per cent, are fairly constant over the entire load range, and are better than those for the converter, particularly for loads less than 75 per cent of rated capacity. The power factor over the greater part of the load range is close to unity.

PROBLEMS

1. A 12-pole, shunt-wound generator has a rating of 3,000 kw, 600 volts, 250 rpm. The armature is lap-wound with single-turn coils and 4 coil sides per slot in each of 252 slots. Each conductor is composed of 3 parallel strands of strap copper 0.05 by 0.56 in. MLT = 92 in.

(a) Calculate the voltage between commutator bars, the weight of copper in the armature winding, the winding resistance at 60°C, the power loss for rated load current, and the armature ampere turns per pole.

(b) Each brush arm holds 15 brushes of 1 by 1.25 in. cross section. What is the current density in the brushes?

(c) What is the time of commutation if the commutator diameter is 59.5 in. and the span of a brush along the commutator is 1.035 in.? Mica thickness between bars is 1/32 in.

(d) Each field spool holds 600 turns of No. 6 wire. The mean length of turn is 65 in. Calculate the weight of copper in the field coils and the maximum ampere turns per pole at 75°C for 600 volts excitation. The coils are in series.

(e) The commutating windings are arranged 6 in series with 2 parallel paths. The resultant resistance is 0.000236 ohm at 25°C. At 60°C, what diverter resistance is required to shunt 20% of normal current from these coils? What is the total power loss in the commutating-field circuit?

(f) Allowing 1 volt brush drop at each arm, what is the generated voltage in the machine for rated terminal conditions? Calculate the required flux per pole.

2. A 6-pole, 1,200-rpm, 250-volt, lap-wound generator has a flux of 2,370,000 lines per pole. If each coil has 4 turns, what is the voltage between commutator bars for 250 volts between brushes? If the speed is reduced 20%, what percentage change in flux is required for the same no-load voltage?

3. A 100-kw, 250-volt, separately excited generator has an armature resistance of 0.05 ohm and an armature reaction effect of 10 volts at full load. If the voltage drop due to armature reaction is directly proportional to the load current, what is the generated voltage at no load? Full-load voltage is to be 250 volts.

4. A shunt generator gives the following data for its saturation curve taken at rated speed:

E_g volts	6	35	100	160	200	230	250
I_f amps...	0	0.2	0.56	0.92	1.23	1.60	2.00

(a) What would you assume as the voltage rating of the generator? Use this value in answering the following questions.

(b) To what voltage will the generator build up if the total field-circuit resistance is 120 ohms?
(c) What is the total field-circuit resistance for rated no-load voltage?
(d) What is the critical value of field resistance?
(e) If the field resistance is 220 ohms, what will be the no-load voltage?
(f) What will be the no-load voltage for 120 ohms field resistance and 80% speed?

5. A 50-kw, 250-volt, 1,200 rpm, separately excited generator has an armature resistance of 0.15 ohm and an armature reaction effect of 20 volts at rated load current and 1,200 rpm. It is coupled to an induction motor which drives the set at a no-load speed of 1,196 rpm and shows a speed regulation of 12% at full load on the generator. For 250 volts at full load and field excitation held constant, what is the voltage regulation of the generator alone and the over-all regulation including the speed regulation of the prime mover? Assume all changes in speed and voltage to be linear.

6. A 50-kw, 125-volt, 1,200-rpm, compound-wound generator has a resistance between brushes of 0.02 ohm, a commutating-winding resistance of 0.0015 ohm, a series-field resistance of 0.003 ohm, and a shunt-field resistance of 20 ohms. If the shunt field is connected directly across the armature terminals, what is the generated voltage of the machine when it is delivering full load at rated voltage? Draw the power flow diagram.

7. The machine of Figure 13.6 is a 5-kw, 125-volt, 1,200-rpm generator with $E_{g0} = 135$ volts. Sketch the saturation curves for 600, 900, and 1,500 rpm. If the field rheostat is left in position for rated conditions, sketch the general shapes of the external characteristics for speeds of 900 and 1,500 rpm. Repeat this last question but with $E_{g0} = 135$ volts for all cases.

8. The machine of Figure 13.9 is a 5-kw, 125-volt, 1,200-rpm, compound-wound generator. Starting from rated voltage at no load, sketch the general shapes of the external characteristics for over-compounding and at speeds of 900, 1,200, and 1,500 rpm. How will these characteristics be affected for speeds of 900 and 1,500 rpm if the starting voltages are 100 and 150 volts, respectively?

9. What would you do to give a generator a magnetizing component of armature reaction? Would your procedure be too satisfactory practically? In what way would the fact that the machine did or did not have commutating poles limit you? A laboratory party finds that a shunt generator actually has a rising characteristic, whereas a second party finds that the same machine has a rapidly falling characteristic. Explain the reasons for both observations.

10. If the machine of Figure 13.9 is cumulatively connected, sketch the reconnections for opposite rotation. The generator is to remain cumulative and the load polarity is not to change. Practically, how would you make certain that a compound-wound generator is cumulatively connected?

11. The neutral lead from a 500-kw, 125/250-volt, 3-wire, compound-wound generator is designed for 25% current. What continuous load can be carried on one half of this system if the other half carries no load? What continuous load can be carried if both sides are loaded, but one side supplies 200 kw?

12. The balance coil on a 60-kw, 120/240-volt, 1,500-rpm, stabilized-shunt (this means a few turns of series-field winding but over-all shunt-generator characteristics), Diesel-driven generator is rated 75 cycles, 6 kva, 85/170 volts. How many poles are on the generator? What is rated current for the neutral lead? For each half of the balance coil? Express your results as percentages.

13. The regulation test on a 3-wire generator requires normal amperes in the heavier-loaded main, rated voltage between outer lines, and the specified percentage current in the neutral. The voltage difference between the two sides of the system shall be no greater than a given percentage of rated voltage between lines.

Give the numerical values of all pertinent currents and voltages for a 300-kw, 125/250-

volt, 3-wire generator limited to 10% rated current in the neutral and a voltage difference between the two sides of the system no greater than 2%.

Repeat for 25% neutral current and no more than 3% voltage difference.

14. The load on a d-c generator, supplying power to a bus in parallel with other generators, is dropped and the machine is allowed to motor on the line. Is there any tendency for the machine to reverse its direction of rotation? What effects are observed on the generator voltmeter and ammeter?

15. What is the effect of increasing the field current of a single generator supplying a load? Of increasing the field current of one of two generators operating in parallel? Practically, how would you shift load from one of several generators operating in parallel and hold the line voltage constant?

16. Two 300-kw, 250-volt, shunt generators, each having 0.004 ohm armature-circuit resistance, deliver 2,000 amp total to the main bus when operating in parallel. The external characteristics of the machines are identical. How does the total load divide between them if they have the same magnitudes of field current? By what percentage must the flux of each be changed to place 1,500 amp on one generator and 500 amp on the other? The load voltage is to remain at 250 volts.

What must be done to hold a 1:1 ratio of armature currents for the generators, yet drop the line voltage to 240 volts?

What must be done to hold a 1:1 ratio of armature currents for the generators, yet drop the line voltage to 240 volts with the same load power as before?

17. Two cumulatively compounded generators are connected for parallel operation. The machines are built from the same specifications but the saturation curve for one is slightly higher than that for the other. What would be the most feasible manner of adjusting the generators so that they would divide load equally?

18. Two stabilized-shunt generators are connected for parallel operation, but they do not divide the load proportionately to their ratings. The air gap of one of them is increased so as to obtain better results. What is the effect of this change, and on which machine should it be made?

19. Successful parallel operation is attained if the load on any generator does not differ more than ±15% of its rated kw load from its proportionate share, based on the generator ratings, of the combined load, for any change in the combined load between 20% and 100% of the sum of the rated loads of all the generators (A.I.E.E. Bull. No. 45, on shipboard installations).

Give the limiting kw loads on each of two 250-kw generators operating in parallel.

Repeat for a 100-kw generator in parallel with a 250-kw machine.

Repeat for three 250-kw generators in parallel.

20. A cargo vessel has two main generators, each rated 300 kw, 240/120 volts, wired to the switchboard for parallel operation. The circuit breakers in the main lines are set for 25% overload. The neutral bus is grounded through a resistance which is to limit the neutral current to 1% of the total current of the power supply for a ground on either the positive or negative main bus. Across this resistor is a circuit breaker which is to trip at 3% of the total current of the power supply. For 25% overload, what is the current setting of the breaker, and what is the resistance of the ground resistor?

21. A 5-hp, 110-volt, 42-amp, 1,150-rpm shunt motor has an armature-circuit resistance of 0.15 ohm and a field-circuit resistance of 55 ohms. What resistance is required in a starting box to limit the armature current to 150% of rated value at starting?

22. A 115-volt shunt motor has a no-load speed of 1,200 rpm. What is the armature current at full load if the speed is 1,140 rpm? The armature-circuit resistance is 0.24 ohm. Neglect armature reaction.

23. A 230-volt shunt motor takes 5 amp line current at no load and runs at 1,400 rpm The field-circuit resistance is 115 ohms, and the armature-circuit resistance is 0.2 ohm

Calculate the speed, delivered torque, and efficiency of the motor for a line current of 45 amp.

24. A shunt motor has a rating of 15 hp, 230 volts, 57 amp, 1,350 rpm. The field-circuit resistance is 105 ohms, and the armature-circuit resistance is 0.21 ohm. Calculate the no-load line current and speed. Calculate the speed and the efficiency when the motor takes 40 amp from the line.

25. A 10-hp, 220-volt, 39.5-amp, 1,500-rpm shunt motor has a field-circuit resistance of 160 ohms, and an armature-circuit resistance of 0.15 ohm. What is the no-load speed if armature reaction decreases the no-load flux to 97% at full load? Calculate the no-load current taken by the motor.

26. A 15-hp, 220-volt, 59.5 amp, 1,700-rpm shunt motor has a field-circuit resistance of 160 ohms and an armature-circuit resistance of 0.16 ohm. The no-load speed is 1,735 rpm. What percentage of the no-load flux is the full-load flux?

27. A 5-hp, 115-volt, 38.7-amp, 1,150-rpm shunt motor has a field-circuit resistance of 85 ohms and an armature-circuit resistance of 8%. Calculate the no-load motor current. If the no-load speed is 1,140 rpm, by what percentage has the flux decreased from no load to full load?

28. What approximate value of field current is required if the motor of problem 26 is to be run as a generator at 1,800 rpm, delivering 60 amp at 230 volts?

29. A shunt motor has a rating of 7.5 hp, 115 volts, 58 amp, 1,700 rpm. The field-circuit resistance is 80 ohms, and the armature-circuit resistance is 0.18 ohm. What resistance should be added to the armature circuit to give a speed of 1,200 rpm when the motor draws 40 amp line current? What is the efficiency for this condition of load? What is the developed torque?

30. A 10-hp, 230-volt, 1,200-rpm compound motor has a shunt-field loss of 420 watts and a stray power loss of 510 watts. The armature-circuit resistance is 0.24 ohm. What are the full-load armature current and efficiency?

31. A 40-hp, 230-volt, 143-amp, 1,100-rpm compound-wound motor has a shunt-field-circuit resistance of 80 ohms and an armature-circuit resistance of 0.08 ohm including 0.011 ohm of the series field. The series field produces 15% of the total flux at full load. What is the approximate no-load speed of the motor? What would be the speed at full load with rated armature current if the series field were short-circuited? What would be the developed torque and efficiency under this condition?

32. How would you ascertain, practically, normal percentage field current for a shunt or compound motor?

33. A shunt motor draws an armature current of 150 amp from a 115-volt line and develops 90 lb-ft torque at starting. For the same torque, what must be the change in armature-circuit resistance if the field resistance is decreased 20%? Assume the saturation curve to be a straight line.

34. A 25-hp, 230-volt shunt motor develops a starting torque of 160 lb-ft with 90 amp armature current and normal field excitation. What torque would be developed with 150 amp in the armature if saturation due to armature reaction at the higher current decreases the flux 5%? The field current is constant. What change is required in the flux if the motor is to develop 200 lb-ft torque with 150 amp armature current? If the armature resistance between brushes is 0.18 ohm, what starting resistance is required to limit the current to 150 amp?

35. A compound motor develops 100 lb-ft torque when starting with rated armature current. The series field produces 25% of the total flux. What approximate starting torque would be developed with 150% of rated armature current? What torque would be developed by the series field alone with 200% of normal current? Assume the saturation curve is a straight line.

36. A vertical pump motor is rated 100 hp, 230 volts, 360 amps, 475 635 rpm, light compound (this means that the series field is not too strong), 3 amp maximum field current. The field-rheostat resistance is 75 ohms. The machine is an adjustable-varying-speed motor.

(a) What is the full-load efficiency, and what are the component losses that you can calculate?
(b) What is the delivered torque at the two speeds, assuming 100 hp for both?
(c) What total armature-circuit resistance will limit the starting current to 150% of rated current?
(d) How is this motor to be adjusted for speed? If the armature-circuit resistance is 0.036 ohm, what are the generated voltages at no load and at full load for a full-load speed of 635 rpm? What is the speed regulation if the no-load value is 710 rpm? By what percentage has the series field changed the flux?
(e) With the full-load conditions of part (d), the shunt field is adjusted to give 475 rpm for 360 amp. What per cent increase in flux has been provided by the shunt field?

37. A 5-hp, 115-volt adjustable-speed motor has a speed range from 400 rpm to 2,000 rpm. It draws 39.5 amp line current at the lowest speed, and 46.1 amp at 2,000 rpm. The field resistance is 35 ohms at the lowest speed, and 205 ohms at the highest speed. Armature-circuit resistance is 0.21 ohm. Calculate the no-load line currents for the 2 speeds of 400 and 2,000 rpm. Calculate the speed regulation in the 2 cases and also the efficiency.

38. A 220-volt series motor delivers 16 hp at 1,200 rpm when taking 60 amp from the line. What are the approximate speed and horsepower output when the motor draws 45 amp? Assume the saturation curve to be a straight line.

39. A 220-volt shunt motor hoists a 600-lb load 500 ft per min when drawing a line current of 45 amp. The field-circuit resistance is 100 ohms, and the armature-circuit resistance is 0.15 ohm. If 0.6 ohm is added to the armature circuit, what weight can be raised at a speed of 300 ft per min? What resistance should be added to the armature circuit if a 400-lb load is to be raised at the speed of 250 ft per min?

40. A 220-volt shunt motor under load draws 52 amp armature current and runs at 1,000 rpm. The armature-circuit resistance is 0.14 ohm. What series armature resistance is required if the motor is to develop the same torque at 750 rpm? What series resistance is required if the motor is to develop the same horsepower at 750 rpm as at 1,000 rpm? What is the armature efficiency in the 3 cases?

41. A 230-volt series motor draws 45 amp while hoisting a 1,000-lb load 300 ft per min. What is the approximate value of resistance which must be added to the armature circuit if 600 lb are to be raised 200 ft per min? The armature-circuit resistance is 0.35 ohm. Neglect armature reaction, and assume the saturation curve to be a straight line.

42. Show the circuit connections for the compound-wound motor of Figure 13.16 for reversed rotation. The motor is to be cumulatively connected in both cases. How would you ascertain whether a compound motor is cumulatively or differentially connected without placing a shaft load on the machine?

43. Under full load, a 15-hp, 230-volt shunt motor draws 57.5 amp line current when hoisting an 1,850-lb load 200 ft per min. Armature-circuit resistance is 0.25 ohm and field-circuit resistance is 92 ohms. In descending with the same load the armature is disconnected from the line and a dynamic-braking resistor of 1.715 ohms placed across it. Calculate the speed at which the load descends (a) neglecting all losses and (b) taking all losses into account.

44. A 40-hp, 230-volt, 1,200-rpm compound-wound motor drives a 25-kva, 440-volt, 3-phase alternator for emergency use on board a cargo vessel. In series with the motor armature is an auxiliary field winding on the generator. Sketch the complete circuit for this motor-generator system and explain the action of the auxiliary generator field winding.

45. A 200-kw, 125-volt, 1,200-rpm constant-speed, compound-wound (short shunt) generator has the following losses at full load: 2,750 watts core loss, 810 watts friction and windage, 2,320 watts shunt-field circuit, 8,840 watts armature and brushes, 1,525 watts series field, and 715 watts commutating field. What is the full-load efficiency? What is the efficiency at half load? What power is required to drive the generator at no load? What is the approximate load at which maximum efficiency is reached? If the machine is to be run as a motor drawing 200 kw from the line at 125 volts, what losses for the motor will be different from those for generator operation? What will be the values of the losses, assuming no saturation? What horsepower rating would you give the machine as a motor under these conditions? Assume 1,200 rpm.

46. A 6-phase, shunt-wound synchronous converter delivers 1,500 kw at 250 volts d-c. What should be the ratings of the transformers of a delta-diametrical bank supplying power to the slip-rings from a 4,000-volt source?

47. A 1-kw, 2-phase synchronous converter has a d-c output voltage of 115 volts What are the ratings of the transformers which supply the power to the 4 slip-rings from a 220-volt, 3-phase source?

CHAPTER XIV

SOME BASIC ASPECTS OF ELECTRONICS

For many years, electron tubes were relegated to the radio engineer and the study of them was not considered essential in basic courses in electrical engineering. Today, in addition to bringing radio and telephone communication to their present state of refinement, electron tubes have assumed an ever-increasing importance in the power, manufacturing, and industrial fields. Their use in control systems for industrial processes has become practically indispensable. Outstanding developments and improvements in the field of measurements have resulted from the use of electron tubes in measuring instruments. There is scarcely a field of engineering or science, including the biological sciences, in which electron tubes do not play some important role. Although a detailed study of electronics is outside the scope of this book, it seems necessary, in view of the importance of the subject to so many fields of engineering, to include a brief treatment of some of the more basic principles. It is only such a rather limited and modest aim that is intended in this and the following chapters. A more thorough treatment must be reserved for those courses specializing in the various phases of the subject.

14.1. Motion of Charges in an Electric Field. Basically, all electron tubes involve the motion of free electrons, in a vacuum or partial vacuum, under the influence of electric and magnetic fields. Electrons can be considered as small physical particles, each having a mass of 9.107×10^{-31} kg and carrying a negative charge of 1.602×10^{-19} coulombs. When subjected to an electric field, a charged particle experiences a mechanical force that is expressed by equation 1.3 of Chapter I, that is,

$$f = q\mathcal{E} \text{ newtons} \tag{14.1}$$

where f is the force, in newtons, acting on the particle; q is the charge, in coulombs, carried by the particle; and $\mathcal{E}$ is the electric-field intensity, in volts per meter. The direction of the force is in the direction of the electric field when the particle carries a positive charge, and is in the opposite direction when the charge carried is negative, as in the case of the electron. The force produces an acceleration of the particle that can be calculated by the ordinary laws of mechanics. The calculation of the trajectory, kinetic energy, time of flight, etc., of the charged particle becomes, to a large extent, a problem in mechanics. The application of these principles in mechanics, with which the reader already should be familiar, to electron tubes has been chosen as the introduction to the subject with the expectation that it will lead to a certain amount of confidence on his part in studying electronic devices.

The simplest form of electron tube is the *diode*, which consists of a *cathode* that serves as a source of electrons produced by thermionic emission, and an *anode*, which is usually placed at a positive potential with respect to the cathode. Under these conditions, an electric field is produced between the cathode and anode. The electric-field flux lines are represented by the dotted lines in Figure 14.1 and the direction of the field intensity is shown by the arrowheads on the flux lines. Lines of electric flux are said to emerge from a positive electrode and to terminate on electrodes that are negative or at a lower positive potential with respect to the anode. Since the electron carries a negative charge, the force exerted on it by the electric field is directed toward the positive plate as indicated in the figure. If the anode potential were to be reversed, the electrons would be driven back into the cathode and no current would flow. This characteristic makes possible the rectifier action discussed in Chapter XVII. For the parallel-electrode arrangement under consideration, the electric-field intensity is fairly uniform in the region between the two electrodes and is given by the expression

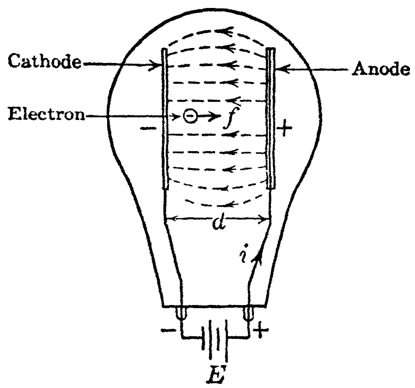


FIG. 14.1. An elementary 2-electrode tube.

$$\mathcal{E} = \frac{E}{d} \text{ volts per meter} \qquad (14.2)$$

where E is the voltage between electrodes and d is the distance in meters between electrodes.

It should be appreciated that this electric field extends all the way to the cathode, and that it is the field in the neighborhood of the cathode that starts the electrons in motion toward the anode. For all the devices considered in this chapter, the electron emission is limited so that all electrons are drawn away from the cathode as rapidly as they are emitted. In some other electron tubes, to be discussed in Chapter XV, there is an abundance of electrons emitted, some of which form a *space charge* in the vicinity of the cathode. The space charge produces effects that require a modification of the treatment given in this chapter.

When an electron is accelerated in an electric field, it acquires kinetic energy. The calculation of the motion of the electron will give us information on the energy, the velocity with which the electron strikes the plate, the current that is flowing in the circuit, and the time of transit from cathode to anode. The start of these calculations is Newton's second law of motion, which may be stated as

$$f = m\frac{d^2x}{dt^2} = m\frac{dv}{dt} \text{ newtons}$$

or

$$\frac{dv}{dt} = \frac{q}{m}\mathcal{E} = \frac{q}{m}\frac{E}{d} \text{ meters per second per second} \tag{14.3}$$

Using MKS units, m is the mass of the electron in kilograms and v its velocity in meters per second. Assuming electrons to be emitted from the cathode with zero velocity, a direct integration gives

$$v = \frac{dx}{dt} = \frac{q}{m}\frac{E}{d}t \text{ meters per second} \tag{14.4}$$

A second integration allows the determination of the position of the electrons and results in the equation

$$x = \frac{1}{2}\frac{q}{m}\frac{E}{d}t^2 \text{ meters} \tag{14.5}$$

where x is measured from the cathode.

Equations 14.4 and 14.5 may be solved simultaneously to give

$$v = \sqrt{\frac{2qE}{m} \cdot \frac{x}{d}} \text{ meters per second} \tag{14.6}$$

The velocity with which the electron strikes the anode is

$$v_a = \sqrt{\frac{2qE}{m}} \text{ meters per second} \tag{14.7}$$

and the kinetic energy associated with this velocity is equal to

$$\text{kinetic energy} = \tfrac{1}{2}mv_a^2 = qE \text{ watt-seconds} \tag{14.8}$$

From earlier considerations in Chapter I, this last equation is just what should have been expected, that is, the work done on the particle is equal to the product of its charge and the difference of potential through which it has moved.

All the energy acquired by the electrons in their flight from the cathode to the anode is supplied by the battery connected to the electrodes. The electrons moving from the cathode to the anode constitute a current, and this current is continuous through the space between the electrodes and around the external circuit through the battery. The convention for direction of *current* flow, adopted many years before electrical current had been identified with the motion of electrons, is opposite to that of electron flow. However, the steady flow of electrons reaching the anode can be considered as flowing into the anode, through the battery and back to the cathode. From these considerations, the power involved can be expressed by the relation.

$$\text{Power} = \frac{d\,(\text{kinetic energy})}{dt}$$

$$= E\frac{dQ}{dt} = Ei \text{ watts} \tag{14.9}$$

where dQ/dt is the rate of arrival of *total charge*, due to many electrons, at the plate, and is exactly the current supplied by the battery.

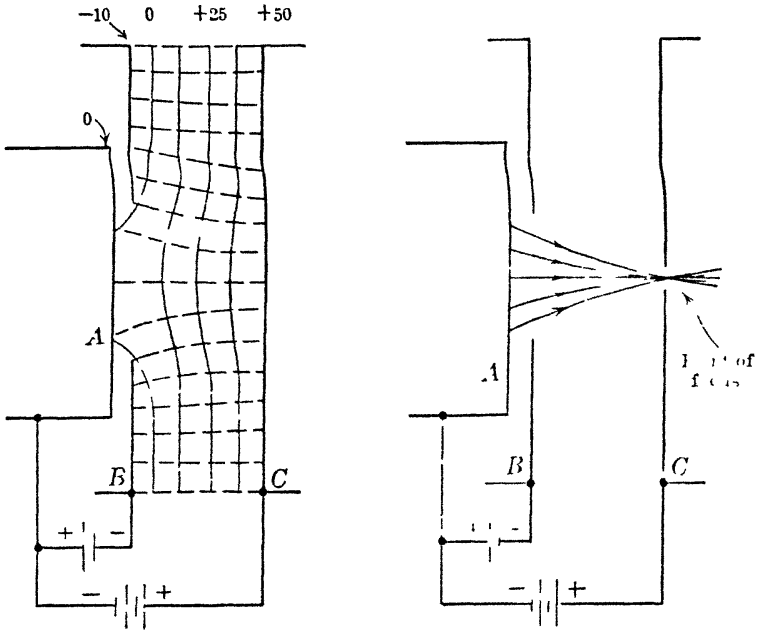
A proper inquiry at this point would be a consideration as to what has become of the energy that was imparted to the electrons by the electric field and that was later shown to have been supplied by the battery source. The electrons impinge upon the anode (also called the plate) with the kinetic energy that is given by equation 14.8. Upon striking the plate, the velocity is suddenly reduced to zero and the energy is converted into heat. This heating of the plate is one of the important factors that limit the power-handling capacity of most electron tubes. Provision must be made to keep the temperature of the plate within a safe operating value. In large power tubes cooling is accomplished by forced air flow or water circulation around the plate. Large water-cooled tubes are commercially built that have a plate dissipation of 100 kw. In later chapters we will see how the majority of this energy can be put to a useful purpose in a circuit connected to the tube.

When the values for the charge and mass of the electron are substituted in equation 14.7, the velocity of an electron starting from rest and moving through a potential difference of E volts is expressed by

$$v = 5.93 \times 10^5\sqrt{E} \text{ meters per second} \tag{14.10}$$

Because of the high ratio of q/m of the electron, the velocity it attains in even moderate fields is very great, and, as a result, transit times in electron tubes are quite small. While equation 14.10 is correct for the voltages used with the majority of electron tubes, it does not hold when the calculated velocity starts to approach the velocity of light, for, when it does, the mass of the electron increases and it becomes necessary to take this relativistic change of mass into account in stating Newton's law of motion.

14.2. Focusing of Electrons by Electric Fields. In the last section we noted the manner in which electrons are acted upon by forces that tend to make them follow electric flux lines, and that the force on a negative charge is in the direction directly opposite that of the flux lines. In many electron tubes, because of the geometrical configuration of the electrodes and the choice of applied voltages, the fields are very much more complex than they are in the basic diode. By shaping the fields properly, it is possible to control the path of the electrons, and, when desirable, to concentrate them in beams of high current density or to focus them to a fine point. Because of the many similarities to the methods of focusing light rays, this phase of electronics is generally referred to as *electron optics* and the focusing field as an *electron lens*. If a flux plot of the electric field can be made, the trajectory of the electron can be calculated. It is often helpful in plotting the flux lines to remember that they always cross the equipotential surfaces at right angles.

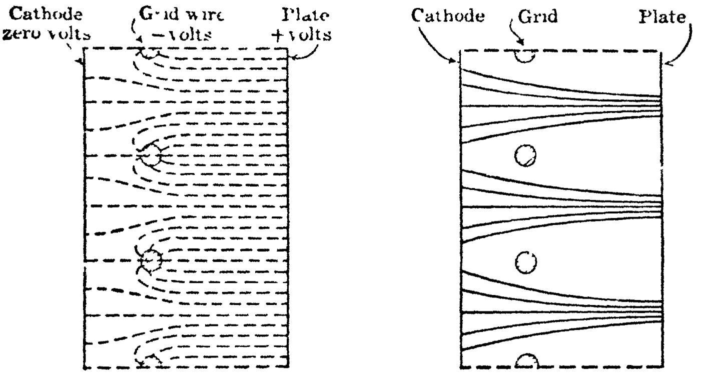


a) Electric field distribution (b) Electron paths

FIG. 14.2. Electric field distribution and electron paths in a simple electron lens

The focusing action of a simple electron lens can be understood from a study of the electric field sketched in Figure 14.2a. The flux lines are shown dotted, whereas the fine, solid lines show the location of the equipotential surfaces with representative values of potential indicated. Electrode A is the cathode and is at zero potential, electrode B has a circular aperture and is at a negative potential of 10 volts, whereas C is at a positive potential of 50 volts. From this distribution of flux, it is evident that electrons not on the axis of the aperture will experience a radial component of force tending to drive them toward the axis as they are accelerated in the direction of electrode C. As a result, the electrons leaving various parts of the cathode move along paths as shown in Figure 14.2b, come to a focus, cross the axis, and then start to diverge. In Figure 14.2b, a small circular hole is shown in electrode C, through which the electrons can pass. The point of focus will actually have a finite cross-sectional area, but it will form an image that is much smaller than the area of the cathode from which the electrons are emitted. This is a very important type of electron lens and affords a means of obtaining an approximate high-intensity point source of electrons. It has a wide application in cathode-ray oscilloscopes.

There is one more point that should be considered in connection with this lens. The flux from the positive electrode C has to reach through the aperture of electrode B, which is at a negative potential, in order to attract the electrons from the cathode. Only the center part of the cathode is in a force field that urges the electrons to leave. The outer portions are, so to speak, in a shadow cast by the negative electrode that produces a field in this region tending to drive the electrons back into the cathode. The size of the shadow is not only a function of the size of the aperture, but it varies also with the magnitude of the negative voltage applied to electrode B. As electrode B is made more and more negative, the area from which electrons are drawn becomes less and less, until finally the electron stream is cut off entirely. This control adjusts the intensity of the beam and is the usual intensity control of cathode-ray tubes.



(a) Electric flux lines (b) Electron paths

FIG. 14.3. Sketch of electric flux lines and electron paths in a triode.

A similar focusing and control action takes place in the triode vacuum tube, which is discussed in the next chapter. In the triode, a series of apertures are formed by the grid wires, as shown in Figure 14.3. Again, the flux from the positive plate must reach through the apertures in order that electrons be urged to leave the cathode. The magnitude of the grid voltage in this case also controls the electric field in the neighborhood of the cathode and, hence, the electron flow. Sketches of the electric field and of the paths of electron flow in a triode are shown in Figures 14.3a and 14.3b, respectively. In the triode tube, it is only the control feature that is sought, the focusing actually being undesirable in tubes designed to handle large amounts of power, because electrons striking the plate in localized areas cause non-uniform heating of the plate with a resulting decrease in the power that the tube can dissipate safely. By proper design, the focusing action at the plate can be minimized.

There are a large number of electron-lens systems that are commonly used, of which Figure 14.2 is a typical example. A second type, consisting of two cylinders placed end to end, is referred to in the next section. Since electrons also experience forces when moving in magnetic fields, electron lenses can be formed by magnetic fields instead of electric fields. A discussion of lenses of this type can be found in many of the books on electronics. Electron lenses find applications in beam-power pentodes, cathode-ray oscilloscopes, electron microscopes, secondary-emission electron multipliers, various types of radio tubes such as the klystron, the cyclotron, the linear accelerator, and a variety of other electronic devices.

14.3. The Cathode-ray Oscilloscope. The cathode-ray oscilloscope illustrates several of the basic principles discussed in the last two sections. It is an increasingly important electron tube for laboratory measurements, monitoring production processes, reproducing television signals, radar systems, and many other applications where a visual presentation of some operation is desired. The speed with which the *cathode rays* (so named before the ray or beam was known to consist of electrons) can respond is limited only by the small inertia of the electrons. The tubes can be used to depict electrical phenomena occuring at frequencies as high as 10^8 cycles per second.

A cathode-ray tube is shown diagrammatically in Figure 14.4. It consists of an *electron gun* that provides a narrow beam of electrons, two sets of deflecting plates by means of which the electron beam can be moved both vertically and horizontally on the screen, and a screen coated with a phosphor which will become luminescent when bombarded with electrons.

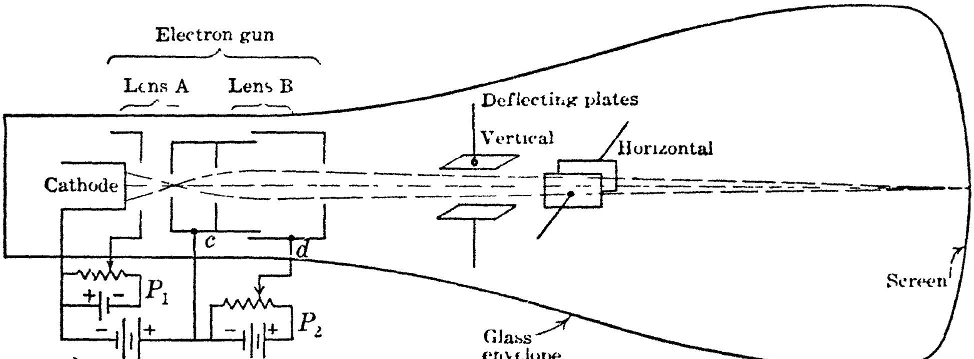


FIG. 14.4. Elements of the cathode-ray tube.

The electron gun contains a cathode that provides a source of electrons and two electron lenses. Lens A is of the type described in the last section and forms a small image of the cathode-emitting area. Potentiometer P_1 adjusts the brightness of the spot that appears on the screen by controlling the number of electrons of the beam, in the manner discussed in the previous section. Lens B is formed by the electric field between the two cylinders and focuses the image formed by lens A on the screen. The focusing control is obtained by means of potentiometer P_2, which adjusts the voltage between the two cylinders. Apertures at c and d stop all electrons that make too large an angle with the axis. This reduces the spherical aberration (electron lenses suffer from this defect as well as optical lenses) and allows a finer spot to be obtained on the screen. The electrons emerge from the aperture at d with a velocity determined by the anode potential, E_A, and can be calculated from equation 14.10, where the value of E_A is substituted for E. The paths of the electrons are indicated by the broken lines in Figure 14.4.

A variety of phosphors are commercially used for screen materials, the choice depending upon whether the tube is to be used for photographic reproduction or visual study, or whether it is desirable for the trace on the screen to persist for some time after the electrons have ceased to strike it. Two of the more commonly used materials are willimite (zinc orthosilicate), which gives a greenish trace of good photographic and visual characteristics, and cadmium tungstate, giving a bluish trace of very short persistence. Commercial tubes can be obtained with screens having a persistence as short as a few milliseconds or as long as several seconds. Screens with a long-persistence characteristic can be used for the visual study of short-time transients since the trace will remain for some time after the transient has been completed.

A signal voltage applied to the vertical deflecting plates gives the electrons a y-component of acceleration as they pass between them. Figure 14.5 aids in obtaining the expression for the deflection of the beam. An electron, after acquiring a velocity v_x from the electron gun, passes between the deflecting plates and emerges with a y-component of velocity v_y, which is related to the y-component of force f_y by the equation

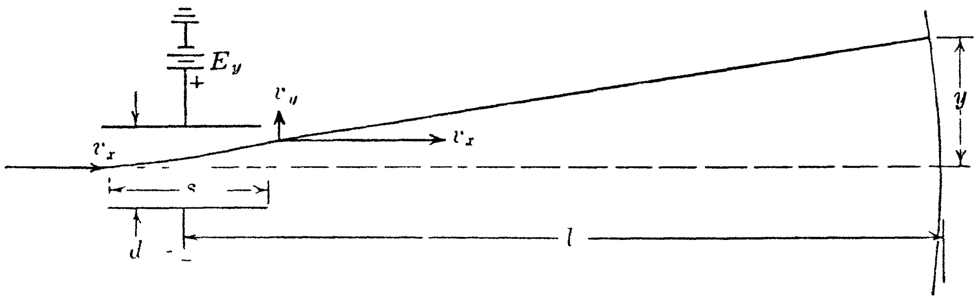


FIG. 14.5. Deflection of an electron beam.

$$f_y t = m v_y \tag{14.11}$$

where m is the mass of the electron and t is the time the electrons spend between the plates and is equal to s/v_x. Substituting for t, and for f_y from equations 14.1

and 14.2, equation 14.11 becomes

$$v_y = \frac{q}{m}\frac{s}{d}\frac{1}{v_x} E_y \text{ meters per second} \tag{14.12}$$

where q is the charge of the electron and the other terms are as indicated in Figure 14.5.

The components of velocity are related to the components of distance that the electron travels by the equation

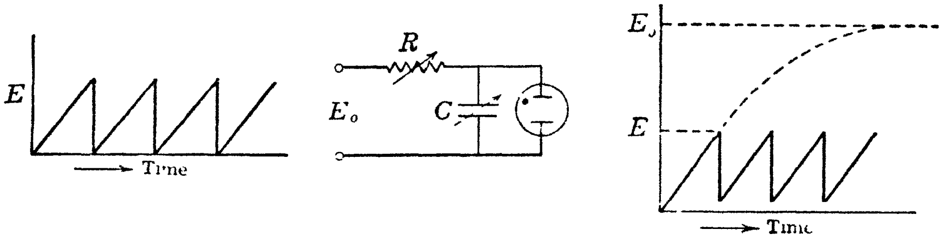
$$\frac{y}{l} = \frac{v_y}{v_x} \tag{14.13}$$

Substituting for v_y from equation 14.12 and for v_x from equation 14.7, the equation for the deflection becomes

$$y = \frac{1}{2}\, l\, \frac{s}{d}\frac{E_y}{E_A} \text{ meters} \tag{14.14}$$

where E_A is the anode voltage of the electron gun and E_y is the deflecting voltage.

The horizontal deflecting plates can be used to obtain a linear-time axis so that the signal trace can be spread across the face of the screen. For this purpose, it is desirable that the spot move horizontally across the screen at a linear rate and, upon reaching the edge of the screen, snap back to the starting point and continue to repeat the cycle. A saw-tooth voltage of the type shown in Figure 14.6a, applied to the horizontal plates, will produce this result, and Figure 14.6b shows a circuit that will give such a wave. When the voltage E_0 is applied to the circuit, the condenser starts to charge at an exponential rate, Figure 14.6c, as was discussed in Chapter III. The first part of the exponential curve is practically linear. When the ionizing voltage E_i of the tube is reached, the gas tube breaks down and the condenser discharges rapidly through it. The voltage drops to the deionizing potential of the tube almost immediately, whence the arc in the tube goes out and the cycle is then repeated. The periodicity is adjusted by varying R or C.



(a) (b) (c)

FIG. 14.6. Saw-tooth sweep voltage and circuit.

As will be evident from the discussion in the next section, the electron beam of a cathode-ray tube also can be deflected by means of a magnetic field. Deflection is accomplished by placing a pair of coils on the outside of the glass envelope so as to produce a magnetic field transverse to the beam. The method is in common use.

14.4. Motion of Charges in Magnetic Fields. When a charged particle moves in a magnetic field, it experiences a force which is always perpendicular both to its direction of motion and to the direction of the magnetic field. The magnitude of the force is proportional to the charge q carried by the particle, its velocity v, the magnetic field density β, and the sine of the angle α between v and β. Expressed in MKS units, the force is

$$f = qv\beta \sin \alpha \text{ newtons} \tag{14.15}$$

which is equation 2.4 of Chapter II. In by far the majority of cases that need to be considered, sin α is unity. In special cases where α differs from 90 degrees it is helpful to think of $v \sin \alpha$ as the component of velocity perpendicular to the magnetic field.

The relation between the directions of magnetic field, velocity, and force for a negatively charged particle are shown in Figure 14.7, where the direction of the magnetic field is into the paper. If the polarity of the charge carried by the particle, or the direction of the magnetic field, or the direction of the velocity, is reversed, the direction of the force is reversed. If any two of these quantities are simultaneously reversed, the direction of the force remains the same.

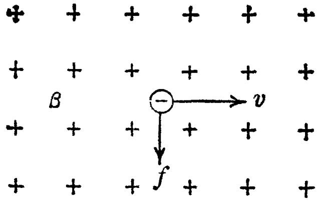


FIG. 14.7 Force on a negatively charged particle moving in a magnetic field.

Since the direction of the force on the particle is always perpendicular to its velocity, no energy can be imparted by the magnetic field, the acceleration is entirely radial, and the particle will tend to move in a circular path. The equation of motion for radial acceleration is

$$m \frac{v^2}{r} = f = qv\beta \sin \alpha \tag{14.16}$$

where r is the radius of curvature of the path, and m is the mass of the particle. From equation 14.16, the radius of curvature, for the condition of sin α = unity. can be stated as

$$r = \frac{m}{q} \frac{v}{\beta} \text{ meters} \tag{14.17}$$

The circular path described by a charged particle injected into a uniform magnetic field is shown in Figure 14.8.

Figure 14.8 depicts the basic components of the *mass spectrograph*. The charged particles are usually accelerated to the velocity v by having them fall through a potential E in a suitable beam-forming gun. The velocity acquired can be obtained from equation 14.7, which, when substituted in equation 14.17, gives the radius of curvature as

$$r = \frac{1}{\beta} \sqrt{2 \frac{m}{q} E} \text{ meters} \tag{14.18}$$

From equation 14.18, it is evident that if a mixture of atoms of different atomic weights are ionized in some manner so as to give them an electric charge, accelerated by a potential E, and then injected into a magnetic field, the various atoms will describe different paths and can be separated by a series of collector pockets. In essence, this is the principle of operation of the mass spectrograph which is proving so useful in the analysis of mixtures. It is also successfully employed in separating the isotopes 235 and 238 of uranium in atomic-energy research and production.

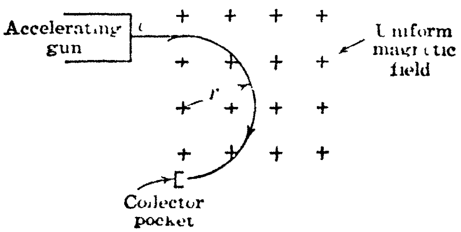


FIG. 14.8. Motion of a charged particle in a uniform magnetic field.

Another application of electric and magnetic force fields is the *cyclotron* developed at the University of California by E. O. Lawrence and his associates. The cyclotron consists of a shallow cylindrical chamber which is divided into two parts as shown in Figure 14.9. Because of the shape of the half-cylinders, they are called *dees*. A high-frequency voltage is applied to the dees and a magnetic field, perpendicular to the faces of them, is produced by a powerful electromagnet. A source of charged particles is located near the center at A. A charged particle will be attracted to one of the dees by the electric field set up in the gap between

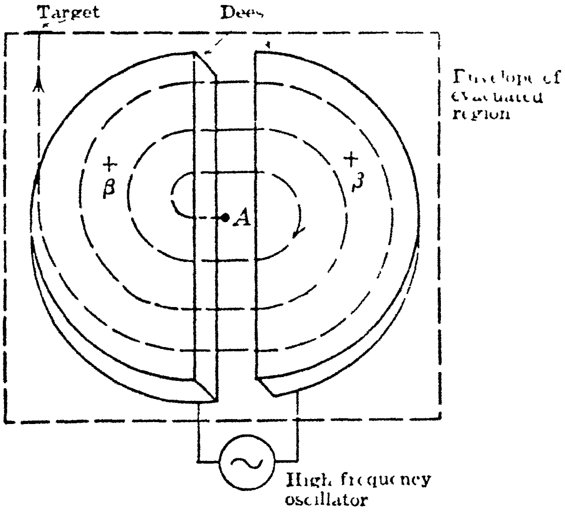


FIG. 14.9. Elements of the cyclotron

the dees by the high-frequency oscillator. Inside the hollow dees the electric field will be small and the particle will be acted upon only by the magnetic field. After passing into the dee, the particle will follow a semicircular path, having a radius of curvature as given by equation 14.17, and again cross the gap as indicated in the figure. If the motion of the particle is synchronized with the frequency of the oscillator, the particle will gain energy from the electric field at each crossing, and its velocity and radius of curvature will increase in steps as is indicated in Figure 14.9. Finally, the high-energy particle is led off through a side tube, where its energy is used to produce nuclear reactions. The angular velocity of the particle can be obtained from equation 14.17, and is

$$\frac{d\theta}{dt} = \frac{v}{r} = \frac{q}{m} B \text{ radians per second} \tag{14.19}$$

The angular velocity is independent of the linear velocity, and the time for each half of a revolution in the cyclotron is, therefore, a constant. As a result, there is no particular difficulty encountered in synchronizing the motion of the particle with a constant-frequency oscillator, within the limits imposed by relativistic considerations, that is, that the change of mass with energy be small.

The main feature of the cyclotron is that it affords a means of obtaining very high-energy particles from moderately high voltages. The high-frequency voltage across the dees is only of the order of 100,000 volts, but when the motion of the particle is properly synchronized with the frequency of the voltage source, the particle always crosses the gap when the voltage will cause acceleration and is at its peak value. Since the number of revolutions completed before striking the target is of the order of one hundred, the energy acquired is equal to that of a particle that has fallen through a potential of the order of 20 million volts.

The combination of electric and magnetic fields also makes possible the magnetron tube, which has proved to be a very efficient and powerful source of high-frequency energy.

14.5. Emission of Electrons. In the last few sections it has been shown that charged particles can be controlled accurately by means of electric and magnetic fields, and a few typical examples have indicated the considerations that are involved in predicting the behavior of these particles. The aim has been to present a physical picture of the basic principles involved. Actually, electronics is in no way a mysterious phenomenon occurring inside an evacuated envelope, but one following naturally from the elementary concepts of electricity and mechanics with which the student already should be familiar.

Up to this point, we have not been concerned with the problems of obtaining the charged particles, but it is the purpose of this section to study the manner in which electrons may be emitted from solids. The fact that electrons can be emitted by various processes from all substances and that all electrons are the same irrespective of their source forms a good share of the evidence upon which the atomic theory of matter is based. In turn, modern atomic theory, based on this and additional evidence, is able to offer a satisfactory explanation of emission phenomena. It will serve our purpose sufficiently well just to state briefly some of the concepts of the theory that relate to the present subject.

In a metal, the atoms are spaced closely in a well-defined geometrical arrangement (*space-lattice*) which can be determined from X-ray analysis. These atoms are held together by interatomic forces that are electromagnetic in character, and it is this set of forces that gives a piece of metal the property of resisting deformation from externally applied forces. An atom consists of a central positive nucleus surrounded by a revolving array of *planetary* electrons arranged in so-called *shells* at well-defined distances from the nucleus. The number of revolving electrons differs for different elements and is equal to the *atomic number* of the element. The nucleus accounts for practically all the mass of the atom and is composed of a closely packed combination of *protons* and *neutrons*. Each proton carries a positive charge equal in magnitude to that of an electron and has a mass approximately 1,840 times that of an electron. The neutron has a mass equal to that of the proton and carries no charge. The number of protons is equal to the number of surrounding electrons, with a resulting net charge of zero for the atom. The sum of the number of protons and neutrons in the nucleus determines the weight of the atom and is equal to its *atomic weight*.

In some atoms the outer shells contain only a few electrons revolving around the nucleus. For example, good conductors, such as silver and copper, have only one electron in the outer orbit. These outermost electrons in a metal are not held very strongly to what we might call the parent atom, and are relatively free to drift from one atom to the next. Under the influence of an electric field (resulting from a voltage being applied across two faces of a conductor), the *free* electrons will acquire a motion that is interrupted at frequent intervals by collisions with atoms, and, at every interruption, will lose kinetic energy which represents the I^2R loss in the conductor. Stated briefly, this is the theory of electrical conduction. Although the free electrons are privileged to move from one atom to another within the conductor, the privilege of free motion is not, in general, extended to leaving the conductor. Within the conductor, the atoms are closely packed and one nucleus will aid a free electron in moving out of the force field of another nucleus. When the electron attempts to leave the surface, however, all the positively charged nuclei are behind it and are exerting forces that prevent it from escaping. In order that the electron overcome this surface restraint, or potential barrier, it is necessary for it to acquire energy over and above that which it normally possesses. The additional energy that must be given it in order for it to free itself of the surface is defined as the *work-function*. The work function ϕ is expressed in *electron-volts*, one electron-volt being the amount of energy acquired by an electron falling through a potential of one volt. The value of the work-function is a characteristic of the particular surface, a low value being desirable for efficient emission. The work-functions of some materials having commercial interest as emitters are given below:

Metal	*Work-function ϕ in Electron-volts*
Tungsten	4.52
Tantalum	4.06
Thorium	3.35
Thorium on tungsten	2.63
Oxide-coated surface	0.5 to 1.5
Caesium	1.36
Potassium	1.55
Sodium	1.82

The various ways in which energy is imparted to the electrons suggest the following classification of the processes of emission:

1. Thermionic emission.
2. Photoelectric emission.
3. Secondary emission.
4. Electric field emission.

The various processes will be discussed briefly in the next few sections.

14.6. Thermionic Emission. At room temperatures, the free electrons do not possess sufficient kinetic energy to escape from the surface. However, if the temperature of the metal is increased, the kinetic energy of the electrons is increased, and at sufficiently high temperatures some of the electrons will fly against the boundary-surface of the metal with enough kinetic energy to surmount the retarding surface effects. They then emerge from the metal with their energy reduced by the amount that was necessary to overcome the surface potential barrier. The energy of the emitted electrons is usually very small, being of the order of 0.5 electron-volts. This value is quite small compared with that acquired from the electric fields in electron tubes.

By dealing with the free electrons in a statistical way, it is possible to deduce theoretically the number of electrons having sufficient velocity to leave the conductor and, hence, calculate the emission current for any temperature. Richardson [1] has shown the emission current per unit area of emitting surface to be given by the equation

$$i_s = AT^2 e^{-\left(\frac{11{,}605\phi}{T}\right)} \text{ amperes per square centimeter} \qquad (14.20)$$

where i_s is the emission current in amperes per square centimeter of emitting surface, ϕ is the work-function of the surface in electron-volts, T is the absolute temperature in degrees Kelvin, and A is a constant characteristic of the metal (for tungsten the value is 60.2).

The exponential term is the more rapidly varying one in Richardson's equation and, hence, the value of the work-function determines the temperature at which the emission becomes appreciable. For tungsten at a temperature of 2,500°K, a reduction of 10 per cent in the value of the work-function would result in an increase of the emission current nearly sevenfold. It is apparent why a low value of work-function is desirable for emitting surfaces.

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[1] O. W. Richardson, "Emission of Electricity from Hot Bodies," Longmans, Green & Co. London, 1921.

The three important thermionic emitters are tungsten, thoriated-tungsten, and oxide-coated metals. Although the work-function of tungsten is relatively high, with a resulting low emission efficiency (defined as milliamperes of emission current per watt of heating power), it is a very rugged emitter in the sense that its emission is not impaired as a result of bombardment by positive ions. Positive ions are always present in high-voltage commercial tubes as a result of the ionization of the small amount of residual gas that is present. Tungsten is used in all vacuum tubes operating with very high plate voltages.

Thoriated tungsten is tungsten containing about 2 per cent of thorium oxide. Under the operating temperature, a molecular layer of thorium is maintained on the surface of the emitter which results in a value of work-function lower than that of pure tungsten. The emission efficiency of thoriated tungsten is much higher than that of pure tungsten, but its monatomic emitting layer is susceptible to ion bombardment. Refined techniques in tube design permit the use of thoriated tungsten with plate voltages up to about 15,000 volts.

Oxide-coated emitters have a higher emission efficiency and a longer life than the other two types, but are not as rugged. They can be employed in vacuum tubes designed for plate voltages of less than 1,000 volts, and because of their high emission efficiency, receiving tubes use them almost exclusively.

Oxide-coated emitters consist of several layers of barium and strontium oxides on a suitable metal. The low value of the work-function at the surface of the oxide coating accounts for the appreciable emission at a temperature much lower than that required for tungsten or for thoriated tungsten. The emission current and emission efficiency of the oxide-coated emitters vary greatly with the type of coating and the operating temperature.

Characteristics of the three emitters are listed in the following table:

	Normal Operating Temperature, Deg. K	*Approx. Emission Current in Ma per Sq Cm*	*Approx. Ma Emission per Watt of Heating Power*
Tungsten	2,500	300	4
Thoriated tungsten	1,900	2,000	100
Barium and strontium oxides coated on metal	1,100	2,000	200

The electron emitter may be in the form of a filament, heated by an electric current passing through it, or a coated metal cylinder that is indirectly heated. The heater type consists of a small-diameter, oxide-coated cylinder, enclosing and heated by a tungsten filament. This type of emitter is widely used in a-c operated radio receivers. Its main advantage over the filament type is a great reduction in the hum caused by the alternating current used for the heating.

The emission efficiency of this type is about the same as that of thoriated-tungsten filaments.

14.7. Photoelectric Emission. When light of a proper frequency strikes a metal surface, electrons are ejected. The modern concept of light quanta, or photons, gives a simple interpretation of the phenomenon. According to present concepts, the photoelectric process runs about as follows: a quantum of light strikes the metal and gives all its energy to a free electron, which then may escape from the metal after having its energy decreased by the potential barrier at the surface. Einstein expressed the picture of the photoelectric effect by the equation

$$\tfrac{1}{2}mv^2 = hf - \phi q \text{ watt-seconds} \tag{14.21}$$

in which $\frac{1}{2}mv^2$ is the kinetic energy of the emitted electron, hf is the energy of the photon, ϕ is the surface work-function, and q is the charge carried by the electron. The energy of a photon of light is proportional to the frequency f. The constant of proportionality h is known as Planck's constant. The energy of the photon must be equal to or greater than the energy that the electron needs to pass through the potential-energy barrier that exists at the surface. In order that an electron escape,

$$hf \geqq \phi q$$

or

$$f \geqq \frac{\phi q}{h} \tag{14.22}$$

The frequency $(\phi q)/h$ is called the *threshold frequency*. If the frequency of the light is less than the threshold frequency, the photon will not possess sufficient energy to allow the electron to escape. The theory does not allow of any accumulation process by means of which an electron can increase its energy until it has acquired a sufficient amount to escape. In order that emission take place, a single photon must be able to supply all the needed energy, and emission occurs instantaneously with impact of the quantum of light. Observations show that the number of electrons emitted is directly proportional to the incident light intensity.

Since

$$f\lambda = c$$

where c is the velocity of light, and λ is the wave length, equation 14.22 can be written

$$\lambda \leqq \frac{hc}{\phi q} \tag{14.23}$$

and the *threshold wave length* λ_0 is

$$\lambda_0 = \frac{hc}{\phi q}$$

$$\lambda_0 = \frac{12{,}400}{\phi} \text{ Angstroms} \tag{14.24}$$

One Angstrom is 10^{-8} cm, and the work-function is in electron-volts. In order that red light (8,000 Angstroms) emit electrons, the work-function must be less than about 1.5. Because of their low work-functions, the alkali metals are the most effective surfaces for photoelectric tubes that are to be used with visible light. Photoelectric emitters having various color sensitivities have been developed for commercial photoelectric tubes. The characteristics of some of these tubes are discussed in the next chapter.

14.8. Secondary Emission. If electrons or ions bombard a metal surface with sufficient velocity, some of the free electrons will be knocked out of the metal. The amount of secondary emission is a function of the velocity with which the primary electrons strike the surface and of the surface material used. Surfaces have been developed that yield 9 secondary electrons for each primary electron impinging on the surface.

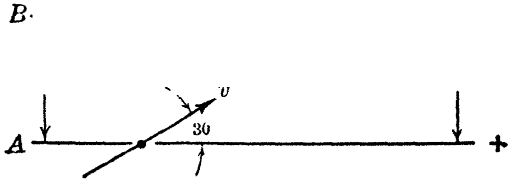
Secondary emission is used to obtain current amplification. In one form of electron-multiplier tube, electrons that are emitted from a photoelectric surface are drawn to an anode by means of an electric field. The secondary electrons that are emitted from the anode are attracted to a second anode, where secondary emission again takes place. This process can be repeated through several stages. One commercial photoelectric tube, the type 1P21, uses 9 stages of secondary emission and obtains by this means a current amplification of 2×10^6.

14.9. Field Emission. It has been found that if very intense electric fields of the order of 10^8 volts per meter exist near the surface of a metal, electrons will be pulled out. Such intense fields can be obtained with moderate voltages by using a fine point as a cathode. The concentration of the electric field at the point will result in a high field intensity. A few commercial tubes make use of this so-called cold-cathode effect.

1. The distance between the cathode and anode of a parallel-plate diode is 0.25 cm and the anode potential is 200 volts. Sketch the acceleration, velocity, kinetic energy, and travel time of an electron as a function of the distance from the cathode. Show that the average velocity is one-half the final velocity. Calculate the final velocity in both meters per second and miles per second. Calculate the transit time. Compare this transit time with the period of a 300-mc voltage.

2. Through what potential must an electron fall to acquire a velocity one-tenth that of light?

3. An electron, having an energy of 1,000 electron-volts, is projected into the electric field between two plates at an angle of 30° with the plates as shown in the figure. Describe the path of the electron. By what distance does the electron miss plate B? What must be the initial velocity of the electron in order that it may just reach B? If the



angle is changed to 90° and the initial energy is 1,000 electron-volts, what is the transit time for the electron to move from plate A to the point of zero velocity?

4. Two concentric cylinders of 1-cm and 5-cm radii, respectively, have a potential difference of 100 volts between them. Sketch the electric flux lines. What is the force, expressed in newtons, on an electron placed midway between them? What maximum velocity would an electron acquire moving from one cylinder to the other?

5. An electron enters an accelerating field of 200 volts per cm with an initial velocity of 6×10^6 meters per sec and travels a distance of 2 cm. Calculate its final velocity. Calculate the time required to travel the distance. What is the total potential, based on the final velocity, through which the electron has fallen?

6. An electron, starting from rest, passes through the following series of consecutive fields: 400 volts per cm accelerating for 1 cm; 150 volts per cm retarding for 2 cm; and 100 volts per cm accelerating for 1.5 cm. Calculate its final velocity and energy. State the general law governing the final energy of a charged particle that has passed through a series of electric fields. Explain what would happen if the second field were increased to 300 volts per cm retarding.

7. An ion of mercury has a mass 3.66×10^5 times that of an electron and carries a positive charge equal in magnitude to that carried by the electron. Calculate the velocity and the transit time of a mercury ion that travels 1 cm through a potential difference of 200 volts. Compare the motion of electrons and mercury ions under the influence of electric fields.

8. A type 6L6 beam-power amplifier has a plate voltage of 300 volts and a plate current of 60 ma. Calculate the kinetic energy, $\frac{1}{2}mv^2$, with which each electron strikes the plate. How many electrons strike the plate each second? What value of energy must the plate be able to dissipate each second? Express this energy in terms of the plate voltage and current.

9. A water-cooled diode rectifier supplies 4 amp direct current. The cooling water shows a difference of 12°C between inlet and outlet temperatures for a flow of 2 gal per min. Calculate the velocity with which the electrons strike the plate.

10. Is the velocity of an electron at the point of focus of the electron lens of Figure 14.2 dependent upon the field distribution over the particular path it has followed?

11. In a 3-inch cathode-ray tube, see Figure 14.5, d is 0.5 cm, s is 1.5 cm, l is 16 cm, and the accelerating voltage E_A is 1,500 volts. Calculate the deflection sensitivity in volts per centimeter. Compare the time that an electron spends between the deflecting plates with the period of a 300-mc voltage. How is the sensitivity changed if the accelerating voltage is decreased to 500 volts? Explain why a cathode-ray oscillograph, using electric-field deflection, cannot be used as a mass spectrograph.

12. A beam of electrons which has been accelerated through 900 volts is projected into a magnetic field of 10^{-4} webers per sq meter. Determine the radius of curvature of the beam. What distance, measured perpendicular to the original direction of motion, will the electrons be deflected by the magnetic field after traveling 16 cm?

13. A mass spectrograph, Figure 14.8, used to separate the isotopes 235 and 238 of uranium, has an accelerating potential of 35,000 volts and a magnetic field of 0.35 weber per sq meter. Calculate the distances between the point at which the ions enter the magnetic field and the two collector pockets. Each ion carries a charge equal in magnitude to that of the electron. The mass of unit atomic weight is 1.66×10^{-27} kg.

14. Describe the basic difference between an electron gun and one that is used for accelerating positive ions.

15. An electron is released with zero velocity at an arbitrary origin. It is acted upon by an electric field of 200 volts per cm in the Y direction and a magnetic field of 4×10^{-3} webers per sq meter in the Z direction. Describe the path of the electron.

16. A 60-inch cyclotron, used to accelerate deuterons (heavy hydrogen ions), has a magnetic field of 1.275 webers per sq meter. A deuteron carries a positive charge equal in magnitude to that of an electron and has a mass of 3.35×10^{-27} kg. What is the frequency of the voltage applied to the dees? Compute the linear velocity of a deuteron after it has been accelerated and has moved out to a radius of 30 in. from the center. Through what potential has it fallen when the radius of its motion is 30 in.? If the peak radio-frequency voltage between dees is approximately 200 kilovolts, how many revolutions does a deuteron make before reaching the 30-in. radius?

17. Calculate the energy in watt-seconds of one electron-volt. Calculate the velocity of an electron having this energy.

18. Calculate the emission from a tungsten filament 2 in. long and 0.01 in. in diameter when heated to 2,400°K. At what temperature would a thoriated-tungsten filament of the same dimensions give the same emission current? The value of A appearing in the emission equation can be taken as 3 for thoriated tungsten.

19. Calculate the relative emission for pure tungsten at 2,000, 2,200, 2,400, and 2,600°K.

20. A sodium photoelectric surface is illuminated by violet light of 4,400 Angstroms wave length. Determine the velocity with which the electrons are emitted. Determine the threshold frequency and wave length. The value of Planck's constant is 6.62×10^{-34} joules-sec.

21. A type 931-A photomultiplier tube uses 9 stages of secondary emission to obtain a current amplification of 260,000. Calculate the ratio of the number of secondary electrons emitted from each surface to the number of primary electrons striking the surface

CHAPTER XV

VACUUM TUBES AND THEIR CHARACTERISTICS

15.1. The Diode: Space-charge Effects. In the electron devices considered in the preceding chapter, a study was made of the forces on electrons resulting from the electric fields produced by voltages applied to the tube electrodes. The electric field created by the electrons themselves was neglected. This approximation in the treatment is a reasonable one as the charge density throughout the regions considered is relatively small, and the inclusion in the analysis of the fields contributed by the charged particles is of negligible consequence. In many electron tubes, however, it is necessary to investigate regions in which the electron-charge density is quite large, and the effects of this *space-charge* should be included in the analysis.

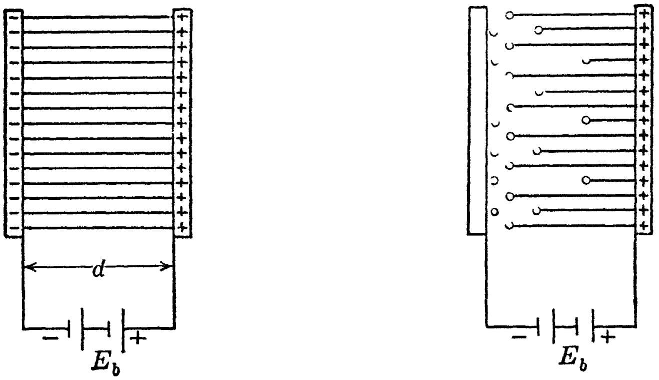
Consider, first, the case of the elementary diode (see Figure 14.1 of the preceding chapter) with no positive potential applied to the anode. When the cathode is first heated, some electrons are emitted into the space between the cathode and plate. The charge density will be relatively high near the cathode and will decrease rather rapidly toward the plate. In the majority of vacuum tubes the greater part of the space-charge is concentrated within the region a fraction of a millimeter from the cathode. These negative charges place the region in front of the cathode at a negative potential with respect to it, and, as a result, electrons will be driven back into the cathode. Equilibrium will be established when as many electrons are returning to the cathode as are being emitted from it. Thus, unless an external field is applied to draw the electrons away, the net continual emission is zero. Equation 14.20 of the preceding chapter shows the amount of current *available* from the cathode and places the limit on the maximum tube current that can flow.

At first thought, it might seem that with a positive potential applied to the anode all electrons emitted from the cathode will be drawn over to it, and that the magnitude of tube current would be equal to the emission current. This is not the case, however. The negative space-charge around the cathode forms the equivalent of an electric shield that prevents the electric flux lines, originating at the plate, from extending all the way to the cathode. As a result, the number of electrons influenced by the applied electric field depends upon the magnitude of that field. Therefore, the magnitude of plate current, when limited by the space-charge effect, is a function of the anode voltage. The equation relating the anode current and voltage can be shown to be

$$I_b = KE_b^{3/2} \tag{15.1}$$

in which K is a constant for any particular tube and can be calculated from its physical geometry.

An approximate analysis of the tube current, as limited by space-charge conditions, can be made with the aid of Figure 15.1. The diode will be considered as two parallel plates with a voltage E_b acting between them. Consider, first, the conditions when the cathode is cold and not emitting electrons. The potential E_b produces positive charges on the plate and negative charges on the cathode. Electric flux lines emerge from the positive charges and terminate on the negative ones as shown in Figure 15.1a. One flux line can be considered emerging for each unit of positive charge. It is pertinent to the analysis to note that for a given physical geometry of the tube the quantity of charge Q_p on the plate is proportional to the voltage, as is expressed by the equation



(a) Electric field distribution when cathode is cold

(b) Electric field distribution when tube current is space-charged limited

FIG. 15.1. Illustrations of electric field distribution in a diode.

$$Q_p = CE_b \tag{15.2}$$

where C is the capacitance between the plate and the cathode.

Next, let the cathode be heated and negative charges (electrons) emitted from the cathode. The flux lines now terminate on the emitted electrons, and if a sufficient number are emitted no flux lines will reach the cathode. The conditions are illustrated in Figure 15.1b. When no electric flux lines reach the cathode, the electric field in this region is zero, and the electrons immediately adjacent to the cathode experience no force urging them to leave. The quantity of negative charge moving toward the plate is approximately equal in magnitude to Q_p, the charge on the plate when the cathode is cold. The value of plate current that results from these moving charges is equal to the ratio of Q_p to the time of transit from cathode to anode. This relation is expressed by the equation

$$I_b = \frac{Q_p}{t}$$

or

$$I_b = CE_b \frac{V_{\text{avg}}}{d} \tag{15.3}$$

where d is the distance from cathode to plate, and V_{avg} is the average velocity of the electrons. The average velocity is approximately equal to one-half the final velocity as given by equation 14.7. Making that substitution, equation 15.3 may be written as

$$I_b = \frac{1}{2}\frac{C}{d}\sqrt{\frac{2q}{m}}\,E_b^{3/2} \tag{15.4}$$

or

$$I_b = KE_b^{3/2} \tag{15.5}$$

The form of the complete plate-current plate-voltage curve is shown in Figure 15.2. The first part of the curve follows the three-halves power law of equation

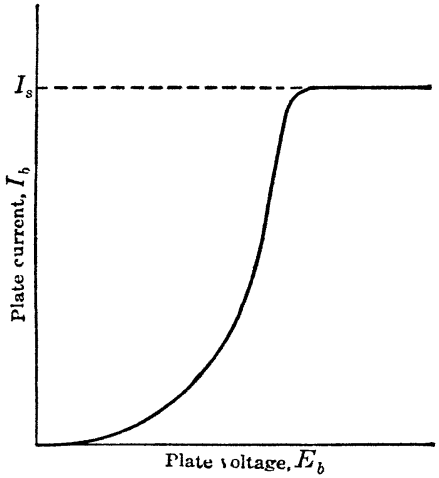


FIG. 15.2. Plate-current characteristics of a 2-electrode tube.

15.1. A limiting current is reached when all electrons emitted by the cathode are drawn to the plate. When the total emission current I_s is flowing, the current is said to be *emission limited;* currents of lesser magnitude are said to be *space-charge limited.* Normal operation of vacuum tubes is almost always with space-charge-limited current.

The curve of Figure 15.3 is the plate characteristic of a type 6X5 full-wave rectifier tube. The tube actually consists of two diodes in one glass envelope as shown in Figure 15.4. The cathodes are indirectly heated by the 6-volt heater filaments. The pin connections to the heater wires and electrodes, as viewed from the bottom of the tube, are indicated in the figure.

Since the diode tube passes current only in one direction, it can be used to convert alternating current into direct current. When the tube is used in this manner, it is classified as a *rectifier tube.* The diode finds wide application for such service. The rectifying characteristic of the diode is used also in vacuum-tube voltmeters, and for the detection of radio-frequency signals. Some of these applications are discussed in Chapter XVII.

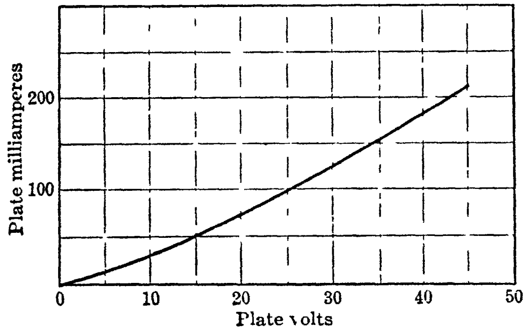


FIG. 15.3. Plate characteristics (one plate only) of the type 6X5 full-wave high-voltage rectifier.

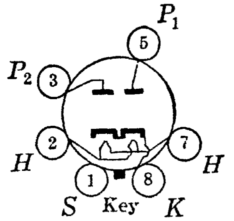


FIG. 15.4. Bottom view of pin connections of the 6X5 tube.

15.2. The 3-Electrode Tube (Triode). In 1907, Dr. Lee De Forest introduced a third electrode, in the form of a coarse-mesh *grid,* between the plate and cathode. As was indicated in the preceding chapter, a voltage between the grid and the cathode can control the magnitude of the electron current that reaches the plate. The introduction of this control grid still represents the greatest single contribution to the development of electron tubes. Without it, the wide variety of applications of electron tubes would be tremendously reduced.

The grid is usually operated at a negative voltage with respect to the cathode and, hence, the electric field in its neighborhood is at a negative potential. As a result, electrons are repelled from the region in the immediate vicinity of the grid wires and the flow to the positive plate is through the central space between the wires. If the grid is made sufficiently negative, the region of negative potential will extend to include all the space between the wires and, hence, there will be no plate current. As an aid in understanding the character of the electric field, equipotential lines for a section of a triode are sketched in Figure 15.5. The distribution is for +100 volts on the plate and −10 volts on the grid. The values of potential are indicated, and the negative potential lines are shown dotted. The electrons leaving the cathode will be drawn to regions of positive potential and will flow in a direction perpendicular to the equipotential lines, that is, in the direction of positive potential gradient. Figure 15.5 also shows the potential distributions between the cathode and plate for a section taken through a grid wire, curve "a," and for one taken between grid wires, curve "b." Because the force that the electrons experience is in a direction of positive potential gradient (positive slope of the potential curves), it is evident that no electrons reach the grid when it is operated at a negative potential. Since the grid voltage can regulate the plate current, relatively large amounts of power can be controlled without the expenditure of any energy in the control circuit.

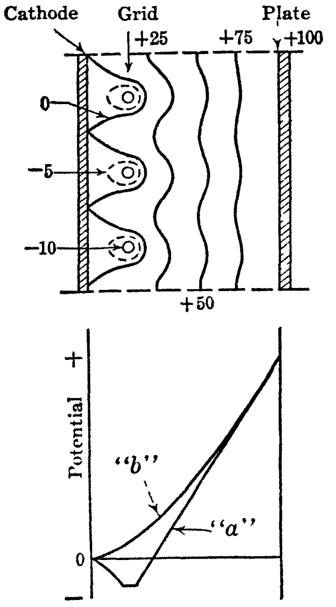


FIG. 15.5. Equipotential lines and potential distribution in a triode. Curve "a" is for a path through a grid wire and curve "b" is for a path between grid wires.

An approximate analysis of the triode can be made by an extension of the concepts discussed in the first section of this chapter. If the velocity with which the electrons are emitted from the cathode is neglected, the space-charge in the diode will adjust itself so that the electric-field intensity at the cathode will be zero. If the field intensity at the cathode were to become positive suddenly, a sufficiently greater electron charge would flow to depress the field potential and, hence, restore zero field intensity. The dominant concept in this picture is that the value of the space-charge-limited current is regulated by the field intensity in the immediate vicinity of the cathode. Since, in the triode, this field will be influenced by both grid and plate potentials, the next step in the analysis is to modify equation 15.1 to include the effects of the grid potential. This suggests rewriting the expression for the plate current of the triode in the form

$$I_b = K(E_b + \mu E_c)^{3/2} \qquad (15.6)$$

E_b and E_c are the respective plate and grid potentials, and K is a constant that is related to the geometry of the tube. The quantity μ is called the *amplification factor*, and it is a measure of the relative effectiveness of a voltage on the grid and a voltage on the plate in producing a field near the cathode. The fields produced by the two potentials are compared, of course, in the absence of space-charge. As previously discussed, enough space-charge current will flow under the influence of these fields to bring the net field intensity at the cathode to zero. For a triode having $\mu = 20$, a potential of 1 volt on the grid produces the same field near the cathode as 20 volts on the plate. If, then, for the tube, the plate potential were made 100 volts positive and the grid 5 volts negative, the two fields would cancel each other and the plate current would be zero. Throughout most of the useful operating range of electrode potentials, μ is nearly constant.

Both K and μ can be calculated, with a reasonable degree of accuracy, from the geometry of the tube. The tube designer, however, makes considerable use of empirical relations and the results of model studies. It is evident, from the considerations involved, that the amplification factor of a tube can be increased by placing the grid closer to the cathode. The value of μ also can be increased by using a finer mesh grid. With this change in grid structure, it becomes more difficult for electric flux from the plate to reach through to the cathode. Thus, the resulting increase in amplification factor results from a decrease in the effectiveness of the plate potential rather than any particular increase in the effectiveness of the grid control.

15.3. Characteristic Curves of the Triode. Equation 15.6 expresses fairly accurately the dependence of the plate current upon the values of grid voltage and plate voltage for commercially available tubes. It is not used very often, however, in obtaining the performance of a tube. Instead, the tube characteristics are usually presented by a series of curves. The two sets of *characteristic curves* that are commonly used are the *grid-plate characteristics* and the *plate characteristics*. A grid-plate characteristic presents the relation between grid voltage and plate current for a specified plate voltage. A family of such curves is obtained by using the plate voltage as a parameter. The graph of plate characteristics shows a series of curves of plate current plotted against plate voltage with grid voltage serving as the parameter. The grid-plate characteristics and the plate characteristics for a type 6J5 triode are shown in Figures 15.6 and 15.7, respectively. It is evident that essentially the same information is presented by each set of curves. The plate characteristics of Figure 15.7 can be obtained from the grid-plate characteristics of Figure 15.6, and vice versa. The plate characteristics are somewhat more convenient in calculating tube performance.

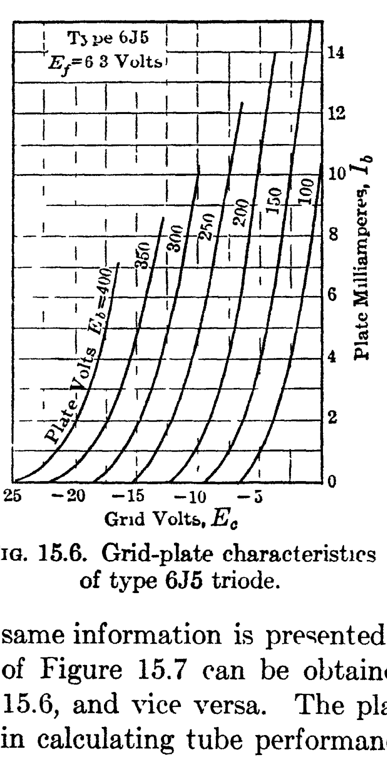


Fig. 15.6. Grid-plate characteristics of type 6J5 triode.

A triode is usually operated with a set of d-c voltages applied to the electrodes and a signal voltage applied in the grid circuit. An elementary circuit arrangement is shown in Figure 15.8. The cathode is indicated to be of the heater type but with the connections to the heater omitted from the diagram. The d-c potentials on the grid and plate, when no signal is present, determine the *operating point* on the characteristic curves. To ascertain the operating point when a load resistance, R_L, is in the plate circuit, consider the values of voltages and resistance as indicated in the diagram. E_{bb} is the plate-supply voltage and E_{cc} is the grid-supply voltage. Since no grid current flows, the value of E_{cc} is also the *quiescent value* (the value when no signal is present) of grid voltage E_c. The quiescent value of plate voltage E_b is, however, less than the value of plate-supply voltage by the I_bR_L drop, where I_b is the quiescent value of plate current. In equation form.

$$E_b = E_{bb} - I_bR_L \tag{15.7}$$

Equation 15.7, and the relations expressed by the characteristic curves, must be satisfied simultaneously. This can be done by plotting Equation 15.7, which is that of a straight line, on the plate characteristic as is shown in Figure 15.7. The line can be constructed by drawing it from a point E_{bb} on the voltage axis and at an angle with the voltage axis such that

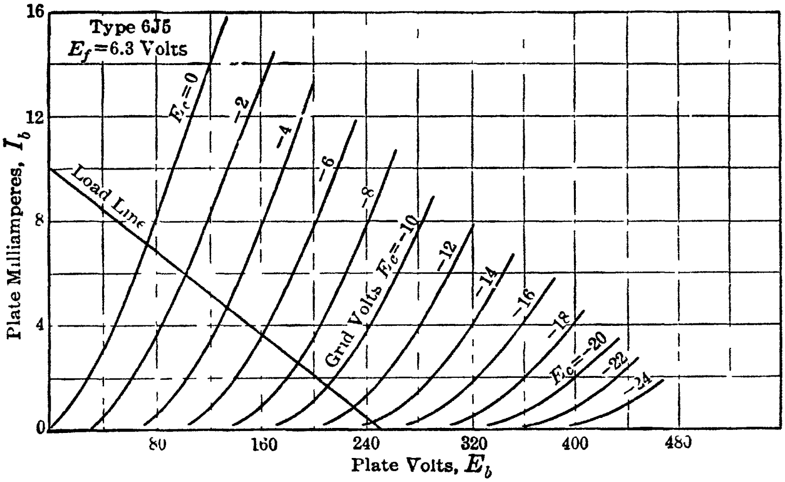


Fig. 15.7. Plate characteristics of type 6J5 triode with a 25,000-ohm load line.

$$\frac{\Delta E_b \text{ (in volts)}}{\Delta I_b \text{ (in ma)}} = \frac{R_L \text{ (in ohms)}}{1{,}000} \tag{15.8}$$

The point of intersection with the specified curve of E_c is the operating point

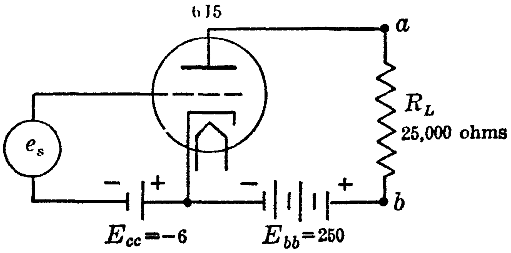


Fig. 15.8. Elementary circuit of a triode with a resistance in the plate circuit.

The straight line drawn on the characteristic curves is called the *load line.* As the grid voltage is altered by a signal voltage, corresponding changes in plate current and plate voltage can be obtained by moving back and forth along this line. For the values given in Figure 15.8, the quiescent values of plate current and plate voltage are 3.6 ma and 163 volts, respectively. If a signal voltage were to change the grid voltage back and forth from -10 to -2 volts, the plate voltage would vary from 212 to 104 volts. This change in plate voltage appears across the load resistance and is the output signal voltage. The amplification obtained from the tube and circuit is the ratio of output signal voltage to input signal voltage, which, for this case, is 108 8, or 13.5.

15.4. The Tube Coefficients. The preceding steps in the analysis of the circuit of Figure 15.8 are typical of the basic considerations involved in the operation of vacuum tubes. First, the operating point on the characteristic curves is determined, and, second, the changes in plate current and plate voltage resulting from changes in grid voltage are noted. From these quantities, the output signal power and voltage can be calculated. Although these calculations can be made directly from the characteristic curves, it is more convenient to express the performance in terms of three tube coefficients which relate the mutual changes in grid voltage, plate voltage, and plate current at the operating point. Finally, the tube and its associated circuit are reduced to an equivalent circuit involving these tube coefficients. The three coefficients are the *transconductance* (sometimes called the mutual conductance), the *plate resistance,* and the *amplification factor.*

The transconductance g_m expresses the change in plate current that results from a change in grid voltage when the plate voltage is held constant. Thus, a change, ΔE_c, in grid voltage will cause a plate-current change, ΔI_b, that is related to the transconductance of the tube by the equation

$$\Delta I_b = g_m \Delta E_c \tag{15.9}$$

The transconductance is, accordingly, defined as

$$\begin{aligned} g_m &= \frac{dI_b}{dE_c}, \text{ for } E_b \text{ constant} \\ &= \frac{\partial I_b}{\partial E_c} \end{aligned} \tag{15.10}$$

The transconductance is, therefore, the slope of the curves in the graph of grid-plate characteristics shown in Figure 15.6. The value of transconductance varies with the operating point and can be shown, from equation 15.6, to be a function mainly of the quiescent value of plate current. This is left as an exercise for the student.

The plate resistance r_p is the value of resistance that the tube offers to small changes in plate voltage. If the grid voltage is held constant, and the plate voltage is changed by an increment ΔE_b, the increment change in plate current, ΔI_b, will be given by the equation

$$\Delta I_b = \frac{\Delta E_b}{r_p} \tag{15.11}$$

The plate resistance is, therefore, defined by the equation

$$r_p = \frac{dE_b}{dI_b}, \text{ for } E_c \text{ constant}$$

$$= \frac{\partial E_b}{\partial I_b} \tag{15.12}$$

The plate resistance is the reciprocal of the slope of the curves in the graph of plate characteristics shown in Figure 15.7. As was the case with the transconductance, the plate resistance also is mainly a function of the quiescent value of plate current.

The amplification factor μ has been discussed in Section 2 of this chapter and can be obtained from the characteristic curves in accordance with the definition

$$\mu = -\frac{dE_b}{dE_c}, \text{ for } I_b \text{ constant}$$

$$= -\frac{\partial E_b}{\partial E_c} \tag{15.13}$$

In effect, equation 15.13 states: The effect of a change ΔE_b in plate voltage on the plate current can be cancelled by a change $-\Delta E_c$ in grid voltage. The amplification factor is nearly a constant for any particular tube, but, because of small variations in grid-wire spacing in practical tubes, all sections of a tube do not reach *cutoff* (the point on a particular characteristic curve at which the plate current is approximately zero) for the same value of grid voltage. As a result, there is a small variation of μ in the vicinity of cutoff.

The three coefficients for a type 6J5 triode are shown in Figure 15.9.

It should be evident from the definitions of the three coefficients that they are not independent, that is, if ΔI_b in equation 15.9 is made equal to $-\Delta I_b$ in equation 15.11, the net change in plate current will be zero, and the ratio of the increments of voltage will be equal to the amplification factor.

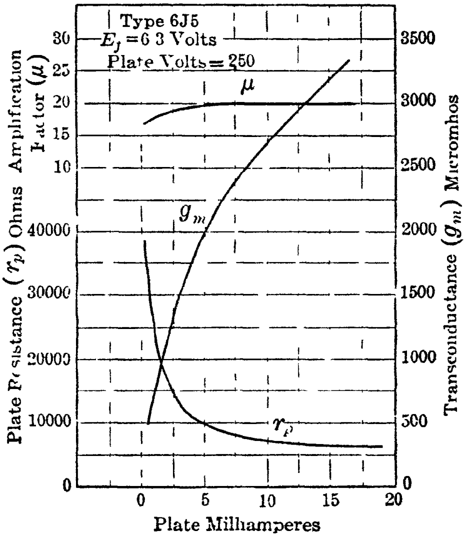


FIG 15 9. Transconductance, plate resistance, and amplification factor for a type 6J5 triode.

Writing these equations,

$$g_m \cdot \Delta E_c = -\frac{\Delta E_b}{r_p}, \text{ for } I_b \text{ constant}$$

and

$$g_m r_p = -\frac{\Delta E_b}{\Delta E_c} = \mu \tag{15.14}$$

Thus, the three coefficients are related by equation 15.14.

15.5. The Equivalent Circuit. Although the tube coefficients are not constant over a wide operating range, it is possible to choose the operating point so that there will be a fairly linear relation between changes of plate current and changes of electrode voltages occurring about this operating point. Under these conditions of operation (called *linear operation*), a sinusoidal signal voltage applied to the grid circuit will produce sinusoidal components of plate current and plate voltage.

The incremental change in plate current resulting from incremental changes of grid and plate voltages is given by the equation

$$\Delta I_b = g_m \Delta E_c + \frac{\Delta E_b}{r_p} \tag{15.15}$$

where the values of g_m and r_p are those calculated at the operating point on the characteristic curves. When the changes are sinusoidal, equation 15.15 can be expressed in terms of the effective values of the a-c components. The equation then becomes

$$I_p = g_m E_g + \frac{E_p}{r_p} \tag{15.16}$$

where I_p, E_g, and E_p are the effective values of the varying components of plate current, grid voltage, and plate voltage, respectively.

For the circuit of Figure 15.8, the varying component of plate voltage is due to the voltage drop across the load resistor and is equal to

$$E_p = -I_p R_L \tag{15.17}$$

Equation 15.16 can then be expressed as

$$I_p = g_m E_g - \frac{I_p R_L}{}$$

Solving for I_p and substituting g_m from equation 15.14,

$$I_p = \frac{\mu E_g}{r_p + R_L} \tag{15.18}$$

The a-c component of load voltage E_L (using the convention $E_L = E_{ab}$) is given by the relation

$$E_L = E_p = -I_pR_L$$

$$= -\mu E_g \frac{R_L}{r_p + R_L} \tag{15.19}$$

Equations 15.18 and 15.19 lead to the equivalent circuit of Figure 15.10, in which only the a-c components appear. The d-c quantities serve the purpose of establishing a proper operating point and set the values of the tube coefficients to be used in the equivalent circuit. The magnitude of signal-voltage amplification can be calculated from the relation

$$A_v = \frac{E_L}{E_g} = \mu \frac{R_L}{r_p + R_L} \tag{15.20}$$

For the voltages shown in Figure 15.8 for the type 6J5 tube, $\mu = 20$ and $r_p = 12{,}000$ ohms. The voltage amplification then is 13.5, which is the same value previously calculated from the graph of the plate characteristics.

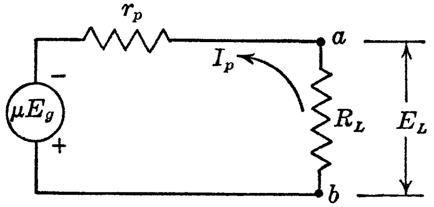


FIG. 15.10. Equivalent circuit of a triode. This circuit is the equivalent of that of Fig. 15.8.

As an aid to the understanding of some of the discussions to follow, it should be noted that the a-c components of grid voltage and plate voltage are 180 degrees out of phase for a tube circuit using a resistance load as shown in Figure 15.8.

The details of amplifiers will be discussed more fully in the next chapter.

15.6. The Tetrode. The triode presents certain difficulties when an attempt is made to use it to amplify radio-frequency signals in an amplifier using tuned circuits, such as is shown in Figure 15.11. In this amplifier the output circuit is coupled to that of the input by the capacitance C_{gp} between the grid and plate electrodes. As a result, radio-frequency power can be fed back through this capacitance to the grid circuit. By such regeneration, the circuit can supply its own input voltage, and sustained oscillations may occur. In fact, the circuit is

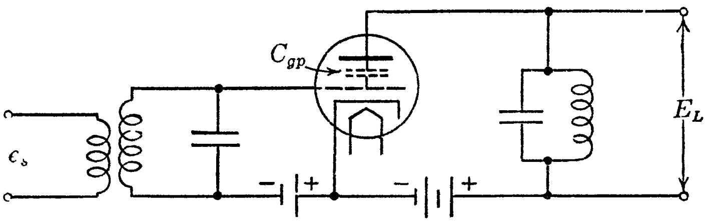


FIG. 15.11. Tuned radio-frequency amplifier with the grid-plate interelectrode capacitance indicated by the dotted line.

a well-known form of oscillator—the tuned-grid tuned-plate type. In radio-frequency amplifiers using triodes, special neutralizing circuits are used to balance out the power fed back through C_{gp}.

The grid-plate capacitance can be reduced to an unobjectionable value by introducing a *screen grid*, in the form of a wire mesh or spiral between the grid and the plate. This grid is held at zero a-c potential and acts as an electrostatic shield between these two electrodes. In addition, a shield on the outside of the anode and a metal shield enclosing the tube further isolate the input and output circuits. The general arrangement is shown in Figure 15.12. By such means, the capacitance between control and output circuits can be reduced by a factor of 1,000. The 4-electrode tube, or *tetrode*, was introduced commercially in this country in 1928 and greatly simplified the problem of radio-frequency amplifier design.

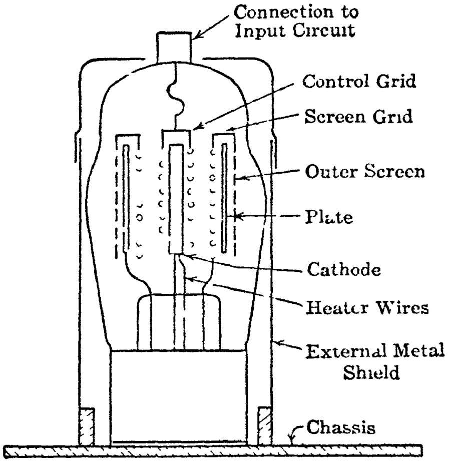


FIG. 15.12. General construction of a tetrode, showing method of shielding between input and output circuits

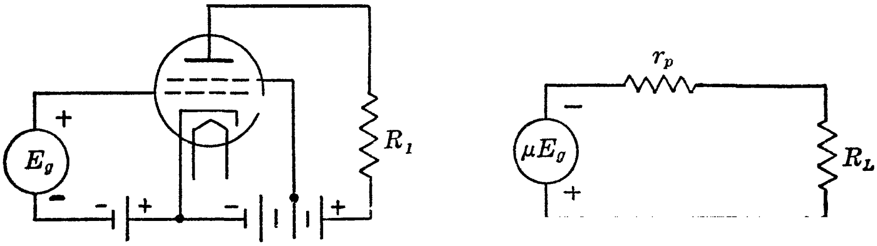
The screen grid not only isolates electrically the plate from the control grid but also shields the plate from the cathode. Relatively few of the flux lines starting from the plate reach through both grids to attract electrons from the cathode. Most of these flux lines terminate on the screen and control grids. Because of this, it is necessary to operate the screen grid at a positive d-c potential in order to draw the electrons past the control grid. Since the screen-grid structure is of small cross-sectional area, most of the electrons pass on through it and reach the plate. Approximately 20 per cent are intercepted by the screen-grid wires.

Because the screen grid reduces the effect of changes of plate voltage on the tube current, performance of the tube is actually improved. The control grid always has a more effective control of the varying component of plate current. This can be understood by reference to equation 15.16, which, for convenience, is rewritten below:

$$I_p = g_m E_g + \frac{E_p}{r_p} \tag{15.16}$$

Since the a-c plate voltage E_p is 180 degrees out of phase with E_g, a reduction of the last term in the equation will increase the a-c component of plate current obtained for a given signal voltage. The value of plate resistance of the tetrode is very much higher than that of the triode.

The tube coefficients of the tetrode are defined in the same manner as those of the triode, with the added qualification that they are defined for the screen-grid potential held constant at some specified value. Using the tube coefficients, determined for a specified operating point, the tetrode can be reduced to an equivalent circuit, as was the triode circuit. The simplified connections and equivalent circuit are shown in Figure 15.13.



(a) Circuit connection of a tetrode (b) Equivalent circuit

FIG. 15.13. Simplified circuit connections and equivalent circuit of a tetrode.

The screen grid increases the value of r_p but does not appreciably change the value of g_m. Since the three coefficients are related by the expression

$$\mu = g_m r_p$$

the amplification factor increases directly with r_p. It does *not* follow, however, that an extremely high voltage amplification can be obtained by designing a tetrode to have a very large value of r_p. This point is brought out by writing equation 15.20 in the form

$$A_v = \frac{\mu}{r_p} \cdot \frac{R_L}{1 + (R_L/r_p)}$$

$$= g_m \frac{R_L}{1 + (R_L/r_p)} \qquad (15.21)$$

In amplifier circuits, there is a practical upper limit to the value of load resistance that can be used. To a first approximation, the ratio R_l/r_p is very much less than unity for a tetrode, or for a pentode type of tube. With this approximation, equation 15.21 becomes

$$A_v = g_m R_L \qquad (15.22)$$

The main point to be emphasized is that for tetrode and pentode tubes, the value of transconductance is of more importance than the value of the amplification factor.

An alternate form of the equivalent circuit can be drawn which is generally more convenient to use when tetrode or pentode tubes are employed. As an aid to the understanding of the second form of the equivalent circuit, equation 15.18 can be written in the form

$$I_p = \frac{\mu}{r_p} E_g \left[\frac{r_p R_L}{r_p + R_L}\right] \frac{1}{R_L}$$

$$= g_m E_g \left[\frac{r_p R_L}{r_p + R_L}\right] \frac{1}{R_L} \tag{15.23}$$

The a-c component of plate current is also the value of load current I_L, or

$$I_L = g_m E_g \left[\frac{r_p R_L}{r_p + R_L}\right] \frac{1}{R_L} \tag{15.24}$$

The a-c component of load voltage, written in its proper phase relation with respect to E_g, is given by

$$E_L = -I_L R_L$$

$$= -g_m E_g \left[\frac{r_p R_L}{r_p + R_L}\right] \tag{15.25}$$

Equations 15.24 and 15.25 suggest the circuit of Figure 15.14. Thus, the vacuum tube, as viewed from the load circuit, can be considered as a source of a-c current, $-g_m E_g$, flowing into the parallel combination of r_p and the load impedance. The circuit of Figure 15.14 is called the *parallel-equivalent* circuit to distinguish it from the series-equivalent circuit previously discussed. The advantage of using the *parallel-equivalent* circuit with tetrodes and pentodes is that the value of plate resistance of these tubes is generally so much greater than the value of load impedance, that, to a good approximation, the plate resistance can be neglected in the equivalent circuit.

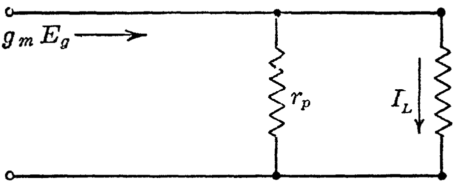


FIG. 15.14. Parallel-equivalent circuit of a vacuum tube amplifier.

A graph of the plate characteristics of a type 24-A tetrode is shown in Figure 15.15. It is noted that when the plate voltage is below the screen potential, dips occur in the curves. These dips in the characteristic curves seriously limit the range of operation of the tube and reduce the voltage output and power output that can be obtained. They are a result of secondary emission from the plate. When the plate is less positive than the screen, the secondary electrons produced at the surface of the plate, by the primary electrons from the cathode striking it, are attracted to the more positive screen. The net current to the plate is determined by the difference between the number of primary electrons arriving at the plate and the number of secondary electrons leaving it. When the plate becomes a few volts higher than the screen potential, the force on electrons near the plate is directed toward it, and, hence, no electrons leave the plate. A curve of screen-grid current, I_{c2}, also is shown in Figure 15.15.

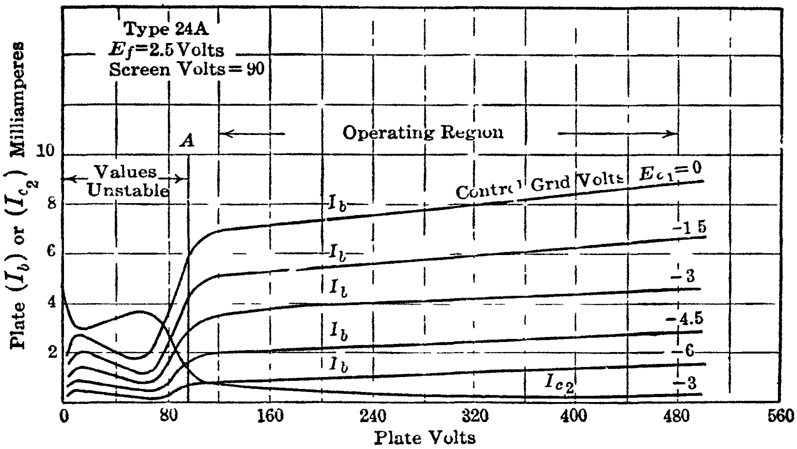


FIG. 15.15. Plate characteristics of a type 24-A tetrode.

15.7. The Pentode. The most effective way to prevent secondary electrons from leaving the plate and producing undesirable dips in the characteristic curves is to depress the space-potential in the region in front of the plate. If the region is depressed to a value below that of the plate, no electrons can move from the plate to other electrodes. One means of accomplishing the result is to place a coarse-mesh grid in front of the plate and connect it to the cathode. Inasmuch as this third grid serves to suppress secondary emission from the plate, it is called a *suppressor* grid. The addition of the suppressor grid makes a 5-element tube, or *pentode*. Although the wires of the suppressor grid are at zero potential, the space between the wires through which the electrons pass is at a positive potential somewhat below that of the plate. Figure 15.16 shows the approximate potential distribution in a pentode along a path between grid wires and also along a path passing through the grid wires.

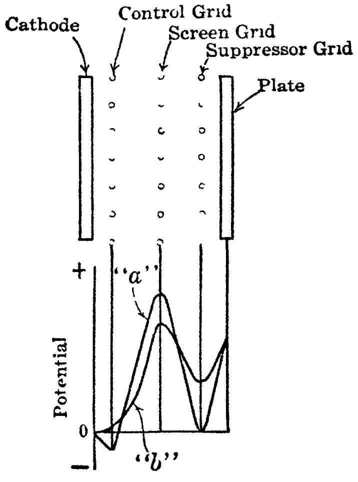


FIG. 15.16. Potential distribution in a pentode. Curve "a" is for a path through the grid wires and curve "b" is for a path passing between the grid wires.

Characteristic curves for the type 6SJ7 pentode are given in Figure 15.17. The effect of the suppressor grid is quite evident. Also to be noted from the figure is the small degree to which the plate voltage influences the plate current. As was discussed for the triode tube, the reciprocal of the slope of the curve at the operating point is equal to the value of r_p that is used in the equivalent circuit. The same equivalent circuit that is used for the triode and tetrode is used for the pentode. Because of the improved performance obtainable, the pentode has replaced the tetrode for all low-power applications. Because of space limitations and manufacturing problems, some of the high-power tubes are limited in design to four electrodes.

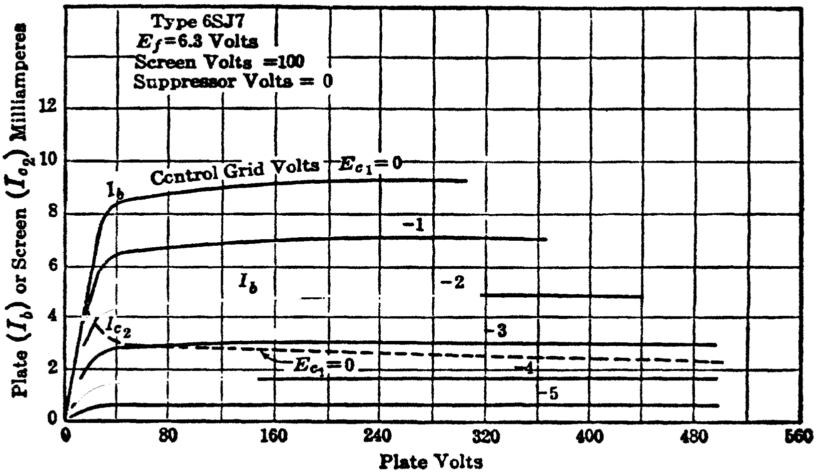


FIG. 15.17. Plate characteristics of a type 6SJ7 pentode (*Courtesy* RCA)

Typical operating values for the type 6SJ7 pentode are given below:

Plate voltage . . .	250
Screen voltage.	100
Suppressor .	Connected to cathode at socket
Grid voltage.	−3
Plate current	3 ma
Screen current . .	0.8 ma
Transconductance	1,650 micromhos
Plate resistance	Approx. 2.5 megohms

15.8. The Beam-power Tube. A second means of depressing the space-potential in front of the plate is to increase the electron space-charge in that region. This is accomplished by concentrating the electrons in beams that produce a high electron density in the vicinity of the plate. In the *beam-power* type of tube, the screen-grid wires are aligned with those of the control grid. The alignment decreases the number of electrons intercepted by the screen grid and also helps to concentrate the electron flow into thin sheets passing between the grid wires. In addition, two beam-forming plates, operated at cathode potential, further increase the concentration of the electron beam. A pictorial sketch of a beam-power tube is shown in Figure 15.18.

The characteristics of the beam-power tube are quite similar to, but somewhat better than, the pentode. The improvement results from a more uniform space-potential formed in front of the plate by the space-charge than can be obtained with a suppressor grid.

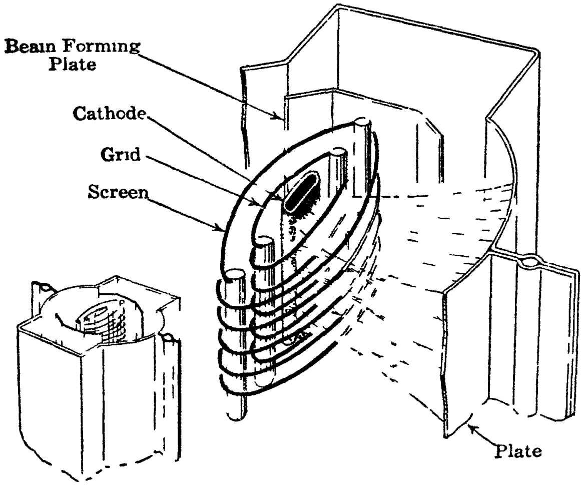


FIG 15 18 Internal structure of a beam power (Courtesy RCA.)

15.9. The Variable-mu Tube.

Some pentodes are purposely made with the spacing between the wires of the control grid non-uniform. As a result, if the value of negative d-c grid voltage (grid bias) is gradually increased, portions of the plate current from different sections of the cathode are not all cut off at the same value of grid bias. The portion of the plate current coming from sections where the grid wires are closely spaced is cut off first, whereas a large grid bias is required to stop the current flowing through the coarser-spaced sections of the grid. Such a tube will have a gradual cutoff characteristic. Tubes having such a characteristic are known by the various names *variable-mu*, *remote-cutoff*, and *super-control* tubes. A cross section of a typical remote-cutoff tube is shown in Figure 15.19.

The remote-cutoff characteristic makes it practical to vary the gain of an amplifier by adjusting the d-c grid bias. This type of control finds wide application in radio-frequency amplifiers. It was shown that the voltage amplification of a pentode amplifier is nearly equal to $g_m R_L$. The remote-cutoff characteristic permits of a smooth variation of g_m over a wide range of values. A comparison of a typical sharp-cutoff tube (type 6SJ7) and a remote-cutoff tube (type 6SK7) is shown in Figure 15.20.

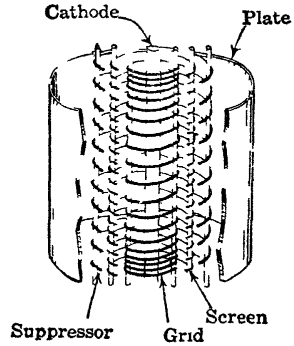


FIG 15 19 Cross-section of a typical remote-cutoff tube, showing the non-uniform spacing of the control-grid wires (*Courtesy* RCA.)

The gradual cutoff characteristic also enables the tube to amplify with minimum distortion fairly large signals over a wide range of grid bias. This feature makes these tubes particularly suitable for use in the radio-frequency amplifiers of radio receivers. Input signals to a radio receiver may range from a few microvolts to approximately one volt. The output level of the receiver can be adjusted by varying the grid bias of the radio-frequency stages of amplification. In practically all receivers, the gain is controlled automatically (automatic-volume-control, abbreviated A.V.C.) by deriving a d-c voltage from the radio-frequency output voltage and applying it to the control grids of the radio-frequency stages. Hence, an increase in the output signal increases the grid bias, which thereby tends to counteract the increase in output by reducing the gain of the receiver. If a decrease in the output signal occurs, the opposite action takes place.

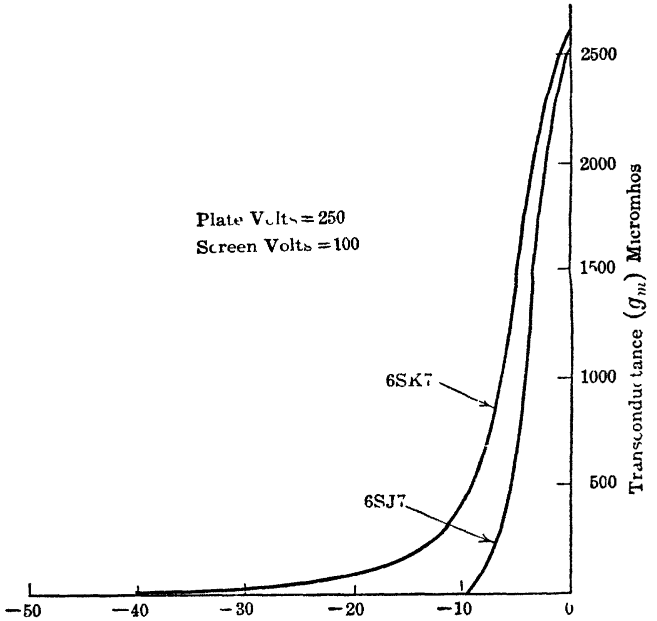


FIG. 15.20. Typical transconductance characteristic of a remote-cutoff type (6SK7) compared with a type (6SJ7) having uniformly spaced grid wires.

15.10. Other High-vacuum Tubes. In addition to the tubes considered in this chapter, there are many special-purpose and multiple-unit tubes. The basic principles of these tubes do not differ essentially from those already described. One especially interesting type of special-purpose tube is the *pentagrid mixer*. The tube has a cathode, a plate, and five grids. One grid is a suppressor grid located near the plate, two grids serve as control grids, and the remaining two function as screen grids to shield the two control grids from each other and from the plate. In such a tube, two individual signals can be applied to the control grids, with the result that the plate current will become a function of the combined effect of the two signal voltages. Tubes of this type are principally used in superheterodyne radio receivers to combine an incoming radio signal with that from the local oscillator of the receiver. The tube mixes the signal frequency with that of the local oscillator to produce the *intermediate* frequency.

To meet the space requirements of equipment designers, tube manufacturers have made available a variety of multiple-unit tubes. These are various combinations of the basic types discussed in the previous sections of this chapter. Some of the more commonly used combinations are the twin-triode, a triode with two diodes, a pentode with two diodes, and a pentode with a diode and a triode.

The tube connections, operating voltages, and complete tube characteristics for all of the various tubes are available in tube handbooks published by the tube manufacturers.

15.11. Photoelectric Tubes. Photoelectric tubes, or phototubes, as they are more commonly called, consist of two electrodes enclosed in a glass envelope. The envelope is sometimes of special glass in order that it will transmit wave lengths in a particular region of the light spectrum. The cathode is the photo-sensitive electrode and often is in the form of a half-cylinder as is shown in Figure 15.21. The anode used with this type of cathode is a slender straight wire located along the axis of the tube.

The basic photosensitive surface consists of a film of an alkali metal such as caesium or sodium deposited on a silver surface. The sensitivity of phototubes has been increased in recent years by the development of composite surfaces made of thin consecutive layers of different emitting materials. In addition, the sensitivity is greatly influenced by the method used in preparing the surface. The surface most widely used commercially today consists of a thin film of caesium superimposed on a layer of caesium oxide which, in turn, is on a silver base. The composition of the cathode surface determines the relative sensitivity to light of various colors as well as the over-all sensitivity. Commercial phototubes are available which have various color-sensitivity characteristics. By using different emitting surfaces, the maximum sensitivity may be shifted to almost any part of the spectrum. The relative spectral responses for two typical commercial surfaces are shown in Figure 15.22. For comparison, the relative response of the eye is also shown in the figure. Since the majority of phototube applications make use of an incandescent lamp having a tungsten filament, the over-all sensitivity of a phototube is generally given for light from a tungsten filament operated at a temperature of 2,870°K. The spectral distribution of tungsten operated at this temperature is shown by the dotted curve in Figure 15.22. Light filters, composed of colored glass or gelatins, can be used with phototubes in such applications as color sorting or matching.

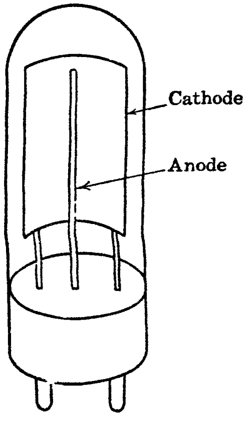


FIG. 15.21. A typical phototube.

The characteristics of phototubes can be shown by a set of curves, just as was done for the vacuum tubes considered in the preceding sections. A typical set of characteristics for a vacuum phototube, with light from a tungsten filament at 2,870°K striking the cathode, is shown in Figure 15.23. The amount of light striking the cathode, measured in lumens, is the parameter, a lumen being the amount of light flux through a unit solid angle from a point source of one candle power. For a constant light intensity, the anode current at first increases as the

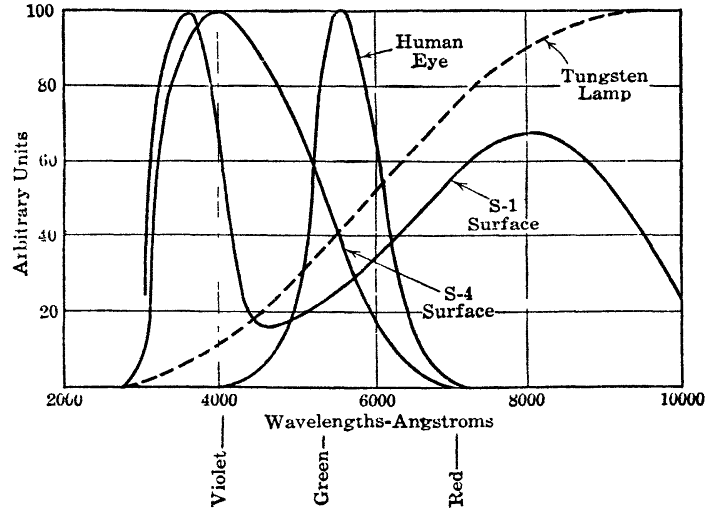


FIG. 15.22. Spectral characteristics of two RCA photoelectric surfaces, the human eye, and a tungsten lamp at a temperature of 2870°K. (Data for plotting this graph were obtained from RCA *Tube Handbook*.)

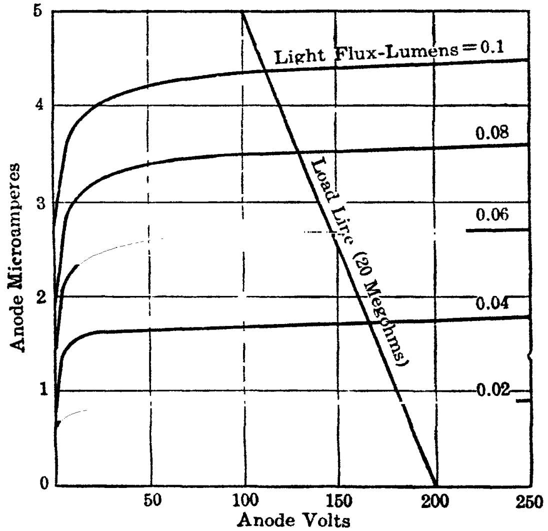


FIG. 15.23. Characteristic curves of an RCA 929 vacuum phototube. (*Courtesy* RCA.)

three-halves power of the anode voltage and then becomes practically independent of it as the value of anode current approaches that emitted from the cathode.

Since the current output of a phototube is only a few microamperes, a vacuum-tube amplifier is generally used with it. Although the magnitude of current is small, a resistance of high value connected in series with the tube will afford the means of developing a relatively high voltage. Figure 15.24 shows a typical circuit used for relay operation. A load line can be constructed on the characteristic curves in the same manner as was done on the plate-current plate-voltage curves of the tubes discussed in the preceding sections of this chapter. It is drawn from the value of anode-supply voltage on the abscissa and with a slope equal to the reciprocal of the value of load resistance. A load line is shown on Figure 15.23 for a supply voltage of 200 volts and a load resistance of 20 megohms, and operation is then along this line. As an example, a light of 0.04 lumen will produce a phototube current of 1.7 microamperes and a change of anode voltage of 34 volts. This change of anode voltage is the value of voltage that appears across R_L and is applied to the grid of the amplifier.

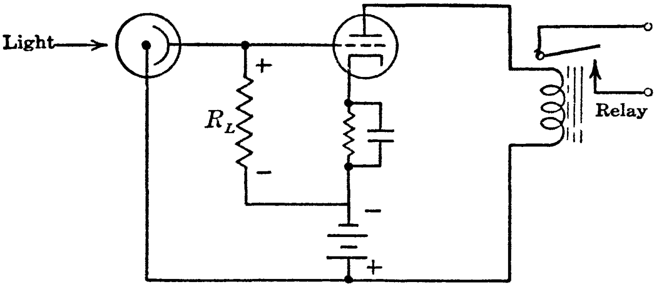


FIG. 15.24. Phototube relay circuit.

For relay operation, the phototube and amplifier tube will give satisfactory operation on a-c power. A typical circuit is shown in Figure 15.25. The condenser across the relay smoothes the a-c pulses, thereby preventing chattering of the contacts.

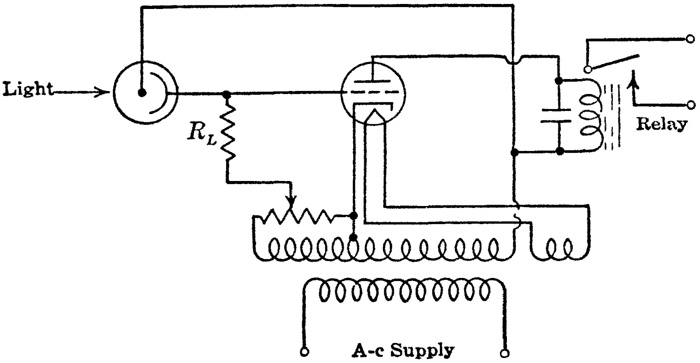


FIG. 15.25. Phototube relay circuit operated from a-c potentials.

The sensitivity of a phototube can be increased by introducing a small amount of an inert gas, such as argon, into the tube. The increase of tube current results from the ionization of the gas by the electrons emitted from the photosensitive surface. An emitted electron, moving toward the anode, strikes a gas molecule and splits it into a free electron and a positive ion. The positive ion will move toward the cathode, whereas the free electron will join the original electron in its travel to the anode. Both electrons may engage in additional ionizing collisions and further increase the current. As a result of these cumulative collisions, the anode current can be increased as much as tenfold. The characteristic curves of a type-930 gas phototube are shown in Figure 15.26.

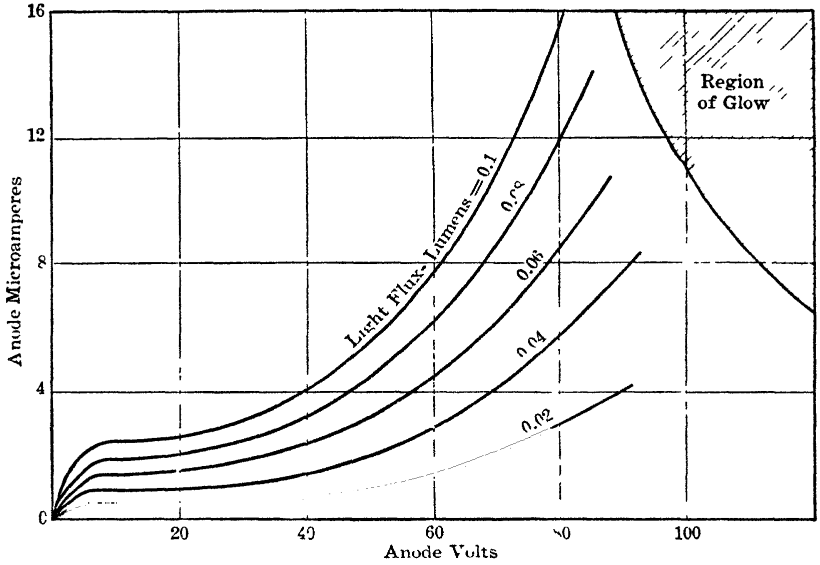


FIG. 15.26. Characteristic curves of an RCA 930 gas-filled phototube. (*Courtesy RCA.*)

Although the sensitivity is increased by the use of gas, there are several disadvantages that result from its use. One is an appreciable departure from a linear relation between tube current and light intensity. This is evident from the curves of Figure 15.26. A second disadvantage is that the tube is not as stable as the vacuum phototube. If for some reason the voltage or light intensity becomes too high, a glow discharge may take place and the tube may be permanently damaged. Gas phototubes are used where the value of light variation is quite small, as in the reproduction of sound on film. Vacuum phototubes, because of their relatively long life and stable characteristics, are preferred for most industrial applications.

It would be impossible to describe here the wide variety of applications of phototubes and, besides, the majority of applications are sufficiently well known so as not to require much discussion. Applications of phototubes can be divided into three general classes: those that depend upon the presence or absence of light and make use of the simple light-relay circuit; those that function from a change in light intensity, a change from either a specified amount, or the continual change that occurs in the modulated light beam used in sound and facsimile reproduction; and those that respond to a change in the frequency spectrum of the light, as in color matching and sorting. A partial list of applications would include high-speed counting, the control of conveyor lines, door openers, automatic weighing, automatic level control, personnel protection as on punch presses, fire detection, measurement of smoke density and the density of chemical solutions, production control, photometry, and various types of inspection and sorting. Such eye-straining tasks as inspection and control of material in paper and textile mills is done much faster and with far more accuracy by photoelectric devices than by the combined work of several employees.

PROBLEMS

1. Check the plate characteristic of the 6X5 diode shown in Figure 15.3 to see how closely it conforms to the three-halves power law.

2. The voltage across a space-charge-limited diode is 32 volts when the tube current is 250 ma. What plate dissipation must have been provided in the tube design if the diode can supply 500 ma safely?

3. A 6J5 triode is to be operated with a plate-supply voltage of 320 volts, a grid bias E_c of -8 volts, and a load resistance of 40,000 ohms. Explain in detail the method by which the operating point is determined. Determine the values of d-c plate voltage and plate current. For a signal voltage of 3 volts rms applied to the grid circuit, plot the instantaneous values of plate current and plate voltage. Determine the peak value of the a-c component of plate voltage. What is the phase relation between the a-c components of grid voltage and plate voltage? Calculate the voltage amplification. Does the value of power dissipated at the plate change when a signal voltage is applied? Compute the various components of power.

4. An amplifier uses a 6J5 triode, a plate-supply voltage of 200 volts, and a load resistance of 25,000 ohms. Plot the plate current as a function of the grid voltage. Over what range of grid voltage is the curve fairly linear?

5. From the characteristic curves of Figures 15.6 and 15.7 for the 6J5 triode, determine the values of μ, r_p, and g_m for the operating conditions of -5 volts grid bias and 175 volts plate voltage. Compare these values with those shown in Figure 15.9.

6. Explain the manner of obtaining a family of plate-current plate-voltage curves for a tube, knowing the amplification factor and one curve of the group.

7. The 2A3 triode has an amplification factor of 4.2. The plate current is 70 ma for a plate voltage of 200 and a grid voltage of -30. Calculate the plate current for a plate voltage of 300 and a grid voltage of -40.

8. Using equation 15.6, express r_p and g_m as a function of I_b.

9. Evaluate the constants of equation 15.6 for the 6J5 triode for $I_b = 5$ ma.

10. In equation 15.6, the values of K and μ for a triode are 0.013 and 8, respectively, with the plate current expressed in milliamperes. From equation 15.6, determine the values of I_b, r_p, and g_m for $E_b = 250$ volts and $E_c = -23$ volts.

11. The d-c operating voltages of a 6J5 triode are $E_c = -4$ and $E_b = 160$. The plate load resistance is 20,000 ohms. Draw the series-equivalent circuit, showing the values of all circuit constants. Calculate the value of voltage amplification. What value of plate-supply voltage must be used? Do you consider that the choice of the operating point was a particularly good one?

12. A 24A tetrode having a value of plate resistance of one megohm and a value of transconductance of 600 micromhos is used as a voltage amplifier with a load resistance of 80,000 ohms. Calculate the amplification factor at this operating point. Draw both the series-equivalent and parallel-equivalent circuits, showing all values. Calculate the voltage amplification for each circuit. What percentage error is introduced by using equation 15.22 for calculating A_t?

13. A 6SJ7 pentode is used in an audio-frequency voltage amplifier with the operating values listed in Section 15.7. The load resistance is 70,000 ohms. Using both the series-equivalent and parallel-equivalent circuits, calculate the voltage amplification.

14. Using the plate characteristics of Figure 15.17, obtain data for plotting the curve of transconductance as a function of plate current for a plate voltage of 200 volts. Estimate the value of plate resistance at several operating points. In general, is it important to know accurately the value of plate resistance of a pentode in making calculations of amplifier performance?

15. Two triodes are connected in parallel. In tube No. 1, $\mu = 10$ and $K = 0.01$ (equation 15.6) when the plate current is expressed in milliamperes. In tube No. 2, $\mu = 4$ and $K = 0.003$. Plot the total plate current as a function of grid voltage for a plate voltage of 100 volts. (This combination of tubes would act as a variable-mu triode.)

16. A type 929 phototube has a plate-supply voltage of 100 volts and a load resistance of 10 megohms. Using the characteristic curves shown in Figure 15.23, plot the voltage developed across the load resistor as a function of the light flux.

17. A 935 vacuum phototube has a sensitivity of 30 microamperes per lumen and a cathode area of 0.82 sq in. Calculate the tube current when the cathode is illuminated by a 16-cp lamp at a distance of 1.5 ft, assuming that the lamp radiates uniformly in all directions.

18. For the circuit of Figure 15.24, an increase of illumination increases the plate current of the triode. Draw a circuit for which an increase in illumination will decrease the plate current of a triode.

19. A type 930 gas-filled phototube is used with a modulated light beam. The supply voltage is 80 volts and the load resistance is 5 megohms. Based on the characteristic curves shown in Figure 15.26, plot the tube current as a function of light flux. If light flux varies sinusoidally from 0.04 to 0.08 lumen, what rms value of voltage is developed across the load resistance? Would the percentage distortion resulting from the nonlinearity of the characteristics be large?

CHAPTER XVI

AMPLIFIERS AND OSCILLATORS

The ability of a vacuum tube to amplify a signal voltage and to control the flow of a considerable value of current with negligible signal power is by far its most important property. A surprisingly large number of electronic devices depend for their operation upon this dominant characteristic. In addition to its obvious application in amplifiers, the amplifying property is basic to oscillators, various types of regulators, industrial control equipment, and a variety of measuring devices. An understanding of the principles of amplifiers is fundamental to the analysis of the majority of equipment employing electronic tubes.

16.1. Classification of Amplifiers. Amplifiers are classified in several ways. One classification conforms to the frequency, or frequency range, for which they are intended and leads to the designations of direct-current, audio-frequency, video-frequency, and radio-frequency amplifiers.

Direct-current amplifiers are designed to amplify electrical signals having a low-frequency limit of zero. The upper frequency requirement depends upon the particular application. One principal use is in the field of measurements, examples of which are the measurement of mechanical stress and strain, turbulent flow in liquids, illumination, and nerve-action potentials. This type also has a wide application in regulator and control systems. Certain design problems connected with the maintenance of stable operation in a system are encountered in high-gain direct-current amplifiers; minor variations in d-c supply voltages are amplified and appear as output voltages of such magnitude as to mask completely the desired signal. In addition, these amplified variations of supply voltage change the d-c operating potentials of the tubes and, hence, change the desired amplifier characteristics.

Audio-frequency amplifiers respond to signals in the frequency range to which the ear is sensitive, that is, from 20 cycles to 20,000 cycles, although, at the extreme limits, the sensitivity is quite low. An audio system is considered to be of *high fidelity* if it will respond, with reasonable uniformity, to a frequency range of 30 to 15,000 cycles. The majority of audio systems use a more limited frequency range than is required for true high-fidelity reproduction, with some sacrifice in naturalness and intelligibility. A frequency range of 200 cycles to 3,000 cycles, such as is used over long-distance telephone lines, gives rather high intelligibility with a fair degree of naturalness. Most of the inexpensive radio receivers do not reproduce frequencies much higher than about 3,500 cycles. The limited frequency range used in commercial audio-frequency equipment is not imposed by the amplifiers but results from economic and engineering considerations involved in the design of long-distance telephone lines and electromechanical devices, such as loud-speakers and microphones. A high value of amplification is easily obtained by using several stages in cascade. The limit of useful amplification is determined by the tube noise of the first stage of amplification. The tube noise results from the fact that the tube current is not strictly continuous in nature but consists of contributions from a large number of individual electrons. This *shot-effect* noise, as it is called, can be expressed as an equivalent noise voltage that is introduced in the grid circuit of the tube. In a well-designed audio amplifier, the equivalent noise voltage referred to the grid circuit will be about one or two microvolts.

A video-frequency amplifier responds to the video signals of a television system. The design of a video-frequency amplifier is similar in many respects to that of an audio-frequency amplifier, but a much greater band width is required to amplify the video signals. To reproduce faithfully a black-and-white 525-line picture, a frequency range of 30 to 5,000,000 cycles is required, although a fairly satisfactory picture can be obtained with a band width of 2.5 mc. A color-television system requires a frequency spectrum approximately three times that of a black-and-white system.

Radio-frequency amplifiers are used to amplify radio-frequency signals. They generally employ a resonant circuit in the plate circuit, which is tuned to the radio-frequency voltage to be amplified. Because of the selective characteristics of the tuned circuit, only frequencies in the vicinity of its resonant frequency are amplified. The degree of selectivity of the amplifier can be controlled through the choice of the constants of the resonant circuit. The interelectrode capacitances, lead inductances, and electron transit time set an upper limit to the frequency at which the tube can amplify effectively. This upper frequency limit for the types of tubes most commonly used is in the vicinity of a few hundred megacycles. Triodes, specially designed for high-frequency operation to give satisfactory performance at frequencies up to 3,000 mc, are commercially available.

In addition, within each of the four divisions discussed in the preceding paragraphs, amplifiers are classified as *voltage* and *power* amplifiers. Voltage amplifiers raise the voltage level of a signal with a relatively insignificant amount of power being involved. In most amplifying systems, the output amplifier is required to deliver electrical power to some load, such as a loud-speaker, a recorder, a radio antenna, etc., and such an amplifier is called a power amplifier. The design considerations of voltage and power amplifiers differ in many respects. In general, any amplifying system will consist of several stages of voltage amplification with a power amplifier in the output stage.

Amplifiers also are classified according to their operating point on the characteristic curves. Under this system, amplifier operation is designated as class A, B, AB, or C.

Under class A operation, the grid bias and alternating voltages applied to the grid are such that the tube performs over the approximately linear region of its characteristics. Under this condition, the wave forms of the plate current and of the output voltage approach closely that of the signal voltage. The distortion introduced by the non-linear tube characteristics can be kept to a very low value, with the result that high-quality speech and music reproduction is possible with the class A amplifier. It is the class of amplifier most widely used and, for this reason, is the one given particular attention in the text.

In class B operation, the grid bias is approximately equal to the cutoff value so that the plate current is practically zero when no signal voltage is applied. When a sinusoidal signal voltage is applied to the grid, plate current flows for approximately 180 degrees of the cycle and consists of a series of half-sinusoids. The plate efficiency (ratio of output signal power to d-c power supplied to the plate circuit) in the class B amplifier is much higher than that of the class A amplifier. Since plate current flows only during the positive half cycle of the signal voltage, distortion is high. It is not objectionable, however, in radio-frequency power amplifiers since the tuned circuits will eliminate the harmonics from the output. Two tubes can be used in a class B push-pull arrangement to amplify audio-frequency signals. Operated in this manner, one tube supplies the positive half cycles while the other tube supplies the negative half cycles.

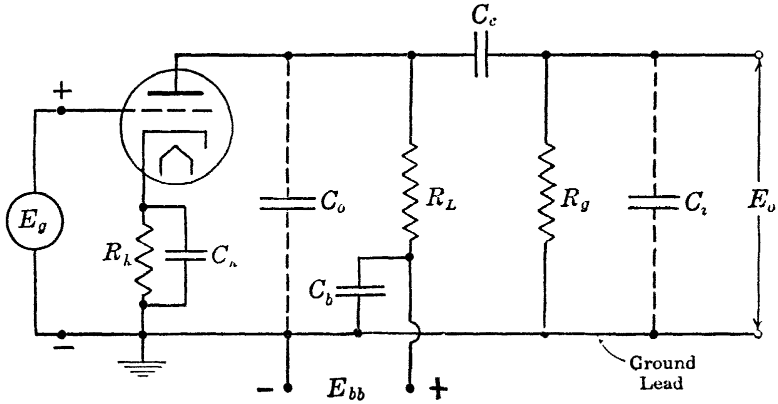
Class AB operation is intermediate to class A and class B. Performance lies then between the two classes of amplifiers, the efficiency being higher than that of class A and the distortion less than that of class B operation.

In class C operation, the grid bias is appreciably greater than the cutoff value so that plate current is zero when no signal is applied. When a signal voltage is applied, plate current flows for only a portion of the positive half cycle. The plate efficiency of this amplifier is higher than that of any of those previously discussed. The class C type is used in radio-frequency power amplifiers and radio-frequency oscillators.

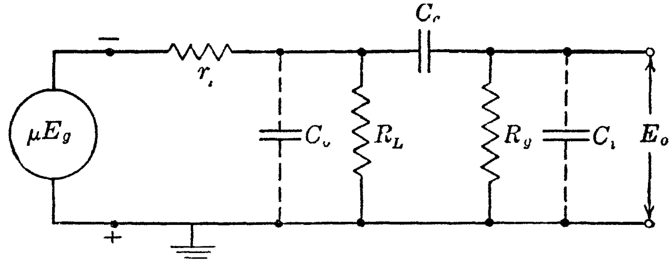
In order to realize a high plate efficiency in class B and class C operation, the grid is driven positive during part of the cycle. As a result, grid current flows and power is required from the source of grid excitation.

16.2. The Resistance-capacitance-coupled Voltage Amplifier. In the application of amplifiers, it is generally necessary to operate several stages of amplification in cascade in order to bring the signal power to the desired level. There are several standard circuit arrangements employed to couple the output of one stage to the input of a following stage. The circuit most widely used for audio-frequency and video-frequency voltage amplifiers employs resistance-capacitance coupling, or, simply, resistance coupling, shown schematically in Figure 16.1a. Since operation is in the linear region of the characteristic curves, μ, g_m, and r_p can be considered as constants, and the tube circuit can be replaced by the equivalent circuit of Figure 16.1b.

The circuit formed by R_k and C_k provides a method used commonly in obtaining grid bias. The d-c component of plate current flowing through R_k makes the grid negative with respect to the cathode by the amount of the I_bR_k drop. The value of R_k can be chosen to give the desired value of grid bias. In order to reduce the voltage which the a-c components of current would otherwise develop across R_k, a *by-pass* condenser is connected in parallel with it. The condenser must have a value of capacitance sufficiently high so that the a-c component of voltage developed across it is negligible at the lowest frequency to be amplified. It is usually of the low-voltage electrolytic type, physically small in size, with a capacitance in the range of 10 to 50 μf.



(a) Circuit of the resistance-capacitance-coupled amplifier. Ground lead is generally connected to amplifier chassis



(b) Equivalent circuit of the resistance-capacitance-coupled amplifier

FIG. 16.1. A single-stage, resistance-capacitance-coupled amplifier and its equivalent circuit.

The condenser C_b provides a low-impedance path between the plate supply circuit and the cathode circuit. It is standard practice in electron-tube circuits to provide these low-impedance paths, in the form of by-pass condensers, so as to confine the a-c components of current to the vicinity of the tube.

The condensers C_o and C_i represent, respectively, the output capacitance of the first tube and the input capacitance of the following tube. They consist of the interelectrode, the tube-socket, and the distributed wiring capacitances and can be neglected except at high frequencies.

The combination of R_L, R_g, and C_c serves to isolate the d-c plate voltage from the grid of the next tube. The capacitive reactance of the *coupling condenser* C_c should be no greater than the value of the *grid-leak* resistance R_g at the lowest frequency to be amplified. The d-c component of plate current flows only through R_L, whereas the a-c component divides between the output impedances in the manner implied by Figure 16.1b.

In the study of the frequency response of the amplifier, the over-all range is generally divided into the following three frequency parts: the *mid-frequency* range where the capacitances C_o, C_i, and C_c can be neglected, the *low-frequency* range where the shunting effects of C_o and C_i can be neglected, and the *high-frequency* range where the reactance of the condenser C_c is small enough so that its effect can be neglected. This division leads to the three equivalent circuits shown in Figure 16.2.

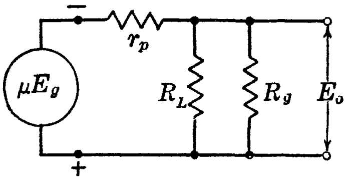
The gain in the mid-frequency range is determined by the tube constants and the values of the resistances in the load circuit of the tube, as indicated by Figure 16.2a. The voltage gain in this frequency range is given by the expression

$$A_{\text{mid}} = \mu \frac{R_{\text{eq mid}}}{r_p + R_{\text{eq mid}}} \tag{16.1}$$

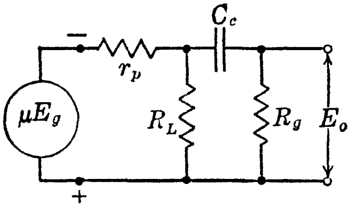
where $R_{\text{eq mid}}$ is the equivalent resistance formed by R_L and R_g in parallel.

At low frequencies, the coupling condenser is of importance, and an analysis of Figure 16.2b will show the gain to be given by the expression

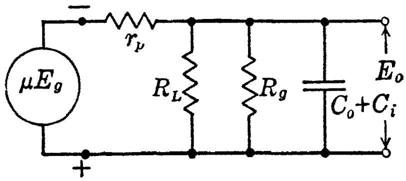
$$A_{\text{low}} = \frac{A_{\text{mid}}}{\sqrt{1 + \left(\dfrac{X_c}{R_{\text{eq low}}}\right)^2}} \tag{16.2}$$



(a) Simplified circuit, accurate at mid-frequencies



(b) Simplified circuit, accurate at low frequencies



(c) Simplified circuit, accurate at high frequencies

FIG. 16.2. Equivalent circuits for the three ranges of a resistance-capacitance-coupled amplifier.

where $X_c = 1/(2\pi f C_c)$ = reactance of the coupling condenser C_c

$$R_{\text{eq low}} = R_g + \frac{R_L r_p}{R_L + r_p}$$

The extent to which the coupling condenser causes the gain of the amplifier to fall with decrease in frequency can be estimated from equation 16.2, which shows that the gain has been decreased to 70.7 per cent at that frequency where the value of capacitive reactance X_c equals the value of the equivalent resistance $R_{\text{eq low}}$.

At high frequencies, the capacitances of the tube electrodes and wiring are the limiting factors in determining the gain that can be obtained. An analysis of Figure 16.2c, aided by the use of Thévenin's theorem, will show the gain at high frequencies to be given by

$$A_{\text{high}} = \frac{A_{\text{mid}}}{\sqrt{1+\left(\dfrac{R_{\text{eq high}}}{X_{oi}}\right)^2}} \qquad (16.3)$$

where $X_{oi} = 1/[2\pi f(C_o + C_i)]$ = reactance of the parallel capacitances

$$R_{\text{eq high}} = \frac{r_p R_L R_g}{r_p R_L + r_p R_g + R_L R_g}$$

Equation 16.3 shows the manner in which the amplification decreases with increase in frequency. The gain has decreased to 70.7 per cent of the mid-frequency response at that frequency where the value of the reactance X_{oi} of the parallel capacitances is equal to the value of the equivalent resistance $R_{\text{eq high}}$.

The gain-frequency characteristic of an amplifier can be expressed by means of a curve that shows the manner in which the amplification varies with the frequency of the signal voltage. A typical frequency-response curve for a resistance-coupled amplifier is shown in Figure 16.3. The constant amplification in the mid-frequency region indicates the range of frequency over which the effects of the capacitances are negligible. The frequencies at which the voltage amplification has decreased to 70.7 per cent of the mid-frequency gain fairly well specify the frequency range for which the amplifier has been designed. The lower and upper frequencies at which the gain has decreased to 70.7 per cent are shown in Figure 16.3. Frequency, plotted as the abscissa, is practically always on a logarithmic scale. The logarithmic distribution corresponds to the musical scale and indicates approximately the manner in which the ear judges changes in frequency.

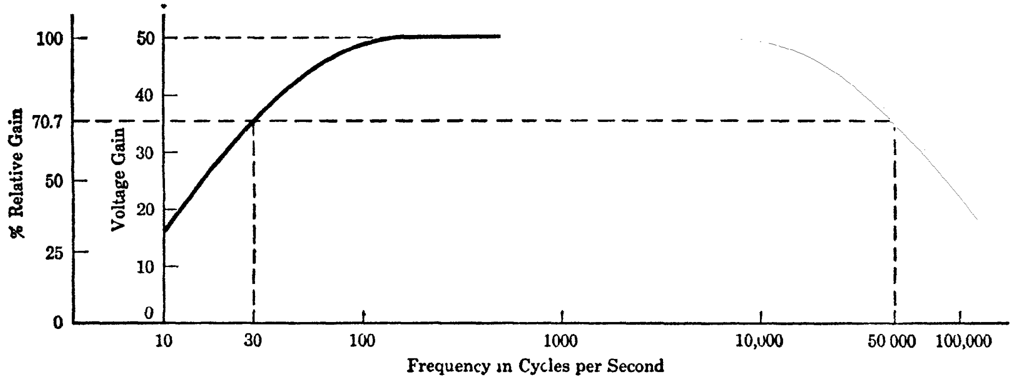


FIG. 16.3. Typical frequency-response curve of a resistance-coupled amplifier.

16.3. The Decibel. The ear also judges changes of sound intensity in a manner approximately logarithmic rather than linear. Since the output of an audio-frequency amplifier is ultimately used to present sound waves to the ear, the variation of gain as a function of frequency is often shown by logarithmic ratios plotted to a vertical linear scale. The logarithmic unit used for this purpose is called the *decibel,* abbreviated *db,* and is defined in character by the equation

$$\text{db} = 20 \log_{10} \frac{E_2}{E_1} \tag{16.4}$$

where E_2 and E_1 are two voltages to be compared in relative magnitude.

The decibel, as originally conceived, was a logarithmic unit for the comparison of two values of power, but common practice is to use it for the comparison of a wide variety of quantities, for example, voltages, currents, sound pressures, sensitivities of microphones, electric field intensities, etc. When used to compare two values of power, P_1 and P_2, the ratio of magnitudes in decibels is

$$\text{db} = 10 \log_{10} \frac{P_2}{P_1} \tag{16.5}$$

It is evident that equations 16.4 and 16.5 are identical when E_2 and E_1 are measured across the same value of resistance.

In using the decibel, as expressed by equation 16.4, to show the gain of an amplifier, E_1 is the value of the input signal voltage and E_2 is the value of the output signal voltage. For the purpose of expressing the frequency-response of an amplifier, the gain in the mid-frequency range can be taken as a reference, the gains at other frequencies then being stated relative to the gain at the reference frequency. In this manner of indicating the frequency response, E_1 is the voltage gain at the reference frequency and E_2 is the voltage gain at the frequency being considered. Figure 16.4 shows the curve of Figure 16.3 replotted, where the ordinate is the relative gain in decibels. A decrease in voltage amplification to 70.7 per cent corresponds to a 3-db loss in amplification from the gain at the mid-frequency. The frequencies at which the gain has decreased by

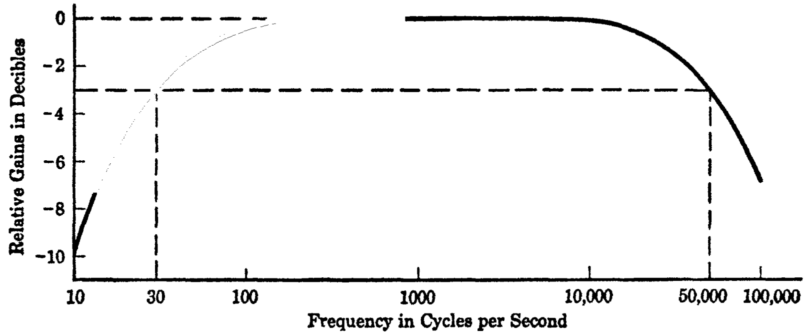
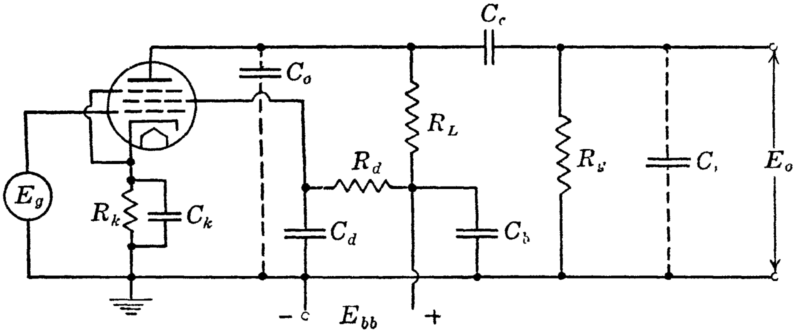


FIG. 16.4. Typical frequency-response curve of a resistance-coupled amplifier. Gain is plotted in decibels relative to the mid-frequency gain.

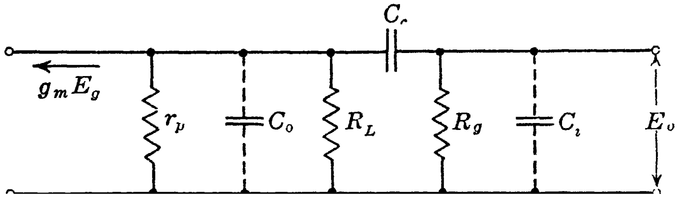
3 db are indicated in Figure 16.4, these frequencies corresponding to the points of 70.7 per cent gain of Figure 16.3.

16.4. The Resistance-coupled Amplifiers Utilizing Pentodes. A higher voltage amplification can be obtained in resistance-coupled amplifiers by the use of a pentode tube in place of the triode, and, because of the increased amplification obtainable, pentodes have largely superseded triodes in audio and video voltage amplifiers. When the same values of circuit constants are used, the frequency response of the pentode amplifier is practically the same as that of the triode amplifier.

The circuit connections of a resistance-coupled amplifier utilizing a pentode tube are indicated in Figure 16.5a. The d-c screen-grid voltage is taken from the power-supply voltage E_{bb} through the resistor R_d. The screen-grid voltage differs from E_{bb} by the drop produced by the d-c component of screen-grid current flowing through R_d, and the value of R_d is chosen so as to obtain the proper value of screen voltage. For best performance of the amplifier, it is essential that the screen voltage does not change when a signal is applied. This is assured by the by-pass condenser C_d, which provides a low-impedance path for the flow of the a-c component of screen-grid current.



(a) Circuit of the pentode resistance-coupled amplifier



(b) Parallel form of the equivalent circuit of a pentode resistance-coupled amplifier

FIG. 16.5. A single-stage pentode resistance-coupled amplifier and the parallel form of its equivalent circuit.

The equivalent circuit, using the parallel form, is shown in Figure 16.5b. It should be emphasized that either the equivalent series circuit or the equivalent parallel circuit can be used to represent an amplifier utilizing either a triode or a pentode type of tube. In general, however, it is more convenient to use the series form, involving μ and r_p, with the triode, and the parallel form, involving g_m and

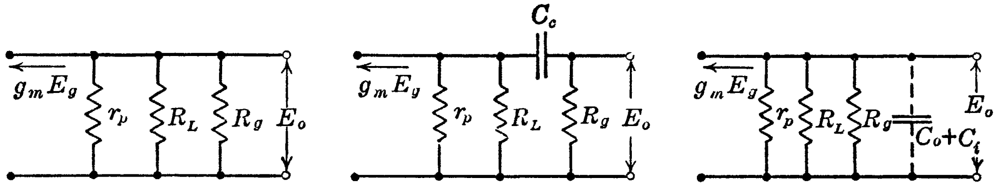
r_p, with the pentode. In fact, the tube manufacturer seldom states the value of μ for a pentode type of tube. Since the value of r_p is generally much larger than that of R_L and R_g, it often is omitted from the equivalent circuit.

The three equivalent circuits that apply to the mid-frequency range, the low-frequency range, and the high-frequency range are shown in Figure 16.6.

The voltage gain in the mid-frequency range is given by

$$A_{\text{mid}} = g_m R_{\text{eq}} \tag{16.6}$$

where R_{eq} is the equivalent resistance formed by r_p, R_L, and R_g all in parallel.



(a) Accurate at mid-frequencies. (b) Accurate at low frequencies. (c) Accurate at high frequencies.

FIG. 16.6. Simplified equivalent circuits, using the parallel form, for the three ranges of a pentode resistance-coupled amplifier.

The student should satisfy himself that equation 16.6 is but a different form of equation 16.1.

The voltage amplifications at low frequencies and at high frequencies, in terms of the mid-frequency amplification, are given by equations 16.2 and 16.3, respectively.

Suitable circuit constants for a variety of conditions and tube types, including both triodes and pentodes, are published in tube handbooks that are available from the tube manufacturers.

16.5. Amplifiers Employing Audio-frequency Transformers. The principal application of an audio-frequency transformer is as an impedance-matching device. In many installations, a vacuum tube receives its signal voltage from a low-impedance circuit and delivers power to a low-impedance load. A vacuum tube is essentially a high-impedance device, and efficient transfer of power can only be obtained by properly matching the tube to the driving-source impedance and to the output load impedance.

The use of input and output transformers is indicated in Figure 16.7. Examples of various types of low-impedance sources are magnetic types of phonograph pick-ups, dynamic and ribbon microphones, and audio-frequency transmission lines. The turn ratio of the input transformer is generally chosen to match 200,000 ohms on the secondary side. Since the impedance is transformed by the square of the turn ratio, a transformer connecting a 500-ohm line to the grid circuit of a tube would have a step-up turn ratio of 20. The signal voltage also is stepped up, of course, by the turn ratio.

The output tube of Figure 16.7 would act as a *power* amplifier. For proper performance, the a-c load presented to the tube should be the value specified by the tube manufacturer, which normally will be of the order of 5,000 ohms. The load might possibly be a 200-ohm or 500-ohm line or a dynamic loud-speaker having a resistance in the range of 2.5 to 50 ohms. As an example, a transformer having a step-down turn ratio of 14.1 would make a 25-ohm voice-coil impedance of a speaker appear as 5,000 ohms viewed from the primary side.

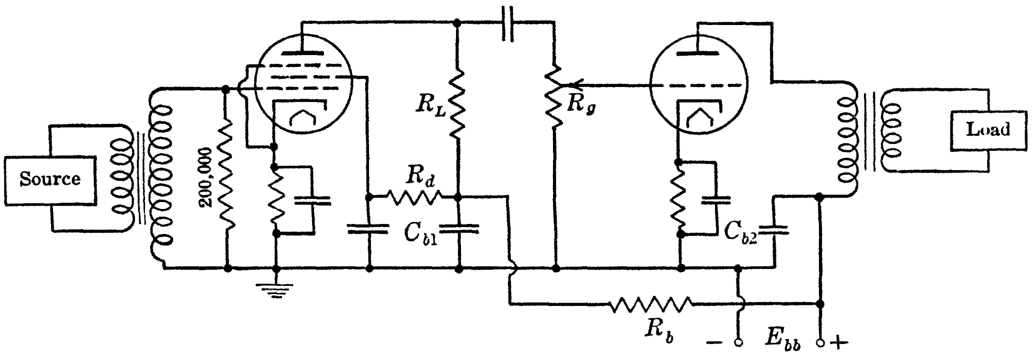


FIG. 16.7. A 2-stage audio-frequency amplifier showing the use of impedance-matching transformers.

When several amplifier stages are operated in cascade, as in Figure 16.7, it is important that a-c components of current do not flow through an impedance that is common to the several stages. If this occurs, voltage from the output stage will be fed back to the first stage and the amplifier may oscillate, with the result that it becomes unusable. One common impedance is the power supply for the d-c voltages to all the stages. By-pass condensers connected across the power supply are effective in some instances, but, if the amplification is high, additional isolation of the stages is necessary. This can be accomplished by resistance-capacitance filters, one such being formed by C_{b2}, R_b. and C_{b1}.

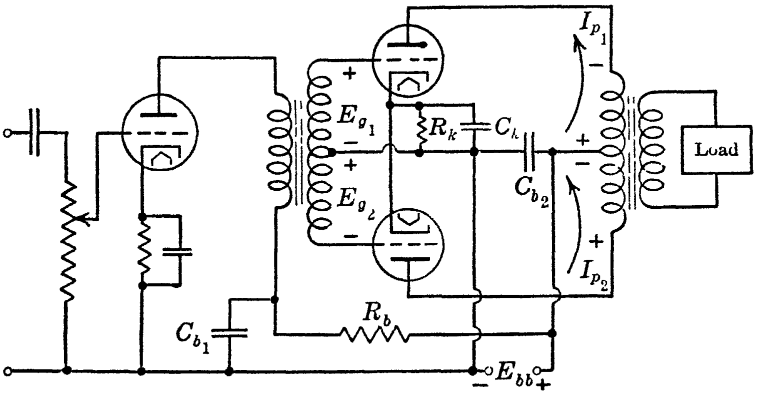


FIG. 16.8. The application of audio-frequency transformers in push-pull amplifiers.

Transformers are also useful in providing the proper grid signals to a *push-pull* power amplifier, see Figure 16.8. As discussed in a later section, push-pull operation reduces the distortion of the signal caused by the non-linear tube characteristic. In push-pull operation, the signal voltages applied to the two grids are 180 degrees out of phase, as indicated by the polarity markings in the figure. The outputs of the individual tubes are combined by the output transformer and delivered to the load.

An audio-frequency transformer also can be used to couple two stages of a voltage amplifier, although this type of service has been largely superseded by resistance coupling. These *interstage* transformers are costly, increase the weight of the amplifier, and give a poorer frequency response than resistance coupling. It is general practice, therefore, to use them only when special requirements suggest certain advantages.

Because of the wide frequency range over which audio-frequency transformers are required to operate, the design features are quite different from those of power transformers. Leakage inductance and the distributed capacitance of the winding must be kept to a minimum if a uniform frequency response to is be obtained. Commercial audio-frequency transformers, which have a satisfactory frequency response, are available for various types of service and for practically all power ratings.

16.6. Radio-frequency Voltage Amplifiers. In a radio-frequency amplifier the load often is supplied by a parallel resonant circuit, as shown in Figure 16.9a. Not only do pentodes give a higher value of amplification per stage than triodes, but the undesirable tube-capacitance coupling between the input and output circuits is negligible when these tubes are employed. For these reasons, pentode tubes are practically always used in r-f voltage amplifiers.

The gain of the amplifier will be high for frequencies in the neighborhood of the resonant frequency of the load circuit, and low for frequencies outside of that region. This characteristic makes it possible to select easily any particular broadcast station by a radio receiver.

The equivalent circuit of the amplifier is shown in Figure 16.9b. The capacitive reactance of C_c is negligible at radio frequencies, and hence C_c need not be included in the equivalent circuit. The stray capacitances, considered in the case of the resistance-coupled amplifier, now provide part of the tuning capacitance. Since the plate resistance of the pentode and the grid-leak resistance R_g are generally high, the equivalent circuit can be reduced to the approximately equivalent circuit of Figure 16.9c. From this latter circuit, it is apparent that the voltage gain varies with frequency in the same manner as does the impedance Z_L of the parallel circuit and is given by the expression

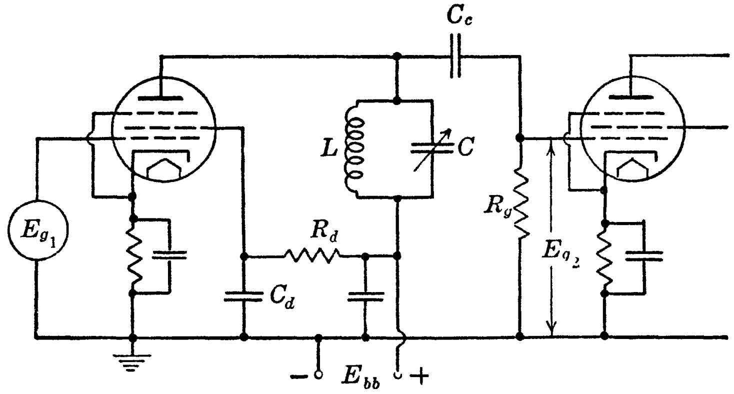
$$A_v = g_m Z_L \qquad (16.7)$$

The gain at resonance can be shown to be approximately

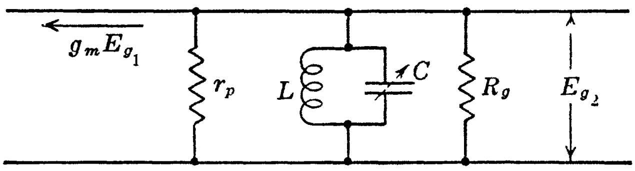
$$A_v \simeq g_m Q \omega_r L, \text{ gain at resonance} \qquad (16.8)$$

where ω_r is 2π times the resonant frequency, L is the value of inductance of the coil, and Q is its figure of merit, discussed in Chapter III.

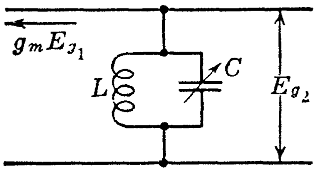
The frequency response of the amplifier near resonance is shown in Figure 16.10. When a radio-frequency *carrier wave* carries a program of speech or music, there are *side-band* frequencies present that extend both above and below the carrier frequency. In A-M broadcasting there are important frequencies extending 5,000 cycles on each side of the carrier. In order to amplify these side-band frequencies, the selectivity of the tuned amplifier, controlled by the proper choice of the Q of the coil, should not be too sharp. It is convenient to express the band width of the amplifier, $f_2 - f_1$ of Figure 16.10, as the frequency range over which the gain does not fall below 70.7 per cent of the maximum gain. An



(a) A radio-frequency amplifier using a parallel resonant circuit



(b) Parallel form of the equivalent circuits of (a)



(c) Simplified equivalent circuit accurate when r_p and R_g are very large

FIG. 16.9. A radio-frequency amplifier using a parallel resonant circuit, and the associated equivalent circuits.

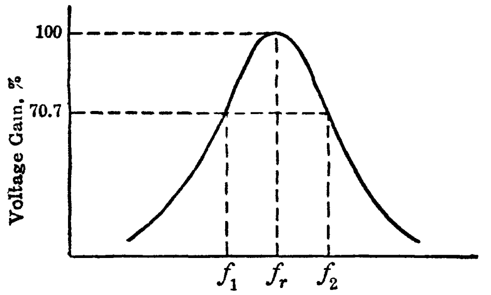


FIG. 16.10. Typical frequency response of a radio-frequency amplifier using a single tuned circuit.

analysis of the equivalent circuit will show that the band width can be given by the equation

$$f_2 - f_1 = \frac{f_r}{Q} \text{ cycles} \tag{16.9}$$

Thus, for a circuit having a Q of 100 and tuned to a frequency of 10^6 cycles, the band width $f_2 - f_1$ would be 10,000 cycles, which would provide fairly well for the amplification of the side bands.

Another type of tuned amplifier, and one quite commonly used, employs transformer-coupling, see Figure 16.11. The characteristics of this amplifier are

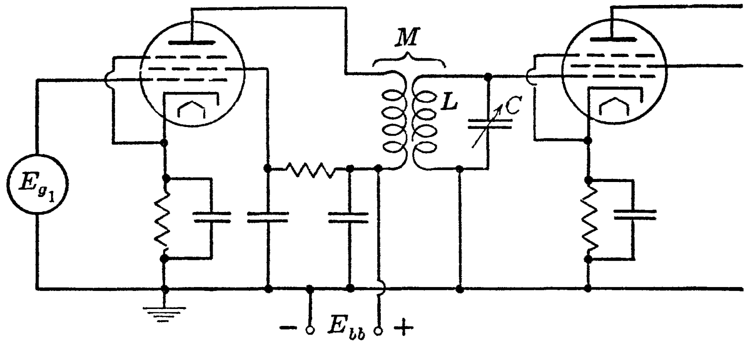
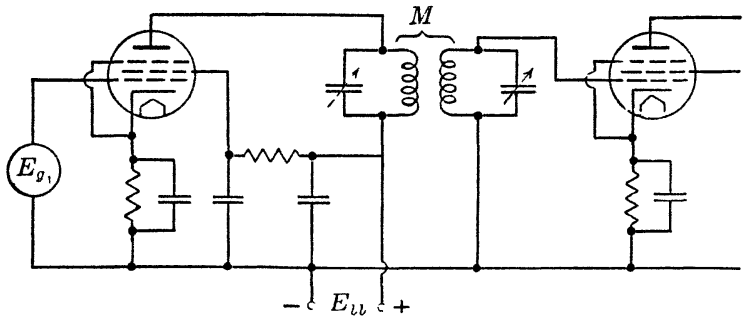
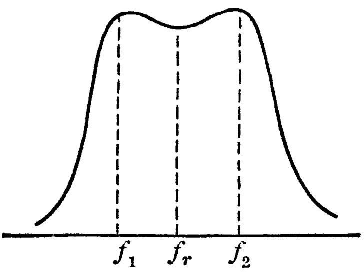


FIG. 16 11. The transformer-coupled radio-frequency amplifier.



(a) Band-pass amplifier



(b) Frequency-response curve of a band-pass amplifier

FIG. 16 12. A band-pass amplifier with typical amplification characteristic.

quite similar to those of the parallel-resonant-load type. The gain at the resonant frequency can be shown readily to be

$$A_r = g_m\omega MQ, \text{ gain at resonance} \qquad 16.10$$

where M is the value of mutual inductance between the two coils, Q is the figure of merit of the secondary coil, and the other quantities are as previously defined. The details of analysis of the transformer-coupled amplifier are left as a problem for the student.

In order to provide an amplification substantially constant for all the essential side-band frequencies of a broadcast wave, the curve of frequency response should have a band-pass characteristic of the type shown in Figure 16.12b. The band-pass characteristic can be obtained by tuning the primary of the radio-frequency transformer as well as the secondary, Figure 16.12a, the method most generally used in the intermediate-frequency amplifiers of superheterodyne radio receivers. Both primary and secondary are tuned to the same frequency f_r, and the coupling coefficient k is adjusted to give the desired value of band width. The band width $f_2 - f_1$ is approximately equal to kf_r. A detailed analysis of the double-tuned circuit can be found in most of the textbooks on radio engineering.

16.7. The Class A Audio-frequency Power Amplifier. In the preceding discussions on voltage amplifiers, no consideration was given to the signal-power output, since it was but incidental to the desired voltage amplification. When power output is not a factor, both the d-c and a-c components of plate current have very small values, operation is confined to a relatively small region of the characteristic curves of the tube, and distortion of the signal is kept to a very low value. In contrast, the power required for operating a loud-speaker, for modulating a radio transmitter, or for some other purpose is the most important consideration in the design of the power amplifier. The d-c power input to the plate circuit, I_bE_{bb}, must be sufficient to supply the desired value of output power. Actually, the signal voltage applied to the grid of the amplifier *converts* part of the d-c plate-circuit power into output signal power, rather than that the tube amplifies, in the strictest sense of the word, the signal power. As previously mentioned, the ratio of the signal-power output to the d-c power supplied to the plate circuit is the plate-circuit efficiency of the amplifier. In order to obtain the largest power output and the highest plate-circuit efficiency for a given value of plate-supply voltage, it is necessary that operation be over a large portion of the characteristic curves. This means that the a-c components of grid voltage, plate current, and plate voltage are large and of the same order of magnitude as the d-c components.

Either triodes, pentodes, or beam-power tubes can be used in power amplifiers. Triodes are characterized by low power sensitivity (the a-c component of grid voltage required for a stated value of power output), low plate-circuit efficiency (about 20 per cent), and low distortion of the signal. Pentodes have a higher value of power sensitivity and a higher plate-circuit efficiency (about 35 per cent) but, in general, introduce more objectionable distortion than triodes.

Beam-power tubes have a somewhat higher plate-circuit efficiency than pentodes, and, hence, are able to deliver a greater power output for a given d-c power input and for a particular size of tube than either triodes or pentodes. For this reason, beam-power tubes are more commonly used in commercial power amplifiers than are the other types. Triodes, however, are still desirable as power amplifiers in high-fidelity systems, when very low distortion is a requirement.

The method of analysis of the power amplifier is much the same for either the triode, pentode, or beam-power tube. In this section, the triode will be used to outline the considerations involved. Although the equivalent circuit also applies to the class A power amplifier, it alone does not indicate the limits of operation, the factors which determine the choice of the best operating point on the characteristic curves, and the optimum value of load resistance to be used. For these reasons, the analysis is made directly from the plate-current plate-voltage family of curves.

Figure 16.13 shows the plate characteristics of a type 2A3 triode. The analysis involves the problem of determining, for a stated value of plate-supply voltage,

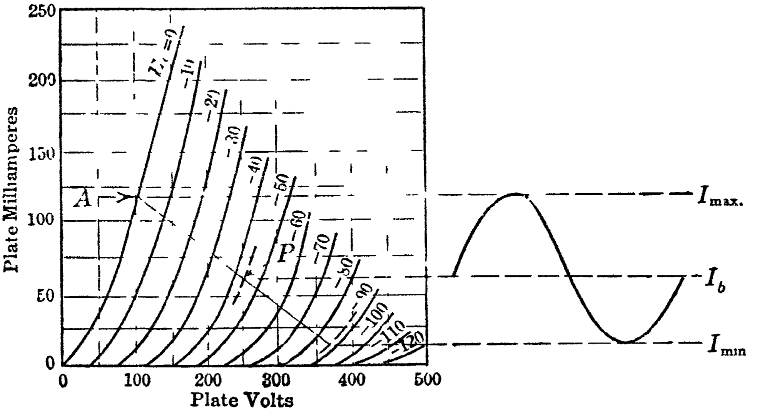


Fig. 16.13 Plate characteristics of the 2A3 triode showing load line and wave shape of plate current. (*Courtesy* RCA.)

the values of d-c grid bias and a-c load resistance that will provide for the greatest output power, consistent with the following limits: the grid is not to be driven positive; to minimize distortion, the tube is not to be operated in the immediate region of zero plate current where the curvature of the characteristic curves is high; and the d-c power input to the plate circuit must not exceed the plate-dissipation rating of the tube. The RCA tube manual suggests the following procedure in analyzing triode power amplifiers: (1) Having decided upon the value of plate voltage E_b, the value of grid-bias voltage E_c is given by the empirical expression

$$E_c = -\frac{0.7E_b}{\mu} \tag{16.11}$$

where μ is the amplification factor of the triode. The values of E_c and E_b set the operating point P and give the value of the d-c plate current I_b. (2) Check to see that the d-c plate power E_bI_b does not exceed the plate-dissipation rating of the tube. If it does, it will be necessary to increase the d-c bias or decrease the plate voltage. (3) Locate a point, A, on the curve for $E_c = 0$ corresponding to a value of plate current equal to $2I_b$. A straight line drawn from A through P gives the load resistance line and the reciprocal of its slope gives the value of the load resistance to be used with the tube.

The RCA tube manual gives the following data for typical operation of the 2A3:

Plate-dissipation rating	15 max watts
Plate voltage	250 volts
Grid voltage	−43 5 volts
Plate current	60 ma
Amplification factor	4 2
Plate resistance	800 ohms
Transconductance	5 250 micromhos
Load resistance	2 500 ohms
Power output	3 5 watts
Second-harmonic distortion . . .	5 per cent

Equation 16.11 would give a grid bias of −41.7 volts. In order not to exceed the plate-dissipation rating of the tube, it is necessary to increase the bias to −43.5 volts. When no signal is applied to the tube, all the d-c power supplied to the plate circuit must be dissipated by the plate, which, for these conditions of operation, amounts to

$$\begin{aligned}\text{plate dissipation} &= E_bI_b\\ &= 250 \times 0\,060\\ &= 15 \text{ watts}\end{aligned}$$

When a sinusoidal voltage of 43.5 volts peak value is applied to the grid circuit, operation is along the load line from $E_c = 0$ to $E_c = -87$; the plate-voltage variation is from 105 volts to 370 volts, with a corresponding variation in plate current of 120 ma to 12 ma. The total variation of plate voltage, 370 − 105 = 265 volts, is equal to twice the peak a-c voltage variation across the load. The effective value of the a-c component of load voltage is, therefore, given by

$$E_L = \frac{265}{2\sqrt{2}} = 93\,5 \text{ volts}$$

The same is true of $I_{\max} - I_{\min}$ of Figure 16.13, and hence the effective value of the a-c component of load current is equal to

$$I_L = \frac{(120 - 12)}{2\sqrt{2}} = 38 \text{ milliamperes}$$

The output signal power is

$$P = E_LI_L = (93.5)(0.038) = 3.55 \text{ watts}$$

which agrees well with the value listed by the tube manufacturer.

The plate-circuit efficiency is given by

$$\text{plate-circuit eff.} = \frac{\text{a-c output}}{\text{d-c input}} \times 100\%$$

$$= \frac{E_L I_L}{E_b I_b} \times 100\%$$

$$= \frac{3.55}{15} \times 100\% = 23.7\%$$

The variation of plate current that results from a sinusoidal grid signal voltage is shown in Figure 16.13. It is to be noted that the variation is not a pure sinusoid about the value I_b and means that harmonics of the signal frequency are present in the output. Any periodic wave can be expressed mathematically as the sum of a fundamental frequency and a series of harmonics. In general, if a sine wave is distorted, it can be expected that both even and odd harmonics will be present. If the distortion is of the type indicated in Figure 16.13, where the upper and lower halves of the wave are not similar, the distortion is due mainly to the second-harmonic term. It can be shown that the per cent of second-harmonic distortion is given approximately by

$$\% \text{ 2nd harmonic} = \frac{I_{\text{max}} + I_{\text{min}} - 2I_b}{2(I_{\text{max}} - I_{\text{min}})} \times 100\% \tag{16.12}$$

For the operating conditions of the 2A3, and using values from the characteristic curves, equation 16.12 gives 5.6 per cent.

Two matched tubes operated in the push-pull circuit of Figure 16.8 will insure a symmetrical wave form for the load current. This is obvious from the fact that as one tube is supplying the top half of the wave shown in Figure 16.13 the second tube is supplying a current to the load in the same direction as that from the first tube, and with a wave shape similar to the bottom half in Figure 16.13. The currents from the two tubes are combined in the transformer, as indicated in Figure 16.14. By making the wave symmetrical, all even-order harmonics are removed, thereby leaving only the odd harmonics which, for the triode, are usually quite small.

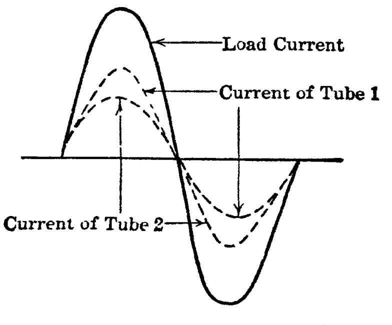


FIG. 16.14. Tube currents in a push-pull circuit combine to give a symmetrical load current.

The method of determining the optimum operating conditions and the power output for a pentode or beam-power tube is much the same as with a triode, although differing in the details of locating the load line on the plate characteristics. It is necessary to locate the load line on the characteristic curves by trial so that maximum power output can be obtained consistent with low distortion and the power-handling ability of the tube The graph of plate characteristics of a 6V6 beam-power tube with the proper load line is shown in Figure 16.15.

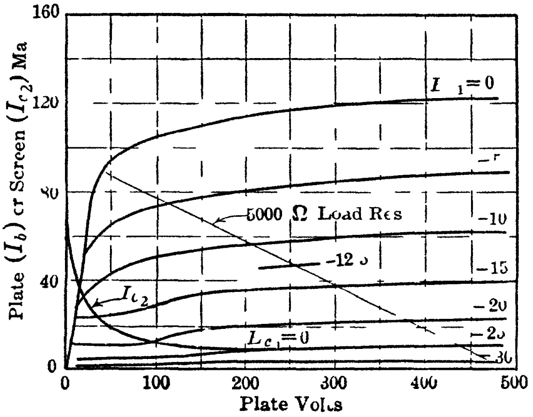


Fig. 16.15 Plate characteristics of the 6V6 beam-power amplifier tube showing optimum load line (Courtesy RCA)

The RCA tube manual gives the following data for typical operation of the 6V6:

Plate-dissipation rating	12 max watts
Screen-dissipation rating	2 max watts
Plate voltage	250 volts
Screen voltage	250 volts
Grid voltage, E_{c1} .	−12.5 volts
Zero-signal plate current	45 ma
Max-signal plate current	47 ma
Zero-signal screen current	4.5 ma
Max-signal screen current	7 ma
Plate resistance	52,000 ohms
Transconductance	3,700 micromhos
Load resistance	5,000 ohms
Max power output	4.5 watts
Total harmonic distortion	8 per cent

It is suggested that the student calculate from the characteristic curves as many tabulated values as are possible and compare them with the similar quantities that pertain to the 2A3 triode

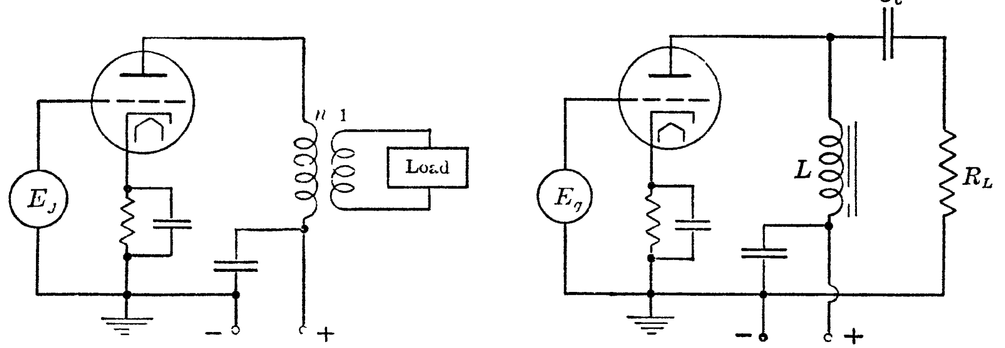
In the power amplifier, the d-c plate voltage is not supplied through the a-c load resistance If this were done, there would be an unnecessary loss of d-c power in the load resistance For example, in the case of the 2A3 triode, a plate-supply voltage of 400 volts would be required in order to supply 250 volts to the plate of the tube through a resistance of 2,500 ohms which carries a d-c plate current of 60 ma. The load resistance is generally connected in the plate circuit by means of a transformer, as shown in Figure 16.16a.

With this arrangement the d-c current flows through the primary winding of the transformer, and since the d-c resistance of the winding is low the voltage from plate to ground with no signal present is approximately equal to the plate-supply voltage. For a-c components of plate current, however, the load resistance in the plate circuit is given by

$$R_L = n^2 R_{\text{sec}} \tag{16.13}$$

where n is the turn ratio of the transformer, and R_{sec} is the resistance connected to the output of the transformer.

When it is not necessary to match the load impedance to the tube by means of a transformer, the *shunt-feed* arrangement of Figure 16.16b is sometimes used.



(a) Transformer-coupled load (b) Shunt-feed method of supplying plate voltage

FIG. 16.16. Methods of connecting the load resistance to a power amplifier.

The inductance L is chosen with a low value of d-c resistance, and an inductive reactance to the a-c components high in comparison with R_L. The coupling condenser should be of such size as to offer negligible impedance to the a-c components of current. Under these conditions, the a-c load resistance reduces to R_L, and the value of d-c voltage from plate to ground is equal to the plate-supply voltage.

Since the grid bias is usually supplied from the plate-voltage supply by means of the cathode resistor R_k, the value of supply voltage should equal the sum of the magnitudes of the d-c plate voltage E_b and the d-c grid voltage E_c.

16.8. Negative-feedback Amplifiers. The performance of an amplifier can be improved decidedly if a portion of the output signal voltage is applied to the input of the amplifier in phase opposition to the input signal voltage. This arrangement for *negative* feedback, sometimes called *degenerative* feedback, improves the frequency response of the amplifier, reduces the distortion introduced by the non-linear tube characteristics, reduces hum voltages introduced by the plate-voltage supply and the a-c heater voltage, and makes the value of amplification substantially independent of small changes of d-c supply voltage, tube replacements, and aging of tubes and circuit elements. The principle of negative feedback has been widely used to improve the quality in electronic systems of all descriptions.

The basic scheme of connections of a negative-feedback amplifier is indicated in Figure 16.17. The relative polarities of the various voltages are shown on the diagram. If R_1 is made zero, there is no feedback voltage present, E_g (the amplifier input voltage) is equal to the signal voltage E_s, and E_0 is given by

$$E_0 = AE_g = AE_s \tag{16.14}$$

where A is the gain of the amplifier labeled *amplifier unit* in Figure 16.17. If a fraction β of the output voltage is developed across R_1, and the relative polarities are as indicated, the voltage input E_g to the amplifier unit bears the following relation to the signal and output voltages:

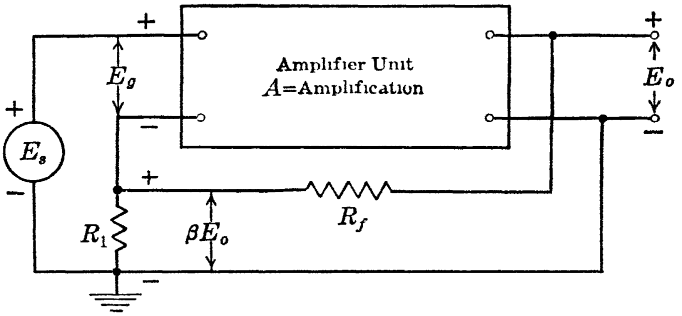


FIG. 16.17. Basic circuit elements of a negative-feedback amplifier.

$$E_g = E_s - \beta E_0 \tag{16.15}$$

where

$$\beta = \frac{R_1}{R_1 + R_f} \tag{16.16}$$

and is the *magnitude* of the ratio of feedback voltage to output voltage. Since the input and output voltages of the amplifier unit are always related by the equation

$$E_0 = AE_g \tag{16.17}$$

whether feedback is present or not, equation 16.15 can be written

$$E_0 = AE_s - \beta AE_0$$

or

$$E_0 = \frac{A}{1 + \beta A} E_s \tag{16.18}$$

Equation 16.18 shows that the over-all amplification $A/(1 + \beta A)$ has been reduced by the factor $1/(1 + \beta A)$ because of negative feedback. This reduction in gain is a disadvantage that is more than offset by the improved performance obtained and can be overcome easily by using an extra stage of amplification. When the product βA is large in comparison with unity, the voltage amplification with feedback A_{fb} is given approximately by the equation

$$A_{\mathrm{fb}} \simeq \frac{1}{\beta} = \frac{R_1 + R_f}{R_1} \tag{16.19}$$

Thus, to the extent that βA is greater than unity, the amplification with feedback is independent of the characteristics of the original amplifier unit and depends only upon the values of R_1 and R_f.

As an example, the amplifier unit might consist of two stages of amplification using 6SJ7 pentodes and have a value of A equal to 5,000. Suitable values of R_1 and R_f would be 1,000 ohms and 250,000 ohms, respectively. These quantities give a value of

$$\beta A = 19.9$$

and a value of amplification equal to

$$A_{\mathrm{fb}} = \frac{5{,}000}{1 + 19.9} = 239$$

$$\simeq \frac{1}{\beta}$$

The extent to which the gain is independent of the characteristics of the amplifier unit can be judged by assuming that, for some reason, the value of A decreases to 4,000. The gain would then become

$$A_{\mathrm{fb}} = \frac{4{,}000}{1 + 15.9} = 237$$

a change of 1 per cent.

The decrease in the distortion introduced by the amplifier results from the fact that the distortion is fed back to the input of the amplifier in such a phase relation that, after it is amplified and reappears in the output, it tends to cancel the original distortion. It can be shown that, for the same value of power output, negative feedback reduces the distortion by the factor $1\ (1 + \beta A)$. For the example just considered, the distortion is reduced to $1/20.9 = 4.8$ per cent of what it would have been without feedback.

16.9. Vacuum-tube Oscillators. As a result of its ability to amplify, a vacuum tube is able to generate sinusoidal alternating-current power. When a fraction of the voltage developed in the output circuit of an amplifier is used to supply the input voltage of that amplifier, the device becomes a self-excited amplifier and is termed an *oscillator*.

The conditions necessary to produce self-sustained oscillations can be understood by reference to Figure 16.17 and equation 16.18. If oscillations are to be produced, the phase of the feedback voltage must be reversed from what it was in the negative-feedback amplifier. The feedback now becomes *positive*, or *regenerative*, and the sign of β in equation 16.18 becomes negative. The equation is then written as

$$E_0 = \frac{A}{1 - \beta A} E_s \tag{16.20}$$

where β is the *magnitude* of the ratio of the positive-feedback voltage to the output voltage.

When the amount of feedback is sufficient to make βA unity, equation 16.20 indicates that the net gain of the amplifier has become infinity, and that a large output voltage will be present regardless of how infinitesimally small the signal voltage may be. When βA is unity, the feedback voltage βE_0 is equal in magnitude to, and is in phase with, the original signal voltage E_s. Under these conditions, the original signal voltage can be removed and the amplifier is capable of supplying its own input, thus maintaining a voltage output. If the feedback is increased until βA is greater than unity, the feedback voltage becomes greater than that needed just to sustain oscillation, and the magnitude of oscillation will steadily increase until limited by the non-linear characteristics of the tube, unless some sort of automatic volume control is provided. In order to maintain a constant amplitude of oscillation and a low distortion, some sort of automatic regulating arrangement always should be included in an oscillator circuit. When βA is equal to, or greater than, unity, it is not necessary to supply a small signal voltage to initiate oscillation; there are always present minute surges of tube currents that will assure the start of oscillation. The necessary and sufficient conditions for maintaining oscillation may be summarized as follows: first, the over-all amplification, including the feedback circuit, must be equal to, or greater than, unity; second, the amplified voltage reintroduced into the input must be in phase with the input voltage.

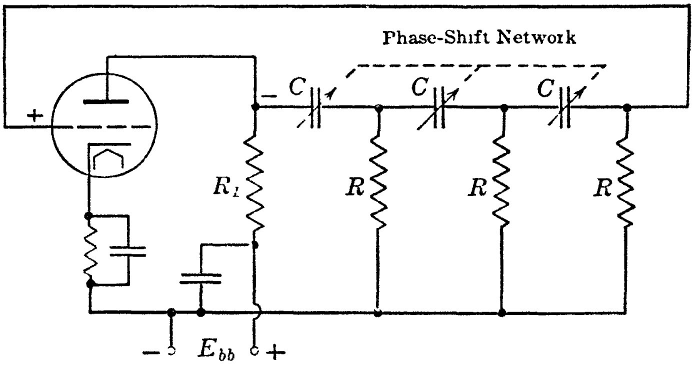
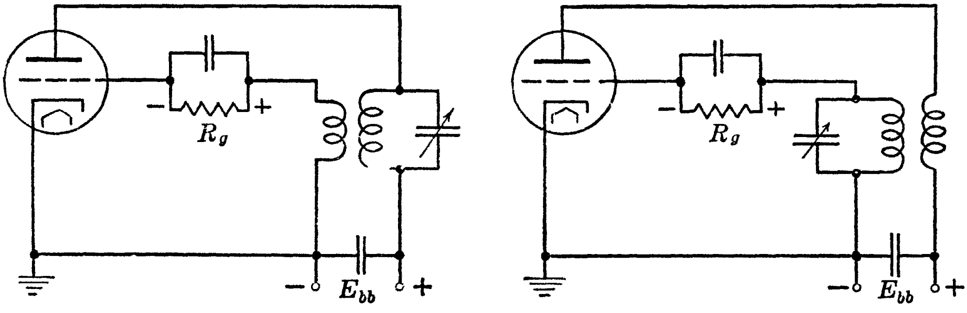


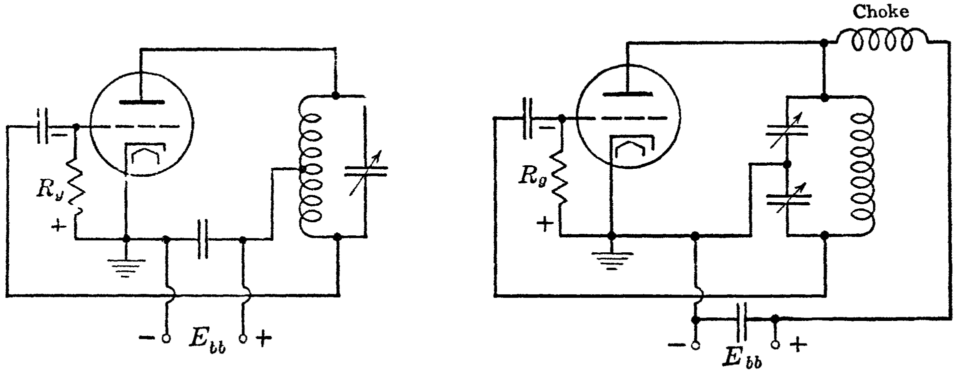
FIG. 16.18. Schematic circuit of a phase-shift oscillator.

A schematic circuit of a basic feedback oscillator is shown in Figure 16.18. It is called a phase-shift oscillator and represents the simplest type of feedback circuit. When provided with automatic volume control, it is a very satisfactory audio-frequency oscillator. The value of R_L is usually made small in comparison with that of R. Under this condition, the a-c component of plate voltage is 180 degrees out of phase with that of the grid voltage, as indicated by the polarity markings on the circuit. The phase-shifting network introduces the additional 180° phase shift that is necessary to produce oscillation. The frequency at which the phase shift of 180 degrees is obtained can be shown to be



(a) Tuned-plate oscillator

(b) Tuned-grid oscillator



(c) Hartley oscillator. Tap on coil of tuned circuit provides means of obtaining feed-back voltage

(d) Colpitts oscillator. Feed-back voltage obtained through split-condenser circuit. The choke has a high impedance to r-f currents

Fig. 16.19. Some typical oscillator circuits.

$$f = \frac{1}{2\pi\sqrt{6}RC} \tag{16.21}$$

Using a variable air condenser, a frequency range of 10 to 1 can be obtained easily with one rotation of the dial. The frequency range can be extended further by using decade steps in the value of R. There are several commercial types of audio-frequency oscillators that use resistance-capacitance tuning, all of them operating under linear (class A) conditions.

Many oscillators—practically all radio-frequency oscillators—use a resonant circuit, an example of which is the tuned-plate oscillator of Figure 16.19a. The frequency of oscillation is practically equal to the resonant frequency of the tuned parallel circuit. It is the frequency at which the voltage amplification is a maximum and at which the amplified voltage induced in the grid coil has the correct phase relation. When power output is a major consideration, oscillators of this type are operated as class C so that the highest possible plate efficiency is attained: the grid is driven positive and d-c grid current flows through the resistance R_g. The d-c voltage developed by the grid current flowing through R_g becomes the grid bias of the tube and acts in a manner to self-regulate the magnitude of oscillation. Regulation results from the fact that if the amplitude of oscillation tends to increase, the grid current increases, which, in turn, increases the d-c bias on the tube. An increase in grid bias decreases the amplification and, hence, tends to decrease the magnitude of oscillation. If the oscillation tends to decrease, the reverse action occurs.

The tuned circuit can just as well be placed in the grid circuit, as shown in Figure 16.19b. Often the tuned circuit is common to both the grid and plate circuits, as indicated in Figures 16.19c and 16.19d, which bear the names of their inventors.

16.10. Applications of Oscillators. Vacuum-tube oscillators have a wide range of application in both the fields of communications and industry. They are capable of generating frequencies from a fraction of a cycle per second to several billion cycles per second. Their use extends from circuits in which the power requirement is only a few milliwatts to applications in which the radio-frequency power demand is several hundred kilowatts.

In the communications field, the oscillator is the heart of the radio transmitter. Low-power radio-frequency oscillators are also used to advantage in superheterodyne radio receivers. They are used in carrier communication systems for generating both the carrier wave and the several individual tone frequencies that modulate the carrier frequency, the scheme making possible the transmission of several communication channels over one electrical line. In addition to the use of carrier current on communication lines, electric-power companies employ it on power transmission lines for relaying, supervisory control, and transmitting meter indications at a distant point to the dispatcher's office. Built for power-line carrier systems, commercial units are available that provide for ten separate channels by the use of ten tone-generating units, each generating a different frequency which is superimposed on the carrier wave. Low-power oscillators, both in the audio- and radio-frequency range, find wide use in laboratory testing.

In the industrial field, low-power oscillators are used in process controls, indicators, and positioning controls. Higher-power oscillators are used extensively in high-frequency heating which is divided into two distinct types, *induction* heating and *dielectric* heating.

In induction heating, the magnetic field from a coil carrying a high-frequency current induces eddy currents in the metal to be heated. Because of skin-effect, the depth of heating can be controlled by the proper choice of frequency, thus making it possible to case-harden desired surfaces of metal parts while leaving the metal underneath in its more malleable condition. Induction surface heating is also widely used in the plating of metals. In some processes, the time of heat treatment has been reduced by a factor of twenty-five by means of radio-frequency heating.

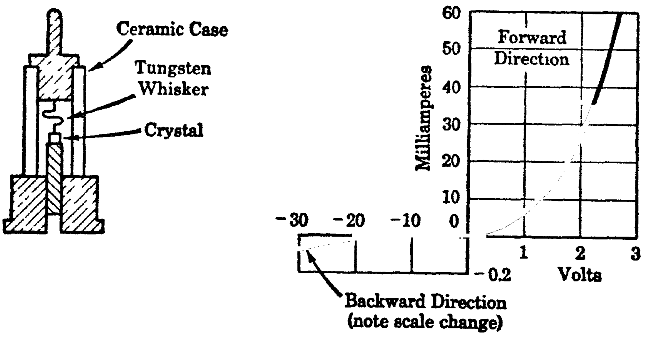
Dielectric heating is used with non-conductors such as plywoods and plastics, which are heated by placing them in the electric field of a condenser. Heating results from the dielectric losses in the material and is due to the periodic displacement of the atoms. Dielectric heating produces uniform heat throughout the volume of the substance subjected to the radio-frequency electric field. The process has a rather long list of applications in industry, the more important, perhaps, being the preheating and curing of plastics, and the bonding of plywood.

Radio-frequency heating units are built in sizes ranging from 1 kw to 200 kw and utilize frequencies in the range of 200 kc to 100 mc.

16.11. Crystal Diodes. In the early days of radio, the crystal rectifier was universally used as a detector in radio receivers. The detector was made by soldering a small piece of a crystalline substance such as galena or silicon in a small receptacle and then making a point contact with the crystal by means of a fine flexible wire (cat whisker). During its early use, the mechanism whereby rectification was achieved was not understood. To obtain reasonable sensitivity it was necessary to adjust the contact point frequently; the early detector could not be considered an efficient or stable rectifier. After the development of the vacuum tube, the crystal detector in radio receivers soon became obsolete.

With the extension of the radio spectrum to higher and higher frequencies, however, the crystal detector was brought back into use. An important reason for the return of the crystal rectifier is its ability to operate at higher frequencies than vacuum tube rectifiers. This comes about because of the much smaller values of electron transit time and capacitance between electrodes in the crystal diode.

A typical crystal unit is illustrated in Figure 16.20a. The crystal is a semi-



(a) Crystal diode

(b) Typical voltage-current characteristic

FIG. 16.20. Crystal unit and rectifying characteristic of a typical crystal detector.

conductor, either silicon or germanium, that has been prepared by a very carefully controlled process. The cat whisker is a fine tungsten wire that makes a very small point contact. The assembly is sealed with wax in a cartridge to give it mechanical and electrical stability. The outside dimensions of the cartridge are approximately $\frac{3}{16}$ inch in diameter and $\frac{1}{2}$ inch in length. A typical voltage-current characteristic is shown in Figure 16.20b. As a result of much research and the development of very complete theories in the field of solid-state physics, it has been possible to develop crystal diodes that have very excellent and stable characteristics. They have the advantage over the vacuum diode of being very small in size, of requiring no heater or filament power, and of being able to operate at higher frequencies. As a result, they have replaced vacuum diodes in many applications.

The theory that accounts for the rectification at the point contact is quite involved, and there is no necessity for considering it in detail in this treatment. It does seem worth while, however, to make a few statements that set forth some of the basic ideas. A semiconductor, such as germanium, is a very poor conductor in its pure state, except at high temperatures. If, however, a very small amount of impurity, antimony for example, is added to the germanium (the result is known as n-type germanium), the bond between the valence electrons and the atoms is changed to the extent that *free* electrons are present. The mechanism of conduction in the *doped* germanium is, therefore, essentially the same as in copper, the difference being that there are far fewer *free* electrons. The condition at the surface (where the cat whisker makes contact) is far different from that in copper and it is this surface condition that results in rectification. In n-type germanium the free electrons are held away from the crystal surface by the internal atomic forces. The depletion of electrons in this region results in a *barrier layer* near the surface that has a net positive charge; it contains no *free* charges and, hence, acts as an insulator. This barrier layer is about 10^{-4} cm in thickness and has a rather large voltage gradient across it like a charged condenser. The barrier layer is eliminated from the back surface by properly treating it and then soldering it to the metal base electrode. The conditions are illustrated in Figure 16.21.

If the metal point is made negative, the free electrons are repelled farther from the surface, thickening the insulating layer so that but little current flow

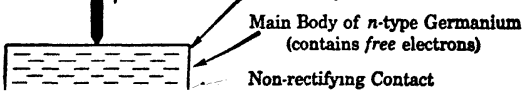

Tungsten Whisker
Forward Direction of Current Flow
Barrier Layer
Main Body of n-type Germanium (contains *free* electrons)
Non-rectifying Contact

FIG. 16.21. Schematic cross section of n-type germanium showing surface barrier.

takes place. Making the metal point positive attracts the electrons, reduces the thickness of the barrier layer. and permits current to flow. Reduced to its barest essentials, the above describes the mechanism of rectification at a surface contact.

16.12. The Transistor. The transistor, Figure 16.22, is essentially a triode form of the n-type germanium diode. Two fine-point contacts are placed in close proximity (about 0.002 in apart) on the surface of a crystal of germanium. One of these, called the emitter, is biased in the highly conducting (forward) direction. The second, the collector, is biased in the low-conduction (reverse) direction. With no current flowing from the emitter, the collector circuit becomes a diode biased in the reverse direction and, for the reasons discussed in the preceding section, the current is practically zero. When a forward emitter current flows, however, the *voltage barrier* in the vicinity of both the emitter and collector point contacts is so changed that collector current flows even though the collector contact is biased in the *reverse* direction. Thus the emitter current is able to control the collector current; the effect is very pronounced, to the extent that amplification of the same order that is obtained with vacuum triodes can be achieved. As a result, the transistor can be used to perform many of the functions of a vacuum tube. As in the case of the crystal diode, the transistor has the advantage of extremely small size and of requiring no filament power. In applications where size and battery consumption are of primary importance, as in hearing aids for example, it has taken over the field completely. The primary limitation of transistor application is the relatively low power rating of the types presently available.

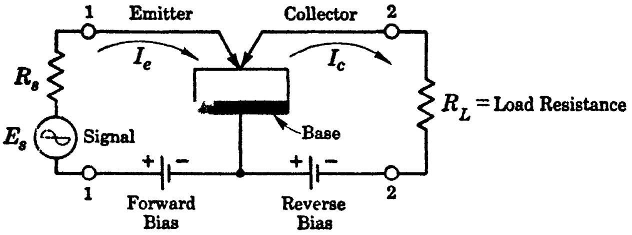


FIG 16 22. Point-contact transistor employing n-type germanium. (R_s is the resistance of the signal source.)

A complete understanding of the phenomena upon which transistor performance depends involves a detailed study of the physics of semiconductors. Its performance as a circuit element, however, can be described in terms of parameters that can be measured at its terminals. As with the vacuum tube, the performance of the transistor can be determined on the basis of an equivalent circuit, the constants of which are determined from measurement. The equivalent circuit of the transistor is shown in Figure 16.23. Currents and voltages that appear in any of the equivalent circuits are the signal components. The parameters R_e, R_b, R_c, and R_m can be obtained from the static characteristic curves in a manner analogous to that used in determining μ, g_m, and r_p for a vacuum tube. At high frequencies, the electrode and distributed capacitances must be included in the analysis, as was done for the tube amplifier of Section 16.2.

In the equivalent circuit, it is to be noted that the control of the collector current by the emitter current is expressed in terms of an equivalent voltage,

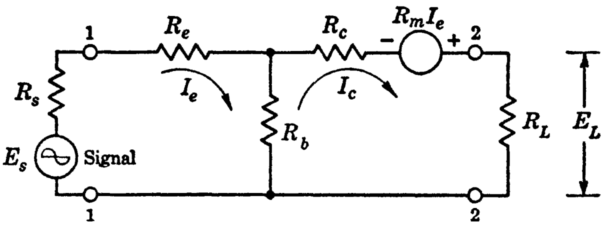


FIG. 16.23. Equivalent circuit of a transistor amplified.

R_mI_e, introduced in the collector circuit. Typical values of the parameters are given in problems 35 and 36.

The characteristics of any device, in which several currents and voltages are interdependent, can be conveniently set forth by one or more families of curves. For the transistor, at least two families of curves are required to express the complete characteristics. The collector characteristics of a typical transistor are shown in Figure 16.24. The similarity to the plate characteristics of a pentode (Figure 15.17) is very marked. The principle difference is that the emitter current now takes the place of the grid voltage control. The amplifier response can be obtained by drawing a load line on the collector characteristics. The treatment is analogous to that used with the triode amplifier. As the emitter current is varied from the quiescent value by a signal, corresponding changes in collector current and collector voltage can be obtained by moving back and forth along this line. This change in collector voltage appears across the load resistance and is the output signal voltage.

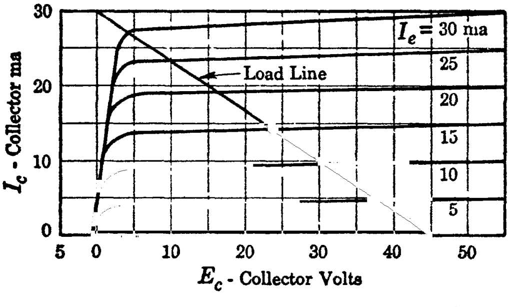


FIG. 16.24. Typical collector characteristics of a transistor (junction type) with a 1,500-ohm load line

The other set of curves that is generally used is the emitter characteristics and shows a plot of emitter current against emitter voltage for a series of constant values of collector current. These curves give information on the input resistance of the transistor. A detailed treatment of the characteristic curves is beyond the scope of this presentation.

The transistor is in many respects similar to the vacuum tube in its operation. A transistor circuit and its analogous tube circuit are shown in Figure 16.25. The emitter is analogous to the tube cathode and the collector is analogous to the plate. Although analogies of this type are useful, it is to be realized that, in the detailed analysis of the circuit, a complete one-to-one comparison is not possible.

Since the introduction of the point-contact transistor in 1948, two other forms have been developed. These are the junction and surface-barrier transistors.

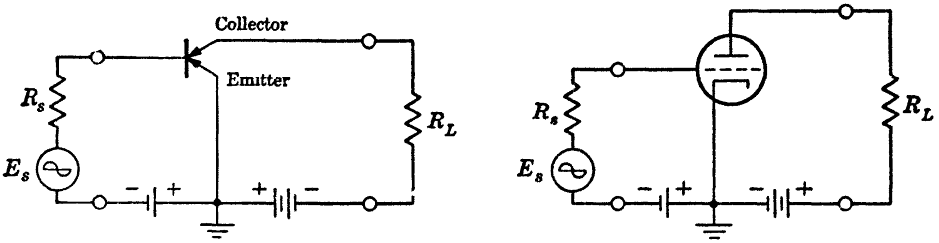


FIG. 16.25. Basic transistor amplifier and its analogous tube amplifier.

The general principles of operation and the equivalent circuits for these types are the same as for the point-contact type The main differences are in the values of the circuit parameters, the high-frequency limit, and the maximum power output obtainable.

PROBLEMS

1. The following data are for a resistance-coupled amplifier in which a 6SF5 triode is used $E_{bb} = 300$ volts, $I_b = 0.5$ ma, $r_p = 100{,}000$ ohms, $g_m = 1{,}000$ micromhos, $R_L = R_g = 250{,}000$ ohms, $R_k = 2{,}600$ ohms, $C_k = 25\ \mu$f, $C_c = 0.01\ \mu$f, and $C_o + C_i = 26\ \mu\mu$f. Calculate the following the values of d-c grid and plate voltages, the impedance of C_k at 100 cycles, the mid-frequency voltage gain, the frequencies at which the gain has decreased to 70.7 per cent, and the peak value of the a-c component of plate current for a 1,000-cycle signal voltage of 0.5 volts rms. Plot the voltage gain as a function of frequency. Express the mid-frequency gain in decibels.

2. Plot the frequency response of an amplifier that consists of three stages, each stage having the circuit and tube of problem 1. State the relation between the over-all gain and the gain of each stage when (a) the gain is expressed as a voltage ratio, and (b) the gain is expressed in decibels.

3. Repeat problem 1 with the following changes in circuit constants: $C_c = 0.002\ \mu$f, and $C_o + C_i = 150\ \mu\mu$f.

4. A 3-stage resistance-coupled amplifier has a voltage gain of 150,000 and an equivalent noise voltage referred to the input circuit of 6.1 microvolts. Calculate the gain per stage. A microphone supplies a signal voltage of 200 microvolts. Calculate the output signal voltage and the number of decibels this voltage is above the noise voltage.

5. Derive equations 16.1, 16.2, and 16.3.

6. A two-stage resistance-coupled amplifier has a mid-frequency gain of 64.1 db. For each stage: $r_p = 500{,}000$, $R_g = 200{,}000$, and $R_L = 100{,}000$. The gain of the amplifier at 40 cycles is 3 db below the mid-frequency gain. Calculate the transconductance of each tube and the frequency at which the gain of each stage is 3 db below the mid-frequency response.

7. Each stage of a 2-stage resistance-coupled amplifier has a voltage gain of 50 at a mid-frequency of 1,000 cycles, and a gain of 35.4 at a frequency of 30 cycles. Calculate the gain in decibels of the amplifier for a frequency of 60 cycles.

8. A triode, with $\mu = 70$ and $r_p = 60{,}000$ ohms, is to be used in a resistance-coupled amplifier. If $R_g = 250{,}000$ ohms, calculate R_L and C_c to give a voltage gain of 40 at the mid-frequency, and a relative gain of 70.7% at 30 cycles.

9. For a type-56 triode, $\mu = 13.8$ and $r_p = 10{,}000$ ohms. The load in the plate circuit is a resistance of 10,000 ohms and an inductance of 1.6 henries connected in series. Draw the simplified equivalent circuit and determine the gain for a 1,000-cycle signal voltage.

10. A 6SF5 triode, with $\mu = 100$ and $r_p = 100{,}000$, has a plate load of 100,000 ohms resistance shunted by a 500-$\mu\mu$f capacitance. Draw the series equivalent circuit. For what frequency is the gain 70.7% of the gain at 100 cycles? What is the phase relation between the a-c components of grid and plate voltages at this frequency?

11. The following values are given for a resistance-coupled amplifier employing a 6SJ7 pentode: $g_m = 1{,}080$ micromhos, $r_p = 2.5$ megohms, $R_L = R_g = 100{,}000$ ohms, $C_c = 0.01$ μf, and $C_o + C_i = 38$ $\mu\mu$f. Draw both the series form and the parallel form of the equivalent circuit, showing the values of all circuit constants. Plot the voltage gain as a function of frequency from 10 to 200,000 cycles. State two methods of improving the low-frequency response. It should be possible, by careful design, to reduce $C_o + C_i$ to 22 $\mu\mu$f. What effect would this change have on the frequency response?

12. In a resistance-coupled amplifier using a pentode, $R_L = 500{,}000$ ohms, $R_g = 500{,}000$ ohms, $C_c = 0.003$ μf, and $C_o + C_i = 25$ $\mu\mu$f. The voltage gain in the mid-frequency range is 183. Assuming the plate resistance to be infinite, calculate the transconductance of the tube, and plot the amplification for the frequency range 10 to 200,000 cycles.

13. A 2A3 triode, with $\mu = 4.2$ and $g_m = 5{,}000$ micromhos, is used in a power amplifier with an output transformer to supply a load of 12.5 ohms. The load as seen by the tube is 2,500 ohms and the grid signal voltage is 25 volts rms. Calculate the transformer turn ratio, the power delivered to the load, the a-c components of plate current and voltage, and the rms value of current in the 12.5-ohm load. If the signal voltage is supplied from a 75-ohm line, calculate the turn ratio of a suitable input transformer. Calculate the rms value of the line voltage and the power supplied by the line.

14. A tuned amplifier uses a 6SK7 tube, with $g_m = 1{,}700$ micromhos and $r_p = 1.1$ megohms, in the circuit of Figure 16.9 for which $R_g = 1.0$ megohms. At the resonant frequency (1,000 kc) the coil has a reactance of 900 ohms and a resistance of 10 ohms. Calculate the inductance and Q of the coil, the range of capacity for tuning from 550 to 1,600 kc, the amplification at resonance, and the band width $f_2 - f_1$. Calculate the gain of an interfering signal 80 kc higher than resonance Plot the response near resonance.

15. A radio-frequency amplifier consists of three stages, each stage using the tube and circuit of problem 14. Calculate the ratio of the gain at the resonant frequency of 1,000 kc to the gain at 1,080 kc.

16. Derive equation 16.8 from equation 16.7. If the coil has a Q of 80, calculate the percentage error introduced by the approximation made in deriving the equation.

17. Assuming the plate resistance of the tubes in Figure 16.11 to be infinite, derive equation 16.10.

18. The circuit of Figure 16.11 uses the tube of problem 14. Both coils have an inductance of 200 microhenries and a Q of 100, and the coupling coefficient is 0.31. Calculate the gain for a resonant frequency of 1,200 kc.

19. The instantaneous value of an amplitude-modulated wave is given by the expression $e = A(1 + m \sin \omega_i t) \sin \omega_c t$, where m is the modulation coefficient, ω_i is the frequency of the modulating signal, and ω_c is the frequency of the carrier. Sketch the modulated wave. By trigonometric expansion show that the wave consists of a carrier and two side frequencies.

20. For the 2A3 triode discussed in Section 16.7, calculate the ratio of the power dissipated at the plate when maximum signal is applied to the grid to the power dissipated when no signal is present. Explain why the power dissipated at the plate decreases when there is a signal voltage applied. Calculate the plate-circuit efficiency when the grid signal voltage is one-half the maximum permissible value.

21. Using the values tabulated in Section 16.7 for the 2A3 triode, calculate the power output for a grid signal of 43 volts peak value by means of the equivalent circuit.

22. The instantaneous plate current, in ma, of an amplifier using a triode with a d-c plate voltage of 300 and a load resistance of 3,000 ohms can be approximated by the expression $i_b = 70 + 1.2e_g + 0.003e_g^2$, where e_g is the instantaneous grid-signal voltage. Plot the wave shape of the plate current for a signal voltage $e_g = 50 \sin \omega t$. Write the expanded expression of i_b for this sinusoidal signal voltage. Calculate the per cent second harmonic, the power in the fundamental component, the plate-circuit efficiency, the power in the second harmonic, and the change in the d-c value of plate current. Is the change in d-c plate current related to the magnitude of the second-harmonic current? Calculate the per cent second harmonic when the signal voltage is reduced to 25 volts peak value.

23. The following values apply to a power amplifier using a triode: $E_b = 250$, $E_c = -45$, $I_b = 60$ ma, $\mu = 4.2$, $r_p = 1{,}000$, and $R_L = 2{,}500$ ohms. When a signal of 31 volts rms is applied, a d-c meter in the plate circuit reads 65 ma. Calculate the plate-circuit efficiency, the per cent second harmonic, and the factor by which the per cent second harmonic is reduced if the signal voltage is decreased to 20 volts rms.

24. Show that if two identical tubes, each having a characteristic expressed by the equation of problem 22, are used in a push-pull amplifier, the per cent second harmonic is zero. (In a push-pull amplifier the instantaneous load current is the sum of the individual tube currents.)

25. Calculate the plate-circuit efficiency of a 6V6 beam-power tube for the operating conditions stated in Section 16.7.

26. Using the values tabulated in Section 16.7 for the 6V6 beam-power tube, calculate the power output for a grid signal of 12.5 volts peak value by means of the parallel equivalent circuit

27. In the circuit of Figure 16.16b, $L = 20$ henries, $C_c = 4$ μf, and $R_L = 5{,}000$ ohms. For a 400-cycle output voltage of 100 volts rms, calculate the power output, the current through R_L, the a-c voltage across C_c, the a-c current through L, and the a-c component of plate current.

28. The following data are for a 2-stage amplifier:

Freq (cycles	20	50	100	500	1,000	2,000	10,000
Voltage gain	800	1,900	2,830	3,920	3,980	4,000	4,000

(a) Plot the frequency response in decibels relative to the gain at 10,000 cycles. (b) Repeat part (a) after degenerative feedback, with a feedback factor $\beta = 0.007$, is introduced.

29. A 2-stage resistance-coupled amplifier has a gain of 8,000. After one of the tubes is replaced, the gain is 6,900. If the gain of the amplifier, using the original tubes, were reduced to 180 by the introduction of negative feedback, calculate the per cent change in amplification resulting from the tube replacement.

30. Design a phase-shift oscillator that uses the basic ideas contained in Figure 16.18. The oscillator is to be a 3-stage regenerative amplifier, each stage introducing a 60° phase shift. Explain in detail the operation of the circuit, and write the expression for the frequency of oscillation.

31. Explain the operation of the Hartley oscillator.

32. For the operating point, $E_c = 24$ volts and $I_e = 15$ ma, on the collector characteristics of Figure 16.24, make an estimate of the transistor collector resistance (analogous to the plate resistance of a vacuum tube).

33. Using the transistor characteristics and the load line of Figure 16.24, calculate a-c components of output voltage and power for an emitter signal of 10 ma peak value. The operating point on the load line is at $I_e = 15$ ma. The input resistance (the ratio of the signal components of voltage across the terminals 1-1, of Figure 16.23, to emitter current) of the transistor is 20 ohms. Calculate the value of the power gain. Express the gain in db.

34. For the equivalent circuit of Figure 16.23, show that:

(a) $I_c/I_e = (R_m + R_b)/(R_L + R_c + R_b)$.
(b) The input resistance, $R_{11} = E_{11}/I_e = R_e + R_b(1 - I_c/I_e)$.

35. The following parameters are given for a point-contact transistor: $R_e = 250$ ohms, $R_b = 250$ ohms, $R_c = 20{,}000$ ohms, and $R_m = 38{,}000$ ohms. Using the equivalent circuit of Figure 16.23, calculate the voltage gain from terminals 1-1 to terminals 2-2 for a load resistance of 20,000 ohms. Calculate the input resistance, R_{11} (see problem 34). Calculate the ratio of the output power to the power input to terminals 1-1. Express this power gain in db. A generator with internal resistance R_s will deliver maximum power to a load when the load impedance R_L equals R_s. In circuit theory, this power is termed the generator available power. For $R_s = 500$ ohms, calculate the ratio of the transistor output power to the available power from the generator. Express this power ratio in db.

36. A junction transistor has the following parameters: $R_e = 50$ ohms, $R_b = 200$ ohms, $R_c = 1.2 \times 10^6$ ohms, and $R_m = 1.15 \times 10^6$ ohms. Repeat the calculations of problem 35 for this transistor using values of $R_L = 50{,}000$ ohms and $R_s = 150$ ohms.

CHAPTER XVII

GAS-FILLED TUBES AND POWER SUPPLIES

17.1. The Thermionic Gas-filled Diode. In the case of high-vacuum diodes, it has been shown that the anode current is limited by the negative space charge produced by electrons from the cathode and that its value is proportional to the three-halves power of the plate voltage. The gas-filled diode differs from the high-vacuum diode in that it contains a small amount of mercury vapor or a chemically inert gas, usually neon, argon, or xenon. The purpose of the vapor or gas is to provide positive ions which neutralize the negative space charge and thus allow a large flow of current for a relatively low value of plate voltage.

The inert gases are used at a pressure of about 0.2 mm of mercury. In the mercury-vapor tube, the vapor is in equilibrium with the small amount of liquid mercury distilled into the tube during the exhaust process. The vapor pressure varies, therefore, with the temperature of the coldest part of the tube, being 1.2×10^{-3} mm of mercury at 20°C and 9×10^{-2} mm at 80°C. In a high-vacuum tube the pressure of the residual gas is between 10^{-5} and 10^{-6} mm of mercury.

A general understanding of the gas-filled diode can be obtained from the following considerations: for low values of plate voltage—less than 10 volts if mercury vapor is used—the electrons are attracted toward the plate just as in the high-vacuum tube, but because of collisions with the gas atoms the plate current will be even less than if no gas were present. As the plate voltage is increased, the electrons acquire sufficient velocity to produce additional free electrons and positively charged ions in many of the collisions with the gas atoms. The electrons resulting from the ionization join the original electrons in their motion to the plate while the heavy positive ions drift slowly toward the cathode. Because of their low velocity, the positive ions spend a relatively long time in the interelectrode space, which makes it possible for one ion to neutralize the space-charge effect of many passing electrons. In this manner, the negative space charge is eliminated and very large electron currents are obtained for low values of plate voltage. Because of their low velocity, the positive ions do not contribute much to the total plate current. In general, the plate potential required is somewhat greater than the ionizing potential of the gas (for example, the ionizing potential of mercury vapor is 10.4 volts) and is relatively independent of the value of plate current. The contrast between the characteristics of the vacuum and mercury-vapor diodes is shown in Figure 17.1. It is evident from the curve that in order to avoid damage to the tube the current must be limited by the external circuit.

The ionization of the gas by the electrons emitted from the hot cathode leads to an arc discharge and the volt-ampere curve shows a characteristic sloping downward, Figure 17.1, that is typical of all arcs. The arc can be maintained only if a continuous source of electrons is provided by the negative electrode and, hence, the gas-filled diode cannot conduct if the anode is made negative. The time from the first ionizing collisions to the complete formation of the arc varies from a fraction of a microsecond to several microseconds, depending upon the tube design and the gas pressure. The time required for the ions to disappear after the plate voltage is removed, termed *deionization* time, usually ranges from several microseconds to a millisecond. These time intervals place an upper limit on the frequency above which gas-filled tubes will not give satisfactory performance in certain applications.

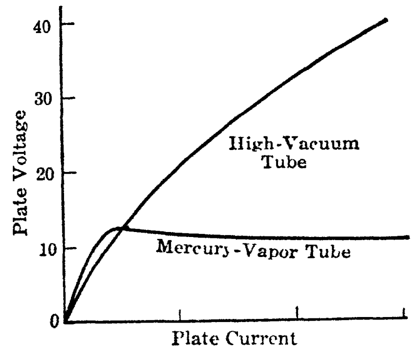


FIG. 17.1. Comparative characteristics of vacuum and mercury-vapor diodes.

When used as a rectifier of a-c power, the mercury-vapor diode has a higher efficiency than the high-vacuum tube. This results from the low voltage drop in the tube, giving a relative low power loss at the plate. For example, a high-vacuum water-cooled diode supplying a d-c current of 4 amp would have a plate dissipation of approximately 6 kw, whereas a typical mercury-vapor tube could supply this current with a plate loss of only 50 watts. The low plate loss greatly simplifies the problems of tube design and rectifier operation. As a result, gas-filled diodes have practically replaced the high-vacuum type in applications where a rectified current greater than about 0.5 amp is required. Furthermore, the small voltage drop in the tube makes possible a rectifier with good output voltage regulation. One disadvantage of the gas tube is the danger of "arcback." During one half of the cycle in rectifier service, the tube is non-conducting. The potential during this half cycle is negative, and if a discharge in the wrong direction through the tube occurs it is termed "arcback." If present-day commercial tubes are properly operated within their ratings, however, failure due to arcback rarely occurs. Gas diodes have been designed that will withstand safely a voltage (maximum peak inverse voltage) of 22,000 volts in the direction opposite that in which the tube is designed to pass current. They are available with d-c current ratings ranging from 0.125 to 75 amp. High-vacuum rectifiers are available that have maximum peak inverse voltage ratings of 150,000 volts.

17.2. The Glow-discharge Diode. Another type of gas-filled diode, one having two cold electrodes, has a characteristic that makes it useful as a voltage regulator. It consists of a cylindrical cathode about one inch in diameter with a slender straight wire along its axis which serves as the anode. A wire stub projects from the cathode to a point near the anode in order that a gaseous discharge can be established in the tube at a reasonably low voltage. The cathode is covered with an oxide of an alkali metal so that electrons can be liberated easily by positive-ion bombardment.

When the voltage applied between the electrodes is increased progressively from zero, a negligible current flows until the breakdown potential of the tube is reached. At this voltage the tube suddenly becomes conducting, the value of voltage across the tube decreases, and the current that flows is determined by the resistance in series with the tube. The voltage across the tube is nearly constant for a range of tube current from 5 to 40 ma. Commercial tubes are available that hold constant potentials of 75, 90, 105, or 150 volts. A voltage about 30 per cent greater than the conduction potential is required to initiate the tube discharge. The first discharge ionizes the gas, producing positive ions, which, in turn, bombard the cathode and liberate electrons by the process of secondary emission. The electrons from the cathode are attracted towards the plate and engage in collisions which maintain the state of ionization.

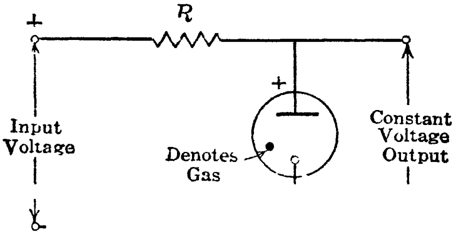


FIG. 17.2. The glow-discharge diode as a voltage regulator.

A typical circuit of a voltage regulator is shown in Figure 17.2. The dot in the tube symbol denotes the presence of gas. Over a rather wide range of supply voltage the tube will maintain a voltage across its terminals that is nearly constant. As an example, consider the type-OC3 VR105 voltage-regulator tube on a supply voltage of 250 volts. The normal d-c operating voltage of this tube is 105 volts, the range of d-c operating current is from 5 to 40 ma, and a change in tube current of 25 ma produces a one-volt change in tube voltage. If R is chosen as 14,500 ohms, the tube current will be 10 ma. An increase in plate-supply voltage to 350 volts will increase the plate current to 16.9 ma, resulting in an increase in tube voltage of $6.9/25 = 0.276$ volts or a change of only 0.26 per cent.

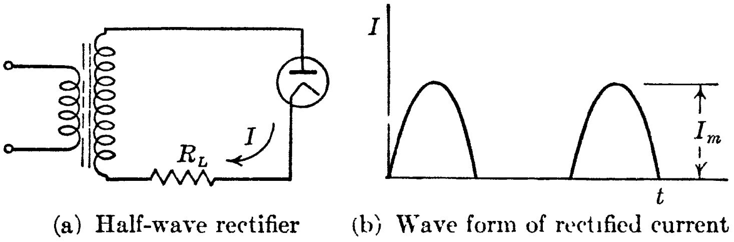
These tubes provide a convenient method of obtaining a constant d-c voltage over moderate variations of line voltage and load current, and they find considerable use as voltage standards in various types of electronic regulators. Several tubes can be operated in series to obtain a higher value of regulated voltage.

17.3. Single-phase Rectifier Circuits. The d-c power required to operate vacuum tubes is in the majority of cases obtained from the a-c power lines by the use of electron-tube rectifiers. Both the vacuum and gas-filled diodes can act as rectifiers since they will pass current only when the plate is positive with respect to the cathode.

A simple half-wave rectifier circuit with a resistance load is shown in Figure 17.3. If a high-vacuum diode is used, the instantaneous value of current, during the half cycle that the tube is conducting, can be written as

$$i_b = \frac{E_m \sin \omega t}{R_p + R_L} \text{ amperes} \tag{17.1}$$

where E_m is the peak value of the a-c voltage applied to the rectifier circuit, R_L is the load resistance, and R_p is an average value of the tube resistance, usually of the order of 300 ohms. Since the tube cannot conduct in the opposite direction, the current is zero for all negative values of equation 17.1. The load current will be a pulsating unidirectional current, Figure 17.3b, which can be analyzed to show a d-c component and a series of a-c components. The d-c component is the average value of equation 17.1 and is equal to



(a) Half-wave rectifier (b) Wave form of rectified current

FIG. 17.3. Circuit of half-wave rectifier together with rectified load current.

$$I_{\text{d-c}} = \frac{E_m}{\pi(R_p + R_L)} \text{ amperes} \tag{17.2}$$

The alternating components can be removed from the load by an appropriate smoothing filter.

If a gas-filled tube is used in the circuit of Figure 17.3, the plate current, for positive values of i_b, is given by the expression

$$i_b = \frac{E_m \sin \omega t - F_T}{R_L} \text{ amperes} \tag{17.3}$$

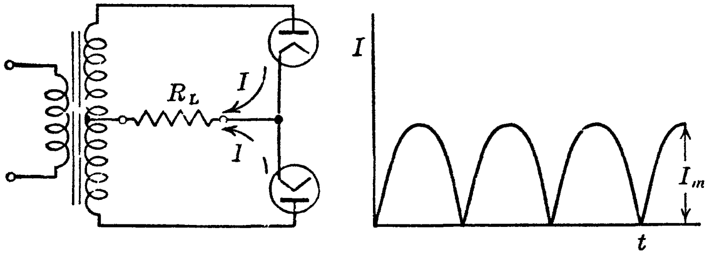
where E_T is the tube voltage. As before, i_b is zero for negative values of the equation. Since the value of tube voltage is generally small in comparison with E_m, the wave form of current differs but slightly from a half sine wave, and the value of d-c current is very nearly equal to

$$I_{\text{d-c}} = \frac{E_m - (\pi/2)E_T}{\pi R_L} \text{ amperes} \tag{17.4}$$

The d-c component of load voltage is given by

$$E_{\text{d-c}} = I_{\text{d-c}} R_L = \frac{E_m}{\pi} - \frac{E_T}{2} \text{ volts} \tag{17.5}$$

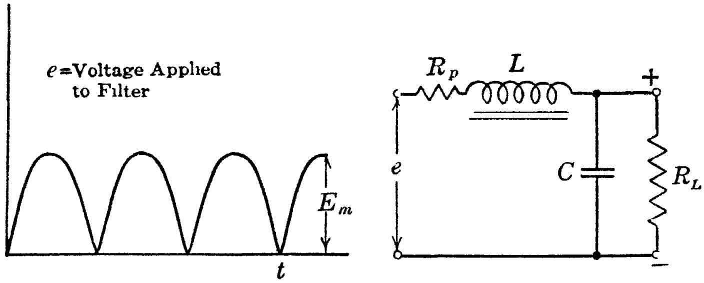
In order to reduce the a-c components of load voltage, use is generally made of two diodes and a center-tapped transformer to obtain full-wave rectification, Figure 17.4a. The diodes conduct during alternate half cycles to supply a pulsating d-c current of the form shown in Figure 17.4b. The values of d-c current from a full-wave rectifier are twice those given by equations 17.2 and 17.4.



(a) Full-wave rectifier (b) Wave form of rectified current

FIG. 17.4. Circuit of full-wave rectifier together with rectified load current.

17.4. Filter Circuits. Because the plate voltage for most tubes must be nearly free of any pulsation, it is generally necessary to insert a filter between the rectifier and its load circuit in order to remove a-c components from that load circuit. In the simple analysis given here for the inductance-input filter, the action of the rectifier tube is assumed to convert the sine-wave supply voltage to the form shown in Figure 17.5a. This pulsating voltage then is smoothed into a steady d-c voltage by means of the *L-C* filter shown in Figure 17.5b.



(a) Rectified sinusoidal voltage (b) Inductance-input filter

FIG. 17.5. Rectified voltage wave form together with inductance-input filter.

Analysis of the wave by Fourier's method shows it to be

$$e = \frac{2}{\pi} E_m \left[1 - \frac{2}{3} \cos 2\omega t - \frac{2}{15} \cos 4\omega t - \frac{2}{35} \cos 6\omega t - \cdots \right] \text{volts} \quad (17.6)$$

where $\omega = 2\pi f$, and f is the frequency of the supply voltage. The voltage contains the d-c component $2E_m/\pi$, and a-c components that are even harmonics of the supply frequency. The d-c component delivers a constant current $2E_m/\pi(R_p + R_L)$ amperes to the load.

To illustrate the effectiveness of the inductance (choke coil) and condenser in eliminating the a-c components, the following example with typical circuit constants is given: $L = 30$ henries, $C = 16\ \mu f$, $f = 60$ cycles, $E_m = 400$ volts, $R_p = 300$ ohms, and $R_L = 5{,}000$ ohms. At the second-harmonic frequency of 120 cycles, $X_L = 22{,}600$ ohms and $X_C = 83$ ohms. The second-harmonic component of voltage e_{2C} that appears across the condenser is

$$e_{2C} = \frac{4E_m}{3\pi}\frac{X_C}{X_L - X_C}\cos 2\omega t$$

$$= \frac{4E_m}{3\pi}(0.0037)\cos 2\omega t \text{ volts}$$

The filter hence has reduced the largest a-c component to 0.37 per cent of what it would be without the filter.

Treating each voltage component in like manner, the total load current for the single-stage filter is

$$I_L = 51(0.94 - 0.0025\cos 2\omega t - 0.00012\cos 4\omega t - 0.000024\cos 6\omega t - \cdots) \quad \text{milliamperes}$$

It will be seen that the magnitudes of the a-c components are very small and that they decrease rapidly with increase in frequency.

Filtering action also is made much more effective by increasing the number of stages of the filter circuit. If two filter sections are used, as shown in Figure 17.6, the load current becomes practically a pure direct current. This circuit is the one most commonly used on single-phase power sources. The two diode units are usually built in one glass or metal envelope such as in the 5U4G, 6X4, etc., types of tube. For most purposes, a sufficiently constant direct current can be obtained with two stages of filtering.

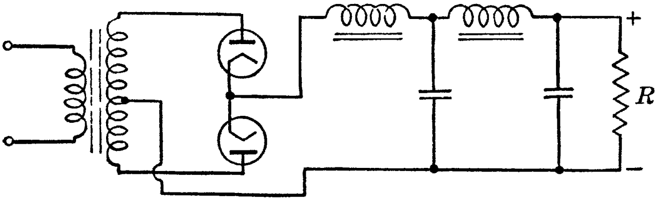


FIG. 17.6. Circuit of full-wave rectifier together with a 2-stage inductance-input filter.

Additional filtering can be obtained by placing a condenser at the input to the filter, which filter is then said to be of the condenser-input type. The power transformer and tube requirements for the condenser-input filter are greater than for the inductance-input filter, and for this reason condenser-input filters are not generally used in high-power rectifier service.

17.5. Regulated Power Supplies. In many applications of electronic devices it is necessary to supply a source of d-c power, the output voltage of which does not vary with the value of load current or with fluctuations in the a-c supply voltage. It is possible to obtain almost any desired degree of voltage stability with relatively simple electronic regulators. Since the regulators respond to sudden voltage fluctuations, they also greatly reduce any ripple voltage which otherwise would appear in the output. The general principles of an electronic regulator are indicated in Figure 17.7. A sample of the output voltage is taken from a potentiometer and compared with a voltage standard or reference. The voltage standard can be a battery or a voltage-regulator tube of the type described in Section 2 of this chapter. A d-c *error voltage,* which is the difference between the sample and standard voltages, is amplified by a d-c amplifier and applied to the grid circuit of a control tube. The grid voltage of the control tube determines the voltage drop from the plate to the cathode of this tube. The regulator operation can be described briefly as follows: *If for some reason the*

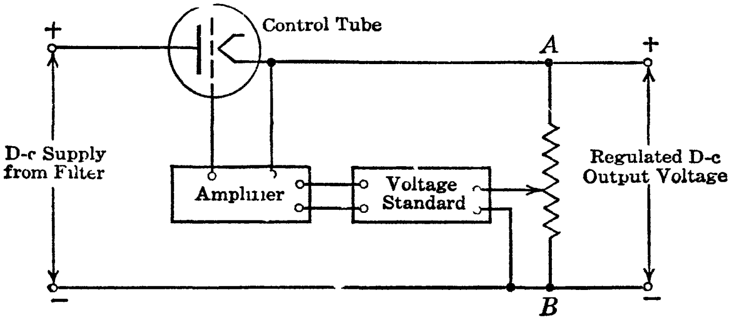


FIG. 17.7. Schematic diagram of a voltage regulator.

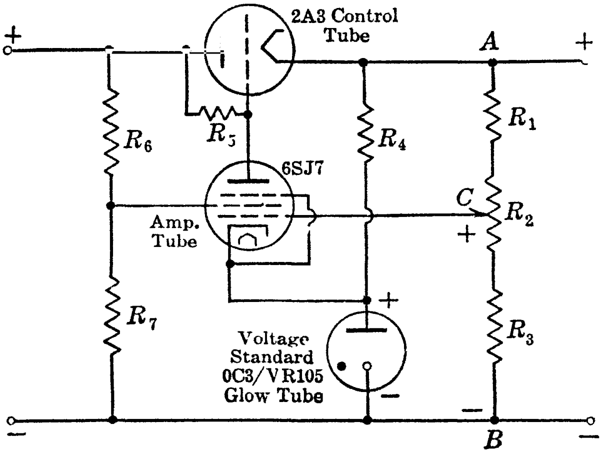
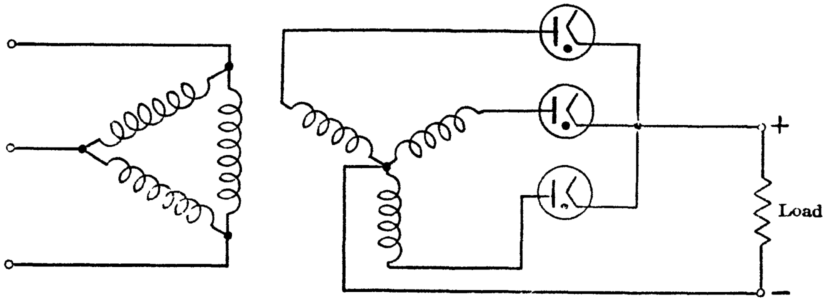
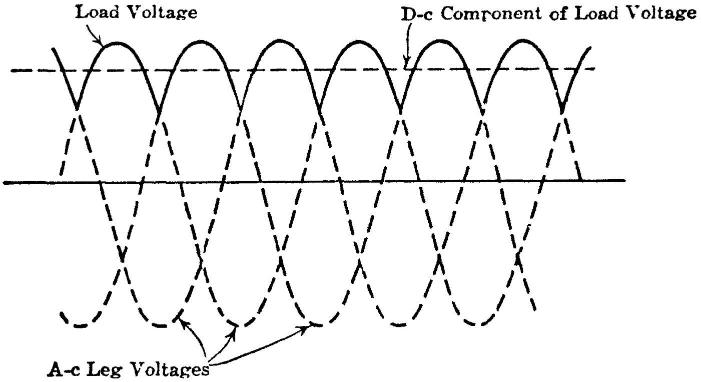


FIG. 17.8. Circuit diagram of a voltage regulator.

voltage between points A and B increases, a d-c error voltage will be applied to the amplifier, the sign of which will contain the information that the output voltage has *increased*. The amplifier then increases the grid bias of the control tube, thereby increasing the voltage drop across that tube, and, hence, reduces the output voltage to within a very small fraction of its predetermined value.



(a) Three-phase half-wave rectifier



(b) Wave form of output voltage

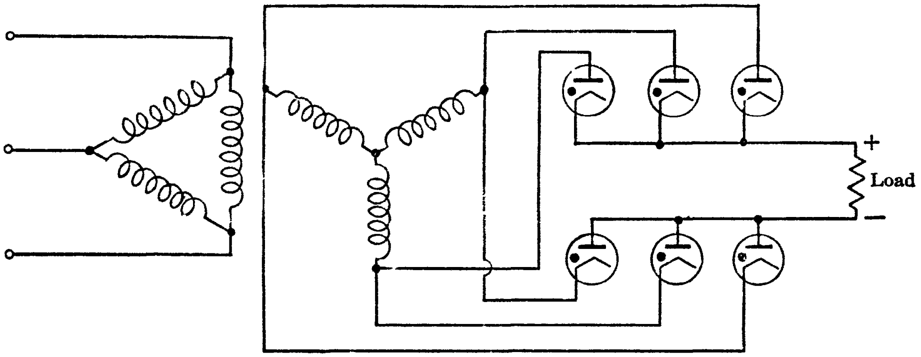
FIG. 17.9. Three-phase half-wave rectifier circuit together with the wave form of the voltage developed across the load.

Rather simple and inexpensive regulators can reduce voltage variations by a factor of several hundred. Somewhat more elaborate regulators have been designed that have a corrective factor of 50,000.

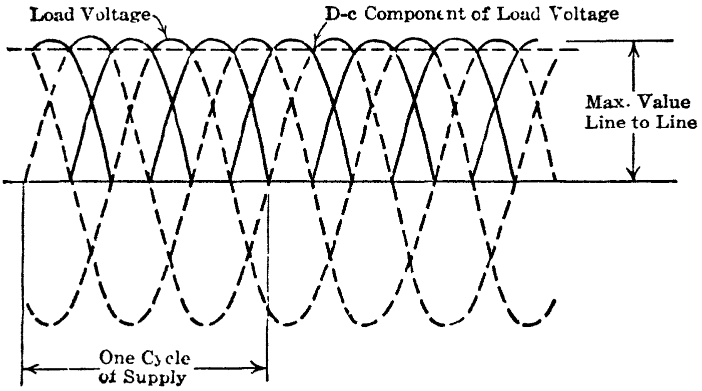
A circuit quite commonly used in laboratory equipment is shown in Figure 17.8. The voltage standard is the OC3/VR105 regulator tube, and R_4 is chosen so as to limit the current through the glow tube to its normal range. The input voltage to the pentode, 6SJ7, is equal to E_{CB} minus the voltage across the glow tube. The output voltage is varied as desired by means of R_2. R_5 is the load resistor of the amplifier tube; R_6 and R_7 provide a suitable screen-grid voltage for the pentode. The circuit is essentially that for a 2-stage degenerative-feedback amplifier.

17.6. Polyphase Rectifiers. Where the d-c power required is more than about 1 kw, it is the usual practice to use rectifiers operating from a 3-phase supply. The percentage ripple from these rectifiers is very much less than from a single-phase supply, with the result that a simpler and less expensive filter can be used.

The simplest polyphase rectifier is the 3-phase half-wave rectifier shown in Figure 17.9a. Each tube carries current for one-third of the time, and the output voltage pulsates at a frequency three times that of the supply, as shown by the heavy line of Figure 17.9b. The d-c output voltage is approximately 83 per cent of the peak value of the secondary leg voltage.



(a) Three-phase full-wave rectifier



(b) Wave form of output voltage

FIG. 17.10. Three-phase full-wave rectifier circuit together with the wave form of the voltage developed across the load.

Six tubes can be connected to comprise a 3-phase full-wave rectifier, as shown in Figure 17.10a. In this circuit the load current pulsates at six times the frequency of the a-c supply, Figure 17.10b. Two tubes conduct in series at any instant, the rectified voltage being identified with the line-to-line voltage of the transformer secondaries. The d-c output voltage is approximately 95 per cent of the peak value of the line-to-line voltage of the secondary windings.

The circuits of Figures 17.9 and 17.10 are but two of several connections that are commonly employed. Other arrangements that are of practical importance are the 3-phase double-Y, the 3-phase zig-zag, and the 6-phase half-wave. Descriptions of these circuits can be found in the majority of books on electronics.

17.7. Thyratrons. A thyratron is a vapor- or gas-filled tube that contains a thermionic cathode, a plate, and a grid, the grid to set the time of *starting* of the plate current. In discussing the gas-filled diode, it was shown that the start of the arc discharge depended upon the electronic current producing the ionization. If no electronic current is present, no ionization can occur and hence the arc discharge does not form. For any voltage on the plate, the electronic current can be held to zero by a grid voltage sufficiently negative. If the grid voltage is gradually decreased, a magnitude will be reached at which electrons will start to the plate and ionize the vapor. After ionization, the grid *loses complete control* over the plate current. The loss of grid control after ionization results from the fact that some of the positive ions form a sheath around the grid structure. The positive field from this sheath neutralizes the negative field of the grid, thereby preventing the grid from having any effect on the magnitude of the plate current. The grid can regain control only by removing the anode voltage momentarily and allowing the tube to deionize. Thus, the grid can be used to prevent or start the plate current but cannot control the magnitude of it. This action is quite distinct from that of the high-vacuum triode.

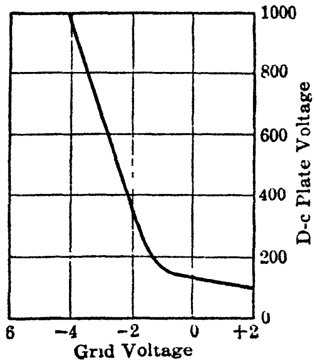
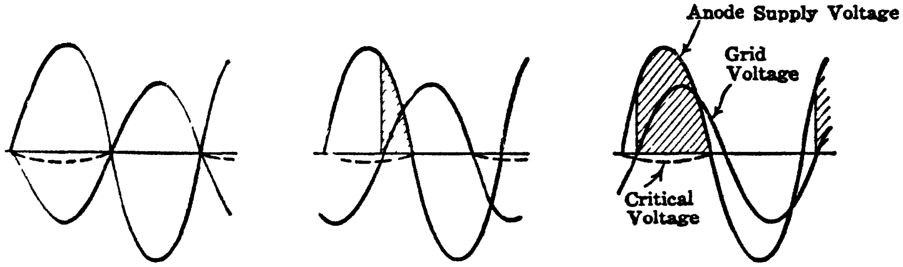


Fig. 17.11. Starting characteristic of Thyratron FG-67, mercury temperature 70°C.

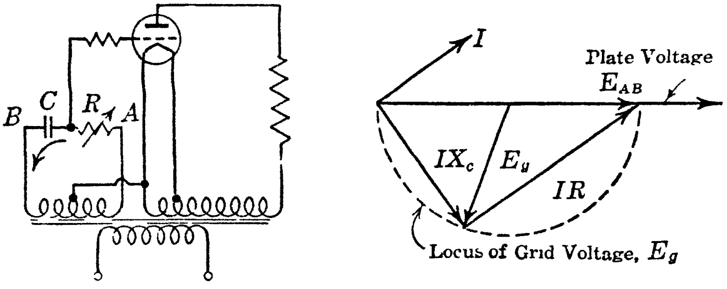
The characteristic of a gas-filled triode is given by a curve called the starting characteristic, as shown in Figure 17.11. The curve shows, for any negative grid voltage, the anode voltage necessary for the arc to start. The starting characteristic of mercury-vapor tubes varies somewhat with the temperature of the condensed mercury since this temperature determines the vapor pressure in the tube. The grid voltage at which ionization will just occur is called the *critical grid voltage.* If the tube is in a non-conducting state and the grid voltage is gradually made less negative until the critical voltage is reached, the anode voltage will drop to about 15 volts as soon as the tube becomes conducting. Both anode and grid currents must be limited to safe values by impedances in their respective circuits.

If a-c voltages are applied to both the anode and the grid, the average value of the anode current can be controlled by adjusting the phase relation between the grid and anode voltages. With an alternating voltage on the anode, the grid is able to regain control once each cycle, that is, when the plate voltage passes through zero. The phase relation between the grid and anode voltages determines the time, in the positive half cycle of the anode voltage, at which the anode current starts to flow. This phase-shift method of control is illustrated by the diagrams of Figure 17.12. In (a), the anode and grid voltages are 180 degrees out of phase and the grid is sufficiently negative during the positive anode cycle that no plate current flows. In (b), the grid voltage lags the plate voltage by about 120 degrees. The tube "fires" when the grid voltage is equal to the critical voltage, the shaded area showing the portion of the cycle during which plate current flows. In (c), the grid voltage lags the plate voltage by 30 degrees and plate current flows for approximately 150 degrees of the positive half cycle. Figure 17.13a shows a circuit commonly used for obtaining phase-shift control.



(a) E_g lags E_p by 180° (b) E_g lags E_p by 120° (c) E_g lags E_p by 30°

FIG. 17.12. Control of plate current by phase control.



(a) (b)

FIG. 17.13. Thyratron phase-shift control circuit and vector diagram.

By means of the capacitance-resistance bridge it is possible to obtain a phase-shift range of nearly 180 degrees. The vector diagram of the circuit is shown in Figure 17.13b.

Thyratrons have found a wide application to industrial problems. A partial list would include control of electric welding, regulated power supplies, speed control of motors, voltage regulators for alternators, sweep circuits for cathode-ray oscillographs, and theater lighting. These tubes can also be used to convert direct current into alternating current. Such a circuit, complete with tubes, is called an inverter.

17.8. Mercury-arc Power Rectifiers. The mercury-arc, or mercury-pool, tube was one of the first gaseous-discharge tubes to be used as a rectifier of a-c current. The tube contains a pool of mercury that becomes the cathode, one or more anodes, and auxiliary or starting electrodes. The pool of mercury serves the double purpose of providing the electrons and of supplying the vapor from which the positive ions, required to neutralize the negative space charge, are produced.

If an arc is established between the mercury pool and an electrode at a positive potential with respect to the pool, the *cathode spot* formed by the termination of the arc at the surface of the mercury becomes a source of practically unlimited electron emission. The cathode spot is an exceedingly strong source of electrons and allows very large values of currents to be drawn from it without any danger of the cathode being damaged as often happens with thermionic cathodes. The mechanism by which the spot emits electrons is not clearly understood, but it is believed that the positive mercury ions form an extremely thin sheath at the surface of the mercury. The very high potential gradient established would account for the large electron emission. Since the mercury pool is not a source of electrons until the arc has formed, some means must be provided to start the arc. This is usually accomplished by a starting electrode which strikes an arc, thus forming a cathode spot to allow the formation of an arc to one of the main anodes.

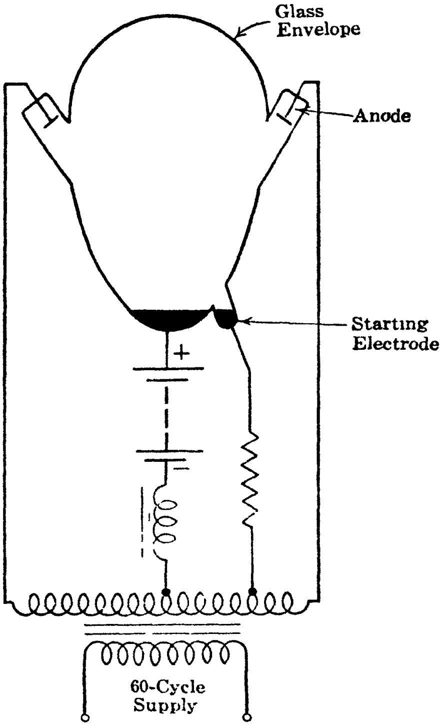


FIG. 17.14. An early form of mercury-pool rectifier for battery charging.

An early form of the mercury-pool rectifier is shown in Figure 17.14 and serves to illustrate some of the basic features that are significant in all mercury-pool tubes. The arc is started by tipping the tube until the two mercury pools are joined. Upon restoring the position of the tube, the mercury path between the two pools breaks and a small arc is formed which is immediately picked up by one of the main electrodes. With the two anodes and the center-tapped transformer, the tube acts as a full-wave rectifier. The function of the reactor is to maintain the arc when the voltage passes through zero, for if the arc goes out it must be restarted by tilting the tube again. The large glass dome is to provide adequate cooling surface on which the vapor can condense and return to the cathode pool. In modern high-current tubes the metal walls are water cooled to prevent excessive temperatures of the mercury vapor. Placing the anodes in arms extending from the main tube reduces the possibility of arcbacks occurring or of an arc forming between the two anodes. This is an important consideration in modern multiple-anode, steel-tank rectifiers. In these units an elaborate baffle arrangement is used around each anode to prevent undesirable arcs from being formed.

Large, steel-tank, mercury-arc rectifiers are built with 6, 12, or 18 anodes. The basic principles of operation of these units are similar to those described for the 2-anode rectifier. The cathode spot is formed by an electrode magnetically operated that is lowered into the mercury pool and then quickly withdrawn to form an initiating arc. Multiple-anode rectifiers always are operated from a polyphase source, and once the arc has started it passes from one anode to another as each becomes more positive than the others during certain parts of the a-c cycle. Tank rectifiers are available in a wide range of voltages and in power-handling capacities up to several thousand kilowatts. Because of the low voltage drop from anode to cathode—about 20 volts—the efficiency of rectification is very high. For an output of 600 volts the efficiency is approximately 96 per cent. Because of the difficulties encountered in the prevention of undesirable arcs, and the higher voltage drop attendant with the use of baffles around the anodes, the use of *ignitrons* is favored over the multiple-anode steel-tank rectifier in the majority of installations.

17.9. Ignitrons. The ignitron, Figure 17.15, is a single-anode mercury-pool tube in which the arc is started by a third electrode called the *ignitor*, a high-resistance material, such as boron carbide, with a pencil-point tip projecting into the pool. When a current of several amperes is passed through the ignitor to the mercury by means of a control circuit, it attempts to leave the high-resistance ignitor and enter the low-resistance mercury at the nearest point of contact. As a result, a small arc is formed at the immediate junction of the two that is sufficient to establish an arc from the main anode to the pool. The ignitor need carry current only for that short time (a few microseconds) required to produce the main arc, and the control circuit can be designed to start the arc at any chosen time during the positive half cycle of the a-c input voltage. The action of the ignitor is therefore similar to that of the grid in the thyratron, and the success of the tube has resulted from this practical method of initiating the arc each cycle.

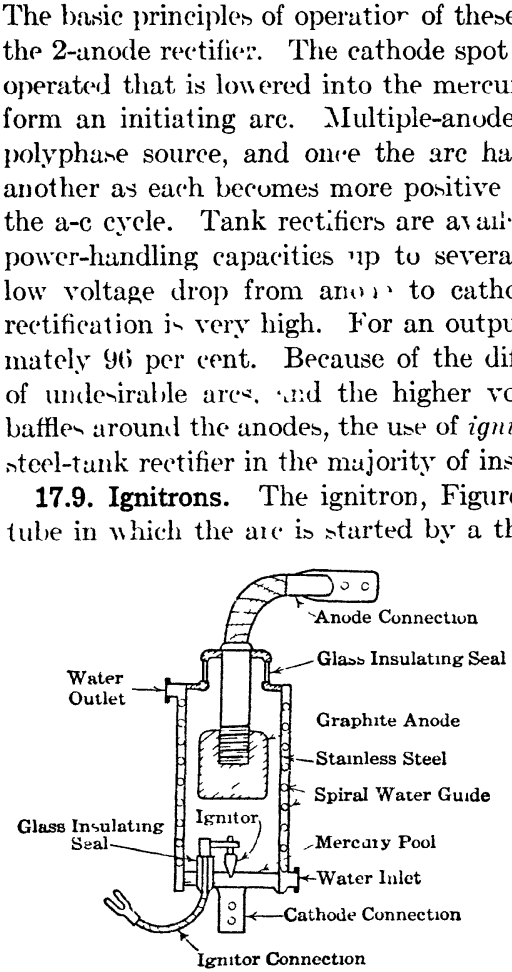


FIG. 17.15. Sketch of the internal structure of an ignitron.

During the negative half cycle of the applied voltage the ignitron carries no current between main anode and pool, whereas in the multi-anode tank rectifier the anodes are at all times in an atmosphere of ionized mercury vapor. Although arcback may occur in the ignitron, the tendency is therefore much less, and the use of several of these single-anode tubes in place of the multi-anode rectifier materially reduces the difficulties and limiting factors resulting from that problem.

The main feature of the ignitron is the extremely high values of current that can be controlled. Sealed-off tubes of relatively small physical size are available that can supply a continuous current of 400 amp. As an example of the power-handling capacities of these tubes, the WL-653-B, which has an over-all length of $29\frac{1}{2}$ in. and a diameter of $5\frac{3}{4}$ in., can be used with voltages up to 900 volts, can carry a continuous current of 200 amp, and sustain a surge current of 12,000 amp for 0.1 sec. The tube drop is approximately 15 volts when conducting. Continuously pumped ignitrons have much higher ratings.

Ignitron unit substations, using sealed-off tubes, are available commercially to supply d-c voltages ranging from 125 to 600 volts and in power ratings up to 1,000 kw. Each unit is a complete substation and includes circuit breakers, transformers, and auxiliary equipment. The efficiency of these rectifying units is quite high compared with that of a motor-generator set. For example, the over-all efficiencies of a 500-kw 600-volt unit are 95, 96, and 95.5 per cent at the respective loads of 25, 50, and 100 per cent. Ignitron units for supplying d-c power are largely supplanting rotating machines in railway service, in factories, in mines, and in chemical plants. During the war, over three million kilowatts of ignitron capacity were installed in aluminum and magnesium plants alone.

PROBLEMS

1. In the circuit of Figure 17.2, the input voltage is 250, the output voltage 105, R is 4,000 ohms, and the tube is a type-OC3 VR105 glow-discharge diode. Explain the operation of the circuit when a 5,000-ohm resistor is connected across the output terminals and calculate the change in output voltage.

2. Express the wave form of Figure 17.3b by a Fourier series.

3. Using equation 17.3, derive the expression for the exact value of the d-c current.

4. The circuit of Figure 17.3 is used with a mercury-vapor rectifier for charging a battery. The load circuit consists of a 125-ohm resistor and a 180-volt battery connected in series. The supply voltage is 220 volts rms and the tube drop is 15 volts. Write an expression for the instantaneous value of current. Calculate the time required for the battery to acquire a charge of 2 ampere-hours.

5. An FG-32 mercury-vapor diode is to be used to charge a 6.3-volt battery, having a resistance of 0.015 ohms, in a half-wave rectifier circuit. The 60-cycle supply voltage is 18 volts rms. In the rating of the diode: tube drop = 12 volts, maximum peak current = 15 amp, and maximum average current = 2.5 amp. Calculate the value of a series resistor that will allow the maximum safe charging rate, and the ampere-hours of charge after 14 hours at this rate of charging.

6. Two rectifier tubes are used in a full-wave rectifier supplying a resistance load, Figure 17.4. Calculate the peak inverse voltage of each tube for a d-c voltage of 400.

7. A full-wave rectifier and filter consists of a 5U4G vacuum rectifier, an inductance coil of 30 henries and 300 ohms resistance, and an 8-μf condenser. The tube resistance is 300 ohms. Calculate the secondary voltage of the supply transformer if the filter is to supply 400 volts, 100 ma. Calculate the percentage second-harmonic ripple voltage. Calculate the d-c output voltage for a load current of 200 ma.

8. A type-83 mercury-vapor tube is used in a full-wave rectifier circuit. The voltage drop across the tube is 12 volts and the rms plate-to-plate supply voltage is 800 volts. Calculate the d-c voltage available from the rectifier.

9. Explain in detail the operation of the circuit of Figure 17.8.

10. Calculate the ratio of the d-c voltage to the peak voltage for a 3-phase, half-wave rectifier, Figure 17.9. Calculate the ratio of the maximum inverse voltage that exists across each tube to the d-c voltage.

11. Repeat problem 10 for a 6-phase, half-wave rectifier.

12. Sinusoidal voltages are applied to both the anode and the grid of an FG-67 thyratron. The rms values of plate and grid voltages are 400 and 5, respectively. If the plate current is to flow for 120° of the cycle, determine the phase relation between plate and grid voltages. Calculate the value of plate current that would be read on a d-c meter for a resistance of 2,000 ohms in the plate circuit.

13. Redraw the vector diagram of Figure 17.13b when the positions of R and C of Figure 17.13a are interchanged. Does this new circuit allow continuous control of the plate current? Explain.

14. In the circuit of Figure 17.13, $C = 1$ μf and the frequency is 60 cycles. Calculate the value of R for the tube to fire at approximately 45° after the start of the cycle.

15. Over what portion of a cycle can control of the plate current of a thyratron be obtained by a d-c control voltage in the grid circuit?

16. A control circuit uses an FG-67 thyratron with a 500-volt 60-cycle voltage and a 200-ohm resistor connected in the plate circuit. A 10-volt 60-cycle voltage, which lags the plate voltage by 90°, and a d-c voltage in series are impressed on the grid. Sketch the wave form of the plate current for the following values of d-c voltage: -16, -6, -3, 0, $+3$, and $+6$ volts.

17. A thyratron passes current for 120° of each cycle through a 200-ohm resistor. The peak value of the current is 5 amp. Calculate the average power delivered to the resistor, and the effective value of the current.

18. The following is for a WL-679 ignitron: peak inverse rating = 900 volts, tube voltage drop = 15 volts, average d-c current rating = 100 amp, and peak anode current rating = 900 amp. Six of these tubes are to be used in a 6-phase, half-wave rectifier unit supplying 300 volts d-c. Calculate the maximum kw rating of the unit, and the efficiency at full-kw output and at half-kw output. What is the maximum inverse voltage to which each tube is subjected? Calculate the kw rating and the efficiency of the rectifier if the tubes and circuit are to be used to supply 125-volt service.

19. Calculate the highest value of d-c voltage that can be safely obtained from a 6-phase half-wave rectifier if WL-679 ignitrons, having the ratings given in problem 18, are to be used.

20. A single WL-679 ignitron is used in series with a 220-volt single-phase supply and a 5-ohm resistor. The tube drop is 15 volts for all values of current. Calculate the d-c current, the average power in the resistor, and the average plate loss for each of the following conditions: the tube fires at 0°, 45°, 90°, and 135°.

CHAPTER XVIII

ELECTRICAL INSTRUMENTS

By general accepted definition, electrical instruments are indicating devices to show the present value of voltage, current, power, etc. Electrical meters are defined as those devices that can give, in addition, a permanent record, either by a chart on which an ink trace shows the continuous variation of the quantity measured or by an arrangement of rotating dials that continue to add units of the measured quantity. The first type of device, the recording meter, applies usually to voltage, current, power, frequency, or the like. The second type, the integrating meter, is applicable in registering energy, active or reactive, for example, the watt-hour meter. The same principles of operation are employed in indicating instruments and in recording meters. While the general distinction between the instrument and the meter is as stated, it is not followed rigorously in practice and the terminologies are often used interchangeably.

18.1. Direct-current Instruments—The D'Arsonval Movement. D-c instruments are the voltmeter and the ammeter and they employ the D'Arsonval type of movement, Figure 18.1. The outer iron body is a permanent magnet which has been given special treatment so that it will retain its magnetism. Soft-iron pole pieces provide a uniform radial flux distribution across the air gaps to a fixed cylindrical soft-iron core. The current-carrying coil is wrapped on a light aluminum frame placed over the core and free to revolve about it. With current flowing in the coil, motor action takes place, the coil on its frame with a pointer attached moving as a unit. Jewel bearings reduce friction, and springs at both ends of the mounting shaft control the motion so as to give a steady deflection for a constant current. The coil sides are always under the poles and in a magnetic field of uniform strength. The torque developed and, consequently, the displacement are directly proportional to the current, thus leading to a uniform scale. Damping is obtained by eddy currents in the aluminum frame.

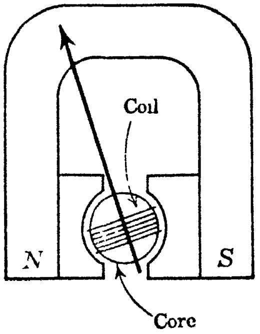
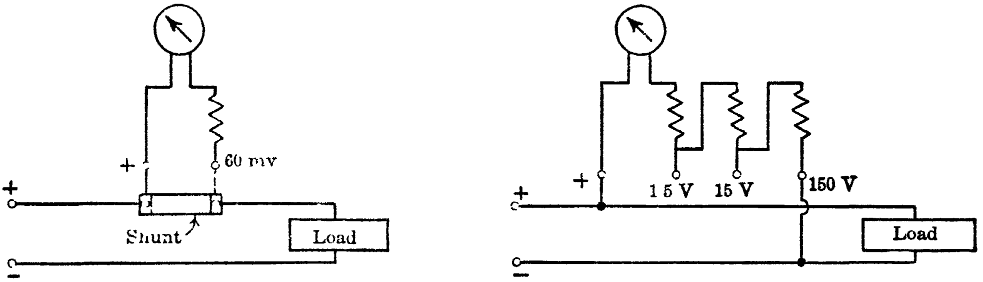


FIG. 18.1. The d'Arsonval movement

The coil is built to carry continuously only a very small current, in the range of 1 to 20 ma (in some instruments only a few microamperes), and the resistance of the element is but a few ohms. Commercial instruments give full-scale deflection for usual voltages of 50, 60, 100, or 200 millivolts on the circuit of the element Small resistances in series with the coil limit the current to its specified value for the millivolts potential to be impressed. Because the instrument is actuated by a few millivolts impressed on its terminals, it usually is called a millivoltmeter.

18.2. D-c Ammeters. If the movement is to be used to measure current other than a few milliamperes, a very low resistance, called a shunt, is placed in parallel with the element, thus diverting the greater part of the current, Figure 18.2a. If the shunt is mounted within the instrument case the entire assembly becomes an ammeter with the scale calibrated directly in amperes. A multirange ammeter has several shunt arrangements within the same case and with circuit terminals connected to binding posts appropriately marked. All of these shunts will have the same number of millivolts across them for the rated currents they are to carry, and the instrument will read full scale in each case.



(a) Ammeter—parallel resistance (b) Voltmeter—series resistance

FIG. 18.2. The millivoltmeter as ammeter and as voltmeter.

The movement only, with no shunts, may be encased with binding posts for one or more millivolt ranges, for example, 60 and 200 millivolts. As such, the instrument is a millivoltmeter, but it becomes an ammeter when used with an external shunt. It can be used with a wide range of shunts, and, if the millivolt rating of a shunt corresponds to that of the instrument, full-scale deflection is obtained for rated current in the shunt. Double scales of 100 and 150 divisions permit convenient conversion of the graduations read with various shunts to actual currents carried in the main line. The moving coil and the shunts are made of an alloy having a negligible temperature coefficient. It should be apparent that the leads connecting a millivoltmeter to a shunt must be included in the calibration of the instrument.

18.3. D-c Voltmeters. A high resistance placed in series with the coil of the element permits use of the device as a voltmeter, Figure 18.2b. The total resistance of the current path must be such as to limit the galvanometer current to normal value for the maximum voltage of the range designated. A multirange voltmeter, for example, one with 1.5-, 15-, and 150-volt terminals, will have within the case three separate series resistances for these ranges, or will have a single resistance tapped at proper points as shown in Figure 18.2. External resistances, called multipliers, in series with the instrument will allow of voltage readings beyond the range of the instrument. In general, multipliers are intended for the 150-volt circuits of instruments and they carry terminal markings indicating the over-all circuit voltage that can be measured, either in terms of volts or as multiples of 150 volts. These statements apply also to multipliers for a-c instruments.

Voltmeters employed with d-c permanent-magnet tachometer generators often have scales graduated in rpm (revolutions per minute) since speed and voltage of these small generators are proportional to each other. Such generators usually produce 6 volts at 1,000 rpm.

18.4. General Comments. Because the magnetic field is created by a permanent magnet, the usual d-c voltmeter and ammeter will give indications only for one direction of current flow in the moving coil. It is necessary, therefore, that the instrument be inserted correctly in the circuit if it is to read up-scale. On-scale deflections are obtained if the positive terminal is met first in tracing the circuit from the positive side of the power supply through the load. These instruments hence may be used as polarity indicators. If the circuit polarity is not known, as is a very usual case, it is an even chance that the instrument will read off-scale, the pointer resting against the left-hand stop. The leads to the instrument must then be interchanged.

Some instruments are set to give the zero reading at the midpoint of the entire scale length. The scales of these zero-center-reading instruments then are to show maximum positive voltage or current at the extreme right, and maximum negative voltage or current at the extreme left of the dial.

The power required to drive the D'Arsonval movement is generally but a small part of a watt, and for many d-c instruments the power may be expressed in milliwatts or even microwatts. The total loss for an ammeter and its shunt, or a voltmeter with its series resistance, will be much higher and may be several watts. Practically all the loss is in the shunt or series resistance.

D-c wattmeters are uncommon. There is generally no necessity for them since the direct product of voltage and current is the power in watts. D-c watthour meters, though, are met frequently in practice.

18.5. Movements of A-c Instruments. The D'Arsonval movement is, of course, not applicable to the measurement of a-c quantities because the current reversals through the coil in the fixed field yield zero average torque. It is therefore necessary to change considerably the form of construction of a-c instruments. Two types are by far the most common, the electrodynamometer and the moving iron vane. Both types are used for a-c voltmeters, the iron-vane type is employed practically always for a-c ammeters, and the electrodynamometer movement for wattmeters. The scales are graduated initially by comparison with *standard* instruments.

18.6. The Electrodynamometer Movement. In the electrodynamometer instrument, Figure 18.3, a magnetic field is produced by two fixed coils, F. In series with them is a movable coil, M, mounted on a spindle with a pointer and restoring springs. Light aluminum vanes, also attached to the shaft, move in a closed chamber and provide air damping.

The reaction between the magnetic fields of the fixed and movable coils produces a torque on the shaft. Since the current reverses in all coils at the same instant, the magnetic torque on the spindle always is in the same direction. This torque is a function of the field strengths of coils F and M, and of the angle between their axes. Both fields are produced by the same current, hence the torque to a great extent is a function of the square of the current. The average torque is proportional to the average square of the current, and a non-uniform scale, usually called a squared scale, results.

The electrodynamometer movement is used for a-c voltmeters, the instruments being calibrated to read effective volts. The maximum current with all coils in series is limited to about 50 ma. hence high resistances are placed in the circuits as is done with d-c voltmeters. The current range in portable ammeters having this type of movement is increased to 10 or 15 amp, the moving coil for currents above one ampere placed across a manganin resistor of very low resistance inserted in the line to act as a shunt.

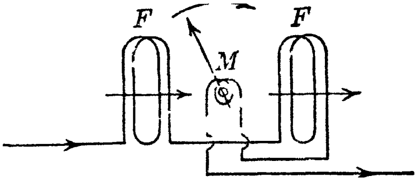


Fig. 18.3. The electrodynamometer movement.

Although a change in frequency will affect the impedance of the coils, errors caused by this factor are very small over a rather wide frequency range on most commercial instruments. The instrument may be calibrated at a standard frequency, although calibration on a d-c circuit is more usual. The electrodynamometer instrument normally is not used as an ammeter with shunts because of the difficulty, due to frequency, of matching shunts with the coils. In general, the instrument can be used on d-c circuits as well as on a-c.

18.7. A-c Wattmeters. The fixed coils of the electrodynamometer instrument may carry the line current and the movable coil may be placed across the line in series with a high resistance. Under this condition the torque on the shaft is proportional to the product of the line current (magnetic field of coils F), the line voltage (magnetic field of coil M), and the power factor which for sine waves is the cosine of the phase angle between the current and the voltage. The error introduced in phase angle by the instrument itself is negligible.

With such a connection of the coils the instrument becomes a single-phase wattmeter, Figure 18.4, calibrated to read average power. Connections, whereby the two fixed coils can be placed in parallel as well as in series, permit a double-current range. Similarly, taps on series resistances in the potential circuit increase the number of voltage ranges. Appropriate multiplying factors for the scale readings of the wattmeter, depending upon these various current and voltage ranges, usually are given inside the cover of the case or on the face of the wattmeter.

The mechanisms of two single-phase wattmeters contained within the same case, and with the two movable coils mounted on a single shaft, comprise a polyphase wattmeter. The terminals and connections of polyphase wattmeters correspond exactly to those of two separate single-phase instruments, and the average torque is the resultant of the torques of the two sets of coils. Polyphase wattmeters are used on 2-phase and 3-phase systems.

A wattmeter has three separate ratings—voltage, current, and power. The current rating is the maximum continuous current which can be carried safely

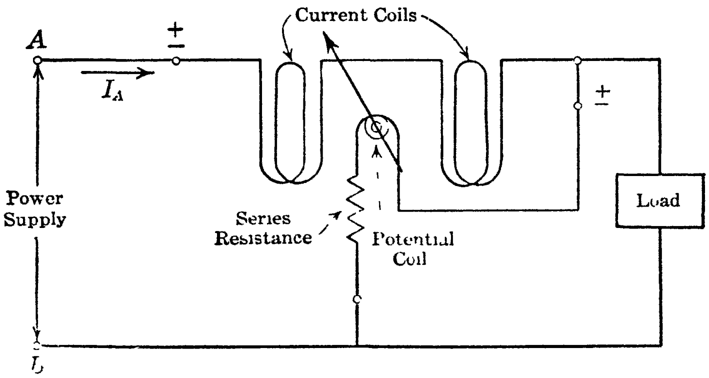


FIG. 18.4. Circuit elements of the single-phase wattmeter.

by the current coils. The voltage rating is the maximum continuous voltage to which the potential circuit can be subjected safely. The power rating is the maximum indication shown on the scale and it may or may not be equal to the product of the voltage and current ratings. Each of these three ratings applies separately, and no one of them may be exceeded, in the selection of a wattmeter for a given duty.

Wattmeters may carry information on *normal* amperes and volts, on *maximum* amperes and volts, or on both. Normal values are of historical interest only and can be disregarded. As just stated, maximum values are those which can be carried continuously by the instrument and consequently are the only ones of usual concern in its selection. Markings on the terminals for currents and voltages may or may not agree with either normal or maximum values given in the instrument cover or on its face. Similarly, the full-scale readings in watts may or may not be the products of combinations of normal, or maximum, or marked volts and amperes. There are no uniform procedures in rating and marking wattmeters that have been adopted by instrument manufacturers, and in selecting a wattmeter one is required to use his own judgment on current and voltage ranges, in choosing a satisfactory scale, and perhaps in calculating and checking necessary multiplying factors.

The great majority of single-phase wattmeters are designed for service where the phase angles between current and voltage may vary from zero to perhaps

65 or 70 degrees. For low-power-factor circuits special design is required in wattmeters, thus making them more expensive. Low-power-factor wattmeters show, by the full-scale reading in watts, only a small part of the direct product of voltage and current given by these respective ranges. In general, such instruments cannot be used on high-power-factor circuits.

The torque on the shaft of a single-phase wattmeter reverses if the connections to either the current or the potential circuit are reversed. Consequently, one current and one voltage terminal are so marked that a proper connection of the instrument, without trial and error, will give an on-scale reading. These polarity markings vary among manufacturers but, in general, are given by (±) or (0) designations. The wattmeter will read on scale if the line current enters the (±) current terminal during the same half cycle that the current in the potential circuit enters the (±) voltage terminal. For a 2-wire circuit the marked voltage terminal should be connected, therefore, to the line containing the current coils. Through this interpretation on the markings, the direction of power flow through the instrument is shown, and, conversely, a wattmeter can be used as a power-flow indicator if desired. A polyphase wattmeter or two single-phase instruments in a 3-phase circuit also will show the direction of power flow.

The (±) terminal of the potential circuit connects directly to the moving coil, thus reducing the voltage between fixed and movable coils to a minimum. Full line voltage exists between these coils if the potential circuit is reversed, and errors in readings may thereby be introduced because of electrostatic effects. It is better to reverse the current circuit should the pointer initially move off scale to the left. Wattmeters with reversing switches interchange connections to the potential coil only, not to the entire potential circuit.

Connection of the (±) voltage terminal to the (±) current terminal gives a power reading which is high by the watts loss in the current coil. Connection of this same voltage terminal to the unmarked current terminal gives a power reading which is high by the watts loss in the potential circuit. The wattmeter always will include one of these losses, depending upon the manner of connection. However, since voltages usually are standard and of constant value, and it is the current that varies widely, many wattmeters are built with compensating coils which automatically subtract *the constant loss in the potential circuit for* the second manner of connection stated above.

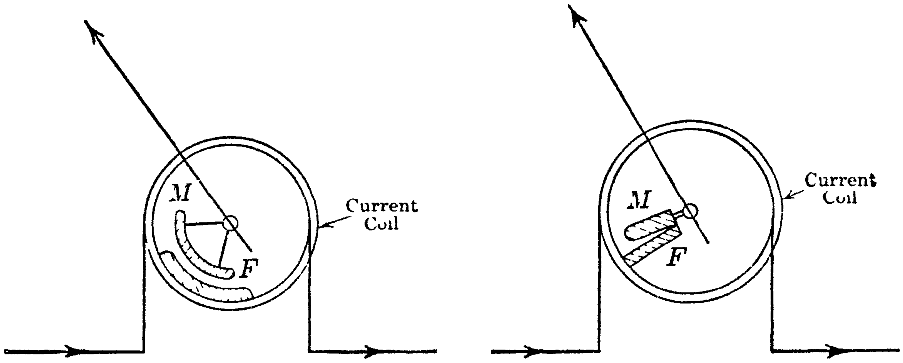
As already stated, the torque on the moving element of any wattmeter depends upon three things: the current in the stationary current, or field, coils; the current in the potential coil on the moving element; and the phase angle between these two currents. Obviously, the pointer will deflect even though the currents in the two sets of coils are obtained from unrelated circuits, provided that their frequencies are the same. With the instrument connected as in Figure 18.4, and with power flow and polarity marks as indicated, the wattmeter hence gives an on-scale reading of

$$\text{power} = kI_A E_{AB} \cos (I_A, E_{AB}) \text{ watts}$$

where k is the multiplying factor, usually an integer, dependent upon the current and voltage terminals used.

Interchange of the current-coil terminals (now $-I_A$ in the above equation), interchange of the potential-coil terminals (now E_{BA}), or reverse power flow (also now $-I_A$) will move the pointer against the left-hand stop. It should be stressed that the manner of writing the general power equation for the instrument is directly dependent upon the terminal markings and the connection of the device. In predicting wattmeter readings from vector diagrams for circuits, for example, in the 2-wattmeter method of measuring 3-phase power, these conventions must be observed.

18.8. Iron-vane Movements. Early moving-iron instruments had as the current-carrying coil a solenoid, sometimes bent into an arc of a circle. A soft-iron core, fastened to the shaft of the device, moved a pointer across a scale as the core was pulled into the solenoid by magnetic action.



(a) Vanes concentric (b) Vanes radial

FIG. 18.5. Mechanism of the iron-vane instrument.

Practically all modern a-c ammeters are of the iron-vane type, the construction allowing measurement of much higher currents than can be used with the electrodynamometer instrument. In the usual form of the iron-vane instrument, Figure 18.5a, the line current flows through a coil within which are two concentric soft-iron bodies. Consequently, these vanes are magnetized in like manner and exert a repelling force on each other. The force always is of the same character because the polarities of the iron vanes change together as the current passes through its alternations. With the outer vane F fixed, the inner vane M then rotates the movable mechanism of shaft and pointer to which it is rigidly fastened. The strength of the magnetic poles of the vanes is determined by the field produced by the current, and the shaft torque therefore is proportional to a certain degree to the square of the current. The vanes are so placed with respect to each other that their magnetic centers of gravity do not lie on the same radius at zero deflection. A change in relative shape of the vanes leads to a somewhat uniform scale over a large part of the range.

Two rectangular vanes placed radially within the coil, Figure 18.5b, one vane fixed and the other secured to the shaft, also provide repelling force action of the character just described. These instruments are the most sensitive of any of the iron-vane types and give a fairly linear scale.

A single iron vane of rectangular shape may be mounted at an angle of about 45 degrees on the shaft of the instrument. The coil also is inclined at about the same angle with the shaft, the axes of vane and coil being approximately 90 degrees apart at the zero position of the pointer. Current in the coil magnetizes the vane and it turns the shaft in that direction which will increase the flux carried longitudinally by the vane, thereby moving a pointer across a scale.

Iron-vane ammeters are built for current ranges from a few amperes to several hundred amperes, although in practice current transformers usually are used for currents above 50 amp. The instruments may be designed with 1- or 5-amp coils, the range being extended as desired by means of the ratios of the transformers. Multirange ammeters sometimes have current transformers within the case—several primary windings on the transformer core and a single secondary coil. Such an instrument, of course, cannot be used on a d-c circuit.

Voltmeters also are made using iron-vane movements, the coils having many turns of fine wire. Series resistances limit the current taken from the line. Although not so accurate as the electrodynamometer type of voltmeter, they are simpler in construction and they cost less. Within an error of perhaps one per cent, iron-vane instruments containing no current transformers can be used for d-c as well as for a-c measurements.

18.9. Rectifier Instruments. Rectifier instruments employ the typical D'Arsonval movement in a bridge circuit of copper-oxide rectifiers so as to permit measurements on a-c systems. Figure 18.6 shows the bridge circuit, the arrows indicating the unidirectional current characteristic of the rectifiers. Each rectifier is composed of two very small plates of copper and a soft metal with a layer of cuprous oxide between them. Current flow is only from the oxide to the copper. The bridge circuit provides for a unidirectional current through the galvanometer element regardless of the external polarity, and deflections, now proportional to the average current carried by the element, give a linear scale.

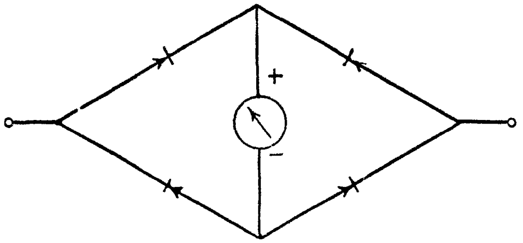
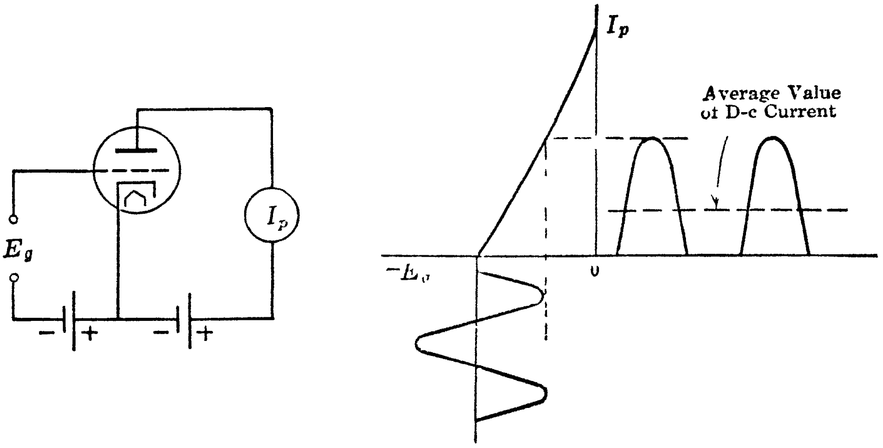


FIG. 18.6. Bridge circuit of the rectifier instrument.

Rectifier instruments are built as ammeters and as voltmeters and are calibrated to show effective values for pure sine waves. Because the average current determines the deflection, errors are introduced in the readings if the wave forms of the measured quantities are non-sinusoidal. The instrument can be used for direct as well as for alternating currents and voltages by a change in the external circuit. A rectifier voltmeter, for example, calibrated to read a-c voltages, will read high on d-c voltages by the factor 1.11. An appropriate resistance, though, to be connected externally in series with the device when measuring d-c voltages, can accompany the instrument and thus make it serviceable for either a-c or d-c circuits. Rectifier instruments are built for actuating currents in the range from a few milliamperes down to a few microamperes.

18.10. Vacuum-tube Voltmeters. The vacuum tube can be used to measure d-c voltages and a-c voltages extending from the lowest audio frequencies up to frequencies of the order of 1,000 mc. In measuring a-c voltages use is made of the non-linear characteristic of the tube. Numerous circuits have been developed for the measurement of both d-c and a-c voltages, one of the more widely used being shown in Figure 18.7a. The grid is biased to cutoff, and a d-c milliammeter is connected in the plate circuit. When an a-c voltage is applied to the grid circuit, a d-c plate current flows, as shown in Figure 18.7b. When properly designed, the instrument can be calibrated on 60 cycles, and the calibration can be relied upon up to very high frequencies. If the grid is not driven positive, the input resistance is extremely high and the power taken from the source is substantially zero.



(a) Simplified circuit (b) Rectifying characteristics

FIG. 18.7. A typical vacuum-tube voltmeter circuit.

Another widely used vacuum-tube voltmeter makes use of the rectifying property of the diode, Figure 18.8. The time constant of the *R-C* circuit is made several times greater than the period of the lowest frequency to be measured. Under these conditions, the d-c voltage of the condenser builds up to a value very nearly equal to the peak of the a-c voltage being measured. The voltage across the *R-C* combination is read by a d-c vacuum-tube voltmeter.

A wide variety of commercial vacuum-tube voltmeters is available, each one having certain special features. In some models the voltage is amplified by a stabilized negative-feedback amplifier before being measured. One instrument obtains a sensitivity in this manner sufficient to give a full-scale reading for a voltage of one millivolt. The input resistance of the commercial models ranges from 1 to 5,000 megohms. One model has substantially no frequency error up to 700 megacycles.

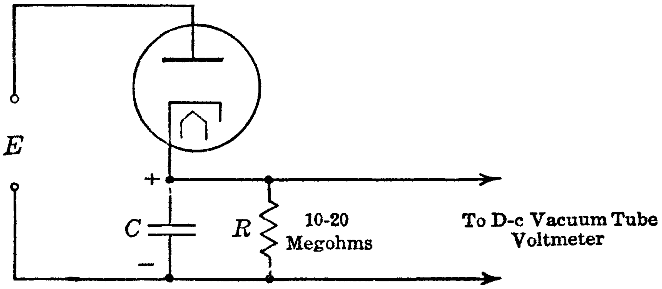
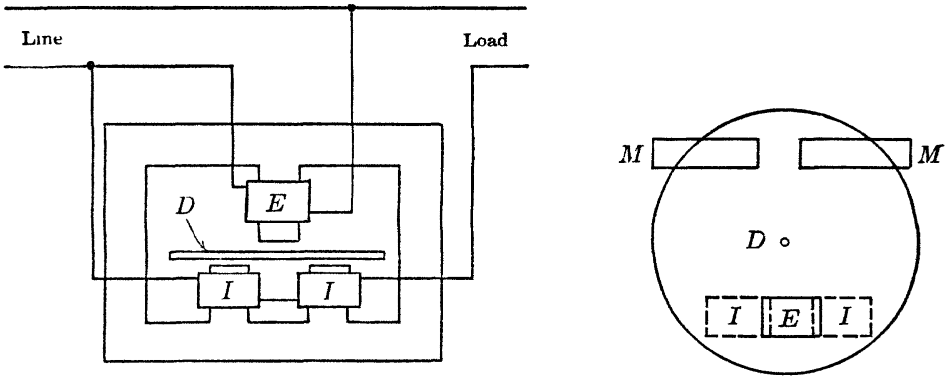


FIG. 18.8. Circuit of a diode vacuum-tube voltmeter.

18.11. Watt-hour Meters. The a-c watt-hour meter, which registers electrical energy, is essentially a small induction (eddy-current) motor, the rotor being an aluminum disk, *D*, Figure 18.9a. The current coils *I* carry the line current and produce a flux between their magnets that traverses the space in which the disk lies. Eddy currents, created in the disk by the potential coil *E*, lie in this field of flux and motor action ensues. The driving torque is proportional to the product of the line current, the line voltage, and the circuit power factor. Because the action is obtained through induction, these measuring devices often are called induction watt-hour meters.



(a) Essential electric and magnetic circuits

(b) Top view showing relative position of coils, magnets, and disk

FIG. 18.9. Simplified diagram of the watt-hour meter.

The disk also rotates between the poles of permanent magnets *M*, Figure 18.9b, and damping is developed by virtue of eddy currents which they produce. The retarding torque is proportional to the speed of the disk, and hence the speed is proportional to the driving torque, or power. A series of pointers, geared to the shaft of the disk and arranged in fixed speed ratios, introduce the time element by indicating total revolutions. This time integration of power, which is energy, is registered on dials over which the pointers move. The graduations usually are given in kilowatt-hours.

As has been stated, the speed of rotation of the disk of the a-c watt-hour meter, or kilowatt-hour meter, is proportional to the product of the line voltage, the line current, and the cosine of the phase angle between them. By making the speed of the disk proportional to the product of voltage, current, and the sine of the phase angle, the meter can be used to register reactive kilovolt-ampere hours. This is achieved by applying to the potential circuit of the meter a voltage at 90 degrees to the normal voltage measured. The particular manner of obtaining such a quadrature voltage depends upon the nature of the circuit. In practice it is from the readings of the kilowatt-hour and reactive kilovolt-ampere-hour meters for a given period of time, a month for example, that an average power factor for a system is calculated.

One type of d-c watt-hour meter is fundamentally a shunt motor and is an electrodynamometer movement with a commutator. The field coils carry the line current, and several armature coils, which are connected to a commutator, are placed across the line in series with a high resistance. There is no iron in the magnetic circuit. The driving torque on the armature, which is free to rotate, is directly proportional to the product of line voltage and current, and the speed is controlled by a disk and permanent magnets as in the a-c watt-hour meter. A series of dials registers energy also in the same manner described for the a-c watt-hour meter.

A second type of d-c watt-hour meter employs a copper disk which is free to rotate in a closed chamber filled with mercury. The line current is carried to the disk through the mercury, entering and leaving at diametrically opposite edges. The current path in the disk lies in a magnetic field which is produced by a coil placed across the line. The field strength hence is proportional to the line voltage, and the driving torque on the movable mechanism is proportional to the circuit power.

Watt-hour meters are integrating meters since they show the total energy transmitted through the circuit up to the time they are read. The energy over a previous interval is obtained by subtracting the reading taken at the beginning of the period from that taken at the end of the interval.

18.12. Recording Meters. Recording, or graphic, meters give the values of voltage, current, power, power factor, frequency, etc., with time so as to form a permanent record. A clock mechanism drives a circular paper chart or a roll under an inking pen, thus to give a continuous trace of the magnitude of the quantity being measured. The circular charts are primarily for 24-hour records, although a few are for 7-day periods. Rolls may cover longer time intervals. Recording meters are fundamentally indicating instruments with the substitution of the inking pen and the chart for the pointer and the dial.

Graphic meters are built for portable or for switchboard use. The majority have recording elements that are actuated by their measuring elements, but in many cases the measuring element controls the power of an auxiliary circuit from which the recording mechanism is driven. Some graphic instruments employ an optical system of control. The charts or rolls may be moved by spring-wound clocks or by small synchronous clock motors.

18.13. Other Instruments. There are many other instruments that are in general use in the electrical field, and a few of them are noted briefly here. Any complete description of these more special instruments is beyond the scope of this discussion, and the reader is referred to electrical handbooks or to manufacturers' publications for further details on construction and operation of such devices.

Frequency is indicated in several ways. The simplest frequency meter comprises a series of steel reeds, each tuned to a slightly different mechanical frequency so that it will vibrate in resonance with a given system frequency as the latter is varied. The excitation is produced by an electromagnet, the coil of which is across the line. The frequency of each reed is marked on a scale opposite the row of reeds, and the system frequency is shown by that reed having the greatest amplitude of vibration. Two other means of indicating frequency are by the induction-type frequency meter, and by an instrument in an impedance-bridge circuit

The single-phase power-factor meter, Figure 18.10, built somewhat on the principle of the wattmeter, shows the cosine of the phase angle between voltage and current regardless of the magnitude of either. The main field is produced by a coil in series with the line and hence depends upon the current. The two smaller coils are fastened at 90 degrees with each other, are free to rotate within the fixed coil and with no restoring torque, and are connected across the lines so as to obtain dependence on the voltage. Their connections are as shown, one circuit containing a high resistance and the other a high inductance. The position of the rotating element depends upon the phase angle between line voltage and current, and the two extremes are those in which the axis of either movable coil lies along the axis of the fixed coil. A pointer attached to the spindle moves over a scale graduated in the percentage equivalents of the cosines of the dis-

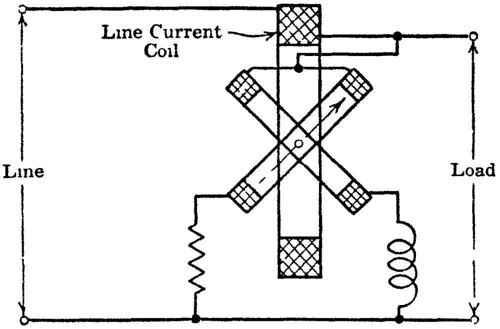


FIG. 18.10. Elements of the power-factor meter.

placement angles with 100 per cent (unity) power factor at mid-scale. Both frequency and power-factor meters may record as well as indicate the quantities they measure.

The synchroscope employs the same principle as the power-factor meter, but the fixed coil is excited from the machine to be synchronized and is placed around a 2-pole magnetic structure surrounding the movable coils. The movable coils and their circuits are connected as before and are placed across the main lines. The three leads to these coils are brought out through slip rings so that the coil assembly can turn as a motor. The direction of motion of a pointer attached to these coils indicates the relative speed of the incoming machine, and the position of the pointer shows the phase position of the incoming machine with respect to the line. The pointer should be stationary, at the position of zero angle, when the machine is synchronized. Synchroscopes serve the same purpose as lamps in synchronizing alternators.

Instruments having the trade names of Clamp-On or Tong-Test ammeters consist of movable, laminated iron jaws which can surround a conductor or a cable and thereby measure the current being carried. In some ammeters of this type that measure alternating currents the conductors enclosed by the jaws become the primary winding of a current transformer. The secondary coil of many turns of small wire is connected to a copper-oxide rectifier bridge circuit A group of resistances in the galvanometer branch provides several current ranges, from a few amperes to several hundred. Another type of tong-test ammeter, suitable for d-c as well as a-c measurements, concentrates the flux of the iron-jaw circuit in an air gap in the instrument. An iron-vane movement in this field is acted upon to carry a pointer over a scale. Clamp-on ammeters indicate the total current passing through the jaws and thus they show only the current being carried by an equivalent single conductor. Where the number of conductors of a cable is known, and all are to carry the same current, such an instrument will detect short-circuited or reversed turns.

Multimeters, or test sets, are very compact units for general testing. They may contain circuits, batteries, and galvanometers so that several ranges each of a-c and d-c voltage and current, resistance, inductance, capacitance, and reactance can be measured with good accuracy. Many types of test sets are available with varying character of measurements that can be made and with varying ranges of those quantities. A single galvanometer may have associated circuits so that the pointer will indicate the magnitudes of all quantities, or the unit may contain two galvanometers, one for d-c and the other for a-c testing.

When measuring resistance, the test set functions as an ohmmeter, and several types of circuit are used in practice. A general method is to place the unknown resistance, if low, in parallel with the galvanometer so as to shunt current from the element. If the resistance is high it is placed in series with the galvanometer element. In either case, readings are obtained directly in ohms. A common type of ohmmeter is a compact Wheatstone bridge, with batteries and zero-center galvanometer, all contained in a small case. Terminals provide for connection to the unknown resistance, which becomes one arm of the bridge, the rheostat arm being a group of decade resistances with dial settings. Bridge ohmmeters have ranges from 0.001 to several million ohms, whereas the other types give readings up to a few thousand megohms.

Meggers also are instruments used in measuring resistance. They contain a small hand-driven magneto and a galvanometer in a permanent-magnet field. The moving element carries two coils mounted at right angles with each other, separate circuits for these two coils being in parallel between the generator brushes. One coil is placed across the line and hence produces a torque proportional to the line voltage. The second coil, in series with the resistance to be measured, also carries a current proportional to the line voltage but inversely proportional to the unknown resistance. The torques of the two coils are in opposition, and, with no springs providing a restoring torque, the pointer takes a position dependent upon the relative effects of the coils. On open circuit the potential coil acts to move the pointer to one extreme position, and on short circuit the effect of the current coil predominates to move the pointer to the other extreme position, 90 degrees from the first. The scale is calibrated directly in ohms from zero to infinity. Resistances in series with both coils limit the currents which they may carry. While normal speed of the crank will produce 500 volts d-c, for reasons indicated above, the readings are independent of the generator speed.

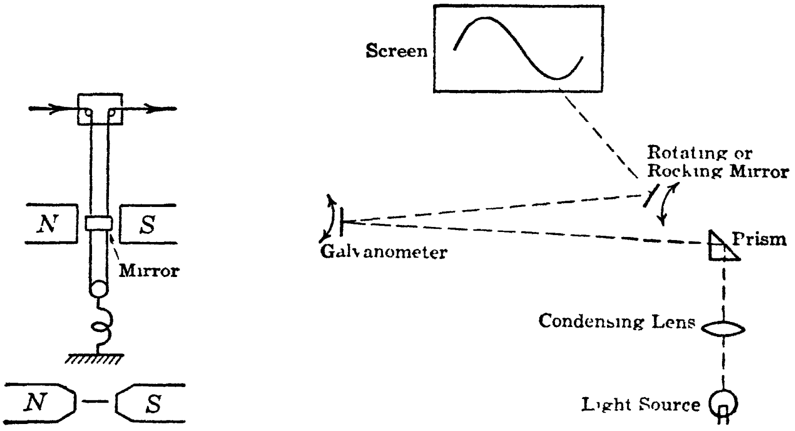
Instruments measuring voltage or current at high frequencies may be of the hot-wire or thermocouple types. In the former, a wire extends with increasing temperature because of the current it carries and allows a spring to rotate a pointer. In the latter, heat produced at the junction of the thermocouple, which is placed in contact with a conductor carrying the line current, gives rise to a thermal voltage which becomes a measure of that line current.

Temperature indicators are of many types. One of the most common is that in which a search coil is one arm of a bridge circuit. Temperature changes alter the resistance of the coil and consequently vary the current through a galvanometer in the network. The instrument is calibrated directly in degrees temperature.

18.14. The Magnetic Oscillograph. The magnetic (sometimes called mechanical) oscillograph is an instrument that shows the time variation of a quantity being studied by means of a light trace on a screen or on a film. This type of oscillograph is distinct in many ways from the cathode-ray oscillograph. Commercial cathode-ray tubes usually have but one beam and therefore can give only one trace at a time. The magnetic oscillograph may have one, two, three, six, or more elements so as to show many wave shapes on the same screen or film at the same time. Again, cathode-ray tubes respond to radio frequencies, whereas the magnetic oscillograph is limited generally to frequencies below 5,000 or 6,000 cycles per second.

The essential parts of an oscillograph are (a) a light source with its sequence of lenses, prisms, and mirrors to direct the path of light, (b) a galvanometer element, (c) a rotating or vibrating mirror, and (d) a reviewing screen. When taking pictures, a stationary film replaces the screen, or a moving film is substituted for the rotating mirror and the screen. The schematic diagram for the oscillograph is shown in Figure 18.11.

Light from a low-voltage lamp is projected onto a tiny mirror glued to a looped conductor in a strong magnetic field. The plate of the mirror is rotated as the current varies through the loop, the angle of deflection being proportional to the strength of the current. The light beam, consequently, is moved back and forth as the galvanometer mirror swings to and fro.



(a) Galvanometer circuit

(b) Light circuit

FIG. 18.11. Essential elements of the magnetic oscillograph.

The light from the galvanometer mirror is projected further to a rotating mirror which moves the beam in a plane at 90 degrees with the first motion. This second mirror provides the time axis, and the consequent result is movement of the light beam in two dimensions. The timing mirror is driven at a synchronous speed so that each repetition of its motion retraces the wave shape and hence shows the wave in a fixed position. The magnitude of the galvanometer current therefore is given, as a function of time, by the displacement above or below a zero or reference axis.

What has been said of the D'Arsonval movement in its application applies equally as well to the oscillograph element. When used for current indications, the element is placed across a non-inductive shunt, the millivolt drop being proportional to the measured current. When used for voltage indications, a high non-inductive resistance is placed in series with the element to keep the galvanometer current within a safe value.

18.15. The Cathode-ray Oscillograph. A cathode-ray oscillograph consists of an assemblage of a cathode-ray tube and its associated power supply, voltage amplifiers, and a saw-tooth-wave oscillator for generating the linear sweep voltage, arranged for convenient and flexible operation. The basic principles of the cathode-ray tube and the saw-tooth sweep circuit are discussed in Chapter XIV. Means for adjusting the rate of sweep along the time axis are provided, most commercial units incorporating a sweep-frequency range of 2 to 30,000 saw-tooth cps. The amplifiers generally have sufficient gain so that voltages of the order of a few millivolts will produce large deflections of the cathode-ray spot along both the vertical and horizontal axes. If desired, the signal voltage can be applied directly to either the vertical or horizontal deflecting plates, thus eliminating the characteristics of the amplifiers. Controls for focusing and adjusting the intensity of the spot and for shifting its position on the screen are normally included.

The pattern which the electron beam traces on the screen is sufficiently well defined so that it can be viewed even in a well-lighted room or it can be photographed The pattern may be projected on a large screen for making a detailed study of a wave form or for demonstration purposes by means of a lens. With one commercial lens, a pattern 3 by 3 in on the tube screen can be projected to give an image 12 ft square.

The principal advantage of the cathode-ray oscillograph are that it responds to extremely high frequencies and that the current it takes from the source under observation is negligible. Although cathode-ray tubes can be used at frequencies up to 100 mc, the design of the amplifiers generally sets a frequency limit of a few megacycles for the more commonly used models.

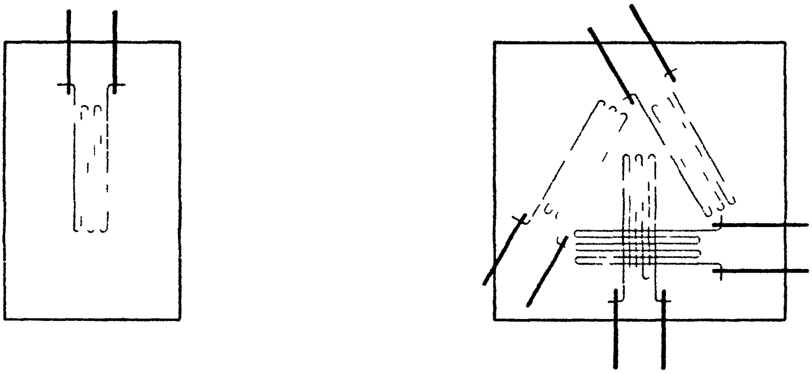
Commercial models are available with various screen sizes, and some units incorporate many special features. For example, one model has two separate and completely independent beams in the one tube, each beam having its own electron gun. With this oscillograph, any two wave forms can be viewed simultaneously.

18.16. Transducers: The Resistance Strain Gage. A transducer is a device that is capable of being actuated by a signal or power from one system and supplying a related signal or power to another system. The signal or power may be constant or varying with time and may be the same type or of different types in the input and output systems, e.g., thermal to electrical (thermocouple), electrical to acoustical (loud-speaker), mechanical to electrical (phonograph pickup, strain gage), light energy to electrical (photocell), electrical to electrical (transformer), etc. Within this definition, an electrical meter can be classified as a tranducer; it translates an electrical signal into a visual indication or records it in some suitable form.

The development of transducers has progressed to the stage where it is possible to translate a signal of practically any type into an electrical quantity, in which form it can be amplified, conveniently controlled, transmitted to any desired location and finally transformed, by means of an output transducer, into a suitable indication, record, or control operation. The flexibility, accuracy, and speed of measurement afforded by appropriate combinations of transducers and electrical circuits have led to their use in the measurement of practically all types of physical quantities. In addition, these systems have had a wide application in the automatic control of various industrial processes.

It is not the purpose of this section to discuss the many and varied electrical methods that are used in the measurement of physical quantities. The resistance strain gage is of sufficient importance and has such a wide range of application, however, that it will be discussed in some detail and, together with its associated circuitry, will serve to illustrate the possibilities of the measurement of non-electrical quantities by electrical means.

The principle of the resistance strain gage is extremely simple. A gage consists of a length of very fine wire (about 0.001 in. in diameter) arranged in one of a variety of patterns and cemented between two pieces of thin paper. In the general-purpose gage the wire is formed into a grid as illustrated in Figure 18.12a. In Figure 18.12b is shown a pattern that can be used for measuring multi-directional strains. Commercial units that have been accurately calibrated are available in a variety of patterns and sizes. In use, the gage is firmly cemented to the member to be tested and experiences the same strain as the test member. The resistance of the wire varies with the strain and a measure of this resistance change is an accurate measure of the strain. The gage responds equally well to either tension or compression.



(a) General-purpose gage

(b) Gage for measuring multi-directional strains

FIG. 18.12 Grid patterns for two typical strain gages.

Under tension, the length of the gage wire increases, its area decreases, and, in general, the specific resistance of the wire material changes. These changes result in a change of resistance that is in accordance with equation 3.3. The *gage factor* is the ratio of unit resistance change to strain, i.e.,

$$\text{gage factor} = F = \frac{\Delta R/R}{\Delta L/L} \tag{18.1}$$

where R and L represent the initial resistance and initial length of the strain gage wire, respectively, and ΔR represents the change in resistance as a result of the change in length ΔL. Commercial units have gage factors in the range of from 1.5 to 3.5 and a nominal resistance between 60 and 1,000 ohms.

The simplest circuit for measuring static strains is the Wheatstone-bridge arrangement shown in Figure 18.13. The bridge is initially balanced by means of R_3 and R_4 with the slide-wire rheostat, S, in the midpoint position (zero setting on the dial). After strain is applied to the *active* gage, the bridge is rebalanced by means of the slide wire. The change in resistance, either plus or minus, can be read from the slide wire setting or the dial can be calibrated directly to indicate the strain in micro-inches per inch. A resistor can be used in the bridge circuit in place of the *dummy* gage. The dummy gage, however, serves to compensate for the effect of temperature changes of the gage wire; both gages are located close together (often at some distance from the bridge) and experience the same temperature changes. Since the strain generally produces an extremely small percentage change in resistance, a highly sensitive galvanometer must be used. With a galvanometer of such high sensitivity in the circuit, the application of this d-c bridge circuit is limited mainly to laboratory service.

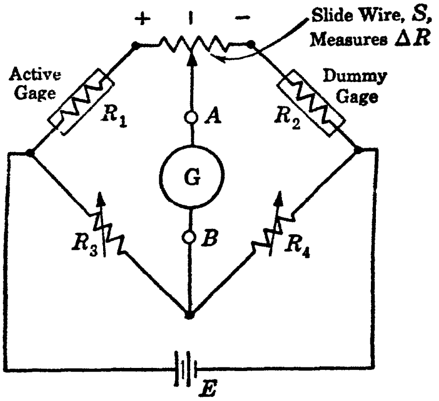


Fig. 18.13. Basic strain-gage bridge using an active gage and a dummy gage.

In one type of portable strain indicator that is widely used, the bridge is excited with an audio-frequency voltage and the bridge output signal is amplified before applying it to an indicating meter. The schematic circuit arrangement is shown in Figure 18.14. The simpler design and greater stability of audio-frequency amplifiers as compared with d-c amplifiers are the principle factors that favor the use of the audio-frequency signal. The a-c output of the amplifier is fed to a *phase-sensitive* rectifier (detector), the output of which actuates a high-resistance d-c galvanometer. Tensile strain causes the galvanometer needle

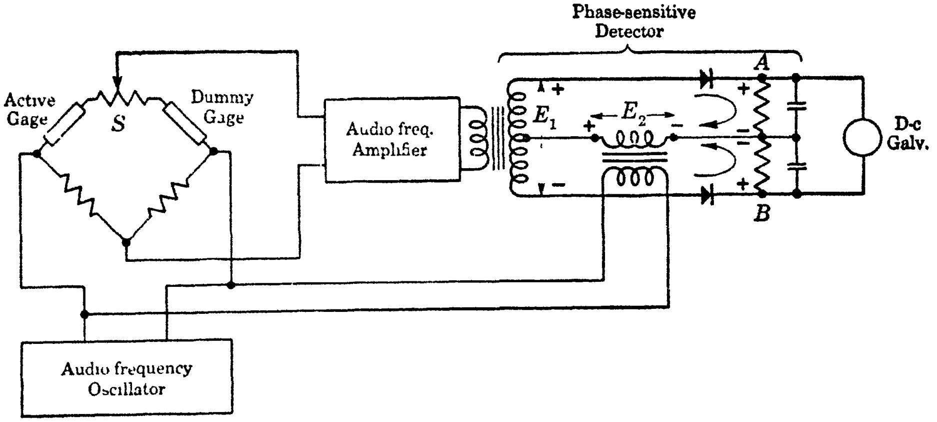


Fig. 18.14. Schematic diagram of a typical commercial strain indicator.

to move in one direction while compressive strains cause the needle to move in the opposite direction. When the bridge is rebalanced by means of the slide-wire rheostat S, the reading on the attached dial indicates the strain. In some models, an electronically driven servo balances the bridge automatically and a pen attached to the servo mechanism records the strain. These latter models will record *dynamic strains*, of relatively low frequency, the frequency being limited by the response time of the servo and recording system.

It is necessary to use a phase-sensitive rectifier circuit with the a-c system in order to determine sign of the strain. If a simple rectifier instrument were connected to the output of the amplifier, the meter would read in the same direction whether the resistance of the strain gage increased or decreased. The bridge output voltage, however, is either in phase or 180 degrees out of phase with the bridge input voltage, depending upon whether the gage resistance is greater or less than the value corresponding to balance. The phase-sensitive rectifier compares the phase of the amplifier output voltage with that of the bridge input voltage and gives an output that is proportional to the strain and of the proper sign. The analysis of the circuit is left as an exercise for the student (see problem 30).

The balanced-bridge method of measuring static or slowly varying strains is easy to use and gives a high degree of accuracy. It is obvious, however, that it would be impossible to follow a bridge balancing procedure if the strain in the test member were varying at several hundred or thousand cycles per second. For dynamic measurements, exactly the same a-c bridge circuit, amplifier, and phase-sensitive detector can be used. The output of the detector is recorded on an oscillograph or pen-and-ink recorder. After the bridge is initially balanced, the oscillographic record gives a measure of the strain In this strain-indicating system, the audio frequency applied to the bridge circuit is known as the *carrier frequency*. The strain-gage signal modulates the carrier frequency, the envelope of the modulated wave being proportional to the strain. For proper operation the carrier frequency needs to be at least five times the highest strain frequency to be measured.

Dynamic strains can be measured, also, by using the d-c bridge, a d-c amplifier, and an appropriate recorder. One commercial unit, using a cathode-ray oscillograph with a special four-gun tube, permits the simultaneous recording of four different strains. The frequency response of 0 to 50,000 cycles per second of this unit makes possible high-frequency and transient stress studies of rockets, jet engines, high-speed reciprocating mechanisms, impact loads, etc.

18.17. The Selection of Instruments. The selection of instruments for a given job depends, of course, solely upon the nature of that job. In no case should any instrument be connected into a circuit without specific knowledge that its range will not be exceeded. In general, this involves but one quantity for each instrument, but for wattmeters it may require knowledge on voltage, current, and power. The first necessary step in choosing any measuring device for a system is a satisfactory preliminary study of known circuit conditions to be met, with thought also given in anticipating others. For example, measurement of the no-load current of a motor ordinarily would require a low-range ammeter, but, if the instrument is in the circuit at starting, the starting current may destroy it. Not even an ammeter of normal range for a motor may be left in the circuit at starting. Protective devices, therefore, enter the preliminary survey as well as do the meters. What has been stated presupposes a working knowledge on the functions of the usual kinds of electrical instruments and the proper manner of connecting them.

Electrical machines and equipment all carry nameplate information which very definitely displays the normal or maximum conditions within which the equipment has been designed to operate continuously. Instruments are no exception. The quantities given in nameplate data of apparatus, expressed either directly or implicitly, and which are of concern in choosing instruments for particular tests or measurements, involve such items as d-c or a-c required supply, number of phases, voltage, current, power, frequency, power factor, speed, etc. The nameplate information of instruments is contained in the manufacturers' stated ranges found on the buttons, terminals, or scales, or within the covers of the cases. Instruments are to be so selected that their ranges will be sufficient for the desired measurements on a particular piece of apparatus or on a given circuit. It should be noted that, although voltmeters and wattmeter potential circuits protect themselves up to the limit of their ratings, ammeters and wattmeter current coils are protected only by the impedance of the load circuit. Instrument ranges are general and cover standard magnitudes which indicate the maximum current or voltage to be used. They do not show the actual current or voltage in the operating coils of the movement.

Since the d-c instrument has a uniform scale, the ease in taking readings is practically the same for all positions of the pointer, and the entire scale generally may be used. However, the range of the instrument should be such that, with due allowance for a reasonable overload, readings of normal quantities are well up on the scale. The a-c ammeter and voltmeter, particularly, have so-called "squared scales" which may be very greatly cramped at the lower ends. This first part, roughly 20 per cent of the entire scale, is seriously limited in accuracy, and readings in this region are generally not satisfactory. An a-c ammeter or voltmeter should be selected to read not lower than mid-scale the most representative values of the quantity measured. Since neither the current nor the potential circuit of a wattmeter must be overloaded, one has to be satisfied with the subsequent power readings even though they may be well down on the scale.

The accuracy of instruments usually is given as a percentage error of full scale, for example, $\frac{1}{4}$ of 1 per cent of full scale at any part of the scale. This means that the possible error of any reading on a 150-volt voltmeter having 150 scale divisions is then within $\frac{3}{8}$ of a division, although it does not imply that all readings may have that maximum error. Obviously, the percentage error of readings may be minimized if the range of the instrument is reasonably comparable to the normal magnitudes of the quantity measured. The meter range should not exceed by very much the highest magnitude to be read.

Generally, an open-circuit test on a generator requires a voltmeter that will read to 150 per cent of rated voltage. For a no-load test on a transformer or a motor, the ammeter will be of very low range, and the highest magnitude to be read probably will have to be estimated. As noted above, protection must be provided in these cases for the ammeter. An ammeter switch is preferred to a short-circuiting switch because the first type of device actually removes the ammeter from the circuit when the switch is closed in one position. Closed-circuit metering jacks, with ammeter plugs, provide the same protection.

An armature resistance test or a short-circuit test on either an alternator or a transformer requires an ammeter reading from 100 to 200 per cent of rated current. The corresponding voltage readings will be low. Good practice is to check the general magnitude of such voltage readings at start by a sufficiently high-range instrument.

Load tests are normally taken at rated voltage and over a current range from zero, or no-load, current, up to 150 per cent of rated current. Field currents of machines have to be estimated. In special tests, where the run is to cover a wide range in current or voltage, it may be advisable to use two or more instruments, each having a different range, or a multirange instrument.

PROBLEMS

1. A milliammeter has a total circuit resistance, including the shunt leads, of 10 ohms. If full-scale deflection is obtained with a 200-amp 60-mv shunt, what is the resistance of the shunt, the parallel resistance of the shunt and the instrument circuit, and the full-scale watts loss in the shunt?

2. A millivoltmeter having terminals for 200- and 60-mv shunts has internal resistances of 10 and 3 ohms, respectively. The scale divisions are 100 and 150. Forty-two amperes are to be measured, using a 50-amp 60-mv shunt.

(a) What is the resistance of the shunt, and what scale deflection would be read?
(b) What current in the element gives full-scale deflection?
(c) Repeat (a) for a line current of 12 amp, using a 15-amp 200-mv shunt.

3. A millivoltmeter, designed for use with 60-mv shunts, has 2 scales, one of 100 divisions and the other of 150 divisions. What factor is required if current is measured by the millivoltmeter and a 75-amp 50-mv shunt?

4. A millivoltmeter having an internal resistance of 5 ohms is to be used as an ammeter with 60-mv shunts.

(a) What should be the resistance of the shunt if 400 amp full scale are to be read?
(b) What should be the resistance of the shunt if the instrument is to read zero at mid-scale and is to read 400 amp for maximum deflection either side?
(c) If the shunt resistance is to be fixed at the value of (a), what else could be done to satisfy the conditions of (b)?

5. Two millivoltmeters, scaled to indicate amperes, have 20-ma movements and 10 ohms resistance in each movement and are used to read up to 100 amp with 200-mv shunts. The shunts are placed in series adjacent to each other in the main circuit with one ammeter directly across its shunt. The other ammeter is located at some distance from its shunt, the leads containing 2.5 ohms. If the distant ammeter is actuated by the voltage drop across its own shunt, what change in cross-sectional area of that shunt will permit both ammeters to read alike? If the leads of the distant ammeter are placed across both shunts in series, what resistance added to this circuit will permit equal readings on both ammeters?

6. A galvanometer element has a resistance of 10 ohms, and it carries 10 ma for full-scale reading as a millivoltmeter. What series resistances are required if it is to have scale readings of 150, 300, and 600 volts?

7. A 10-ma galvanometer movement has a resistance of 15 ohms and is to be used as a voltmeter with 5-, 15-, and 150-volt ranges. Calculate the series resistances for each range. What resistances in a multiplier will permit measurements on 300- and 750-volt systems?

8. A voltmeter having a 10-ma movement and a resistance of 15,000 ohms is connected in series with another instrument having a 15-ma movement and a resistance of 20,000 ohms. What is the range of each voltmeter? What will each instrument read if the combination is placed in series across a 350-volt supply?

9. Inside the cover of a 150-volt d-c voltmeter is the information "100 Ohms Per Volt." A multiplier for the instrument has 4 separate resistances and 5 terminals marked 0 (common terminal), 2, 3, 5, and 8 which are multiplying factors. What is the ohmic value of each resistance? What additional voltage ranges are provided by the multiplier?

10. A 150-volt d-c voltmeter contains a total resistance of 14,550 ohms. A multiplier for it has 2 resistances, one of 14,550 ohms and the other of 43,650 ohms, connected to a common terminal. What should be the factors stamped on the terminals of the multiplier?

11. An a-c voltmeter has 3 voltage ranges of 150, 300, and 750 volts. The respective resistances at 25°C are 4,666, 9,326, and 23,332 ohms. What is the power loss in the instrument when placed across 230-volt lines? When placed across 450-volt lines?

12. An a-c voltmeter has a triple rating of 75, 150, and 300 volts. If the power loss is 3.92 watts when the instrument reads 117 volts on the 150-volt range, what values of resistance should accompany these three voltage ranges?

13. A d-c voltmeter and a millivoltmeter to be used with 60-mv shunts are arranged to measure the power taken by a pure resistance load. The voltmeter reads 150 volts full scale and contains 100 ohms per volt. If the load resistance is 120 ohms, and the millivoltmeter reads full scale with a 1-amp shunt, and the voltmeter is placed across both shunt and load, what is the percentage error in power by taking the direct product of the readings of voltage and current? Repeat for the voltmeter placed inside the shunt. Repeat for a load resistance of 0.8 ohm, the millivoltmeter reading full scale with a 150-amp shunt.

14. A wattmeter has current connections for 10 and 20 amp. The voltage terminals are marked for 50 and 100 volts, but the potential circuits can carry a 50% overload indefinitely. A multiplier having 2 terminals marked 0 and 300 volts, respectively, contains 3,774 ohms. What resistances should accompany the 50-volt and 100-volt potential circuits? What is the loss in the potential circuit when it is placed across 230-volt lines?

What is the loss with full current in the current coils if their resistance is 0.023 ohm for the 10-amp connection, and 0.0045 ohm for the 20-amp connection?

A single scale is graduated for 500 watts, and the multiplying factor is 1 for the 10-amp and the 50-volt ranges. What are the multiplying factors for the other combinations of current and voltage ranges, including the multiplier?

15. A wattmeter has current terminals for 2.5 and 5 amp, and voltage terminals for 75 and 150 volts, although the potential circuits can carry double voltage indefinitely. The resistances of the potential circuits are 4,958.4 and 9,916.8 ohms, respectively, at 25°C. What is the loss in the potential circuit when it is placed across 120-volt lines?

If the current circuits have resistances of 0.12 ohm for the 2.5-amp range, and 0.03 ohm for the 5-amp range, what is the loss when the instrument carries 1.8 amp? When it carries 4.5 amp?

A single scale is graduated for 500 watts, and the multiplying factor is 1 for the 5-amp and the 75-volt ranges. What are the multiplying factors for the other combinations of current and voltage ranges?

16. A wattmeter is rated for 30 60 amp maximum and 200/400 volts maximum although the voltage terminals are marked 150 and 300 volts, respectively. Three scales are graduated for 3, 6, and 12 kw. Tabulate the combinations of current and voltage ranges and the corresponding power scales to be used.

17. A low-power-factor wattmeter is rated 20 40 amp and 75/150 volts, with a multiplier for 300 and 600 volts. Full scale is 600 watts direct reading for the 20-amp and 150-volt ranges. Tabulate the multiplying factors for the other combinations of current and voltage, including the ranges covered by the multiplier.

What is the maximum power factor for 600 watts scale reading and for 20 amp and 150 volts on these ranges? For 600 watts scale reading and for 40 amp and 300 volts on these ranges? The power factor for 130 watts scale reading and for 11.8 amp on the 20-amp range and 14.5 volts on the 75-volt range?

18. A voltmeter and a wattmeter are arranged to measure the voltage and power of a circuit. The voltmeter has a rating of 75 150 300 volts and has resistances of 2,560, 5,120, and 10,240 ohms, respectively, in its potential circuits. The wattmeter is rated 5 10 amp and 150 300 volts, with resistances of 18,867 and 37,710 ohms, respectively, in its potential circuits. The voltmeter and wattmeter potential circuit are on the load side of the wattmeter, and the voltage is 144 volts. What value of power is indicated by the wattmeter with the load current zero?

An ammeter placed ahead of the wattmeter and voltmeter indicates 2.74 amp when the load is drawing current. What current is the load taking? The load voltage is still 144 volts.

19. A rectifier-type voltmeter is calibrated to read a-c volts and has a rating of 75 150 300 volts. Information on its scale states 1,000 ohms per volt. What current does it draw on 235 volts a-c? What indication will the instrument give on 120 volts d-c? What series resistances added externally to the voltmeter will allow one to use it for d-c voltages?

20. A rectifier-type voltmeter, calibrated to read a-c volts, has a rating of 75/150 300 600 volts and has a circuit resistance of 400 ohms per volt. What is the equivalent series resistance of a 500-ohm resistor having the 150-volt circuit of this instrument in parallel with it? Repeat for the voltmeter of problem 19. Repeat for an electrodynamometer-type voltmeter containing 3,000 ohms resistance. State the conclusions you draw with respect to the change in impedance of a circuit element having a voltmeter in parallel with it.

21. In order, between a power supply and a load, are a wattmeter current coil of 0.028 ohm resistance, the wattmeter potential circuit of 9,800 ohms resistance, an ammeter having 0.036 ohm resistance, and a voltmeter having 3,960 ohms resistance. The ammeter reads 9.65 amp and the voltmeter reads 117.20 volts. What is the loss in each instrument, and by what number of watts will the wattmeter read high?

22. The saturation curve on a 3-phase, 50,000-kva, 14,500-volt turbogenerator requires test results carried to 150% of rated voltage. A potential transformer with a 120:1 ratio and a 150 300-volt voltmeter are available. What will be the maximum reading on the voltmeter? What transformer ratio would permit use of only the lower scale of the voltmeter? What is the minimum ratio that can be used? What ratio is required of a current transformer for a short-circuit test on the generator reaching to 200% of rated current?

23. A special current transformer for testing purposes has primary terminals marked ± (common), 0.5, 1, 2, 10, and 25 amp. The secondary coil has a 5-amp rating. Give the current ratios as fractions or multiples of the output current. If an ammeter is to measure from 0 to 7.8 amp line current, with scale graduations below 1.5 amp unsatisfactory to read, what primary terminals should be used?

24. A switchboard wattmeter is designed for 5 amp and 100 volts. The scale reads full line kw, 1.0 pf, when used with a 20:1 potential transformer and a current transformer having a 60-amp primary. What is the full-scale reading in kilowatts? If the instrument is used without instrument transformers for regular laboratory work what multiplying factor is needed? If a new scale is made so as to give correct readings directly what is the full-scale value of power?

25. A 5-amp ammeter has an accuracy within $\frac{1}{2}$ of 1% of full scale at any part of the scale. What might be the possible percentage accuracy in reading 1 amp on this instrument? A wattmeter having 3 scales marked, respectively, 30, 60, and 120 hektowatts is stated to be correct within 0.4 of 1% without a correction scale, and to be correct within $\frac{1}{5}$ of 1% with a correction scale. What might be the possible error in readings at $\frac{1}{3}$ full-scale deflection?

26. A type SR-4 strain gage having a resistance of 120 ohms and a gage factor of 2 is cemented to a steel bar that has a modulus of elasticity of 30×10^6 psi. If the bar is subjected to a stress of 20,000 psi, calculate the change in resistance of the strain gage.

27. The d-c bridge circuit of Figure 18.13 is initially balanced. Each arm of the bridge has a resistance of 120 ohms and the battery voltage is 6 volts. When R_1 is increased by 0.2 of an ohm, calculate the open-circuit voltage developed across terminals A and B; if the galvanometer has a resistance of 200 ohms, calculate the galvonometer current (the use of Thévenin's theorem will simplify the solution of this part).

28. Two type SR-4 strain gages are used in a d-c bridge (Figure 18.13). Each gage has a resistance of 350 ohms and a gage factor of 2. The bridge is initially balanced with S at the midpoint of its dial and $R_3 = R_4 = 500$ ohms. After strain is applied to the active gage, balance is restored by moving S so that the resistance in one arm is increased by 0.04 and the resistance in an adjacent arm is decreased by the same amount. Calculate the strain.

29. A plastic rod $\frac{1}{4}$ in. in diameter supports a 150-lb weight. The modulus of elasticity of the plastic material is 450,000 psi. A gage of the type referred to in problem 28 is used to measure the strain. Calculate the change in resistance due to the strain.

30. For the phase-sensitive detector shown in Figure 18.14, show the following:

(a) When the voltage E_2 is zero, the d-c voltage across terminals A and B is zero.

(b) When the voltage E_1 is zero, the d-c voltage across terminals A and B is zero.

(c) When the voltages E_1 and E_2 are in phase, the d-c voltage of terminal A is positive with respect to terminal B.

(d) Reversing the phase of E_1 reverses the d-c polarity of the terminals A and B.

APPENDIX I

THE MAGNETIC CIRCUIT

Through his studies on electricity and magnetism the student by this time should have a knowledge of magnetism and magnetic effects, the terminology and symbolism pertaining to the magnetic field and the magnetic circuit, and some introduction to the procedures in making calculations on simple magnetic circuits. It is not the purpose of the following discussion to duplicate past subject matter but merely to present the few basic laws of the magnetic circuit from a general viewpoint and from them to approach typical engineering applications.

A1.1. Force on a Current Element. In 1820, Oersted made the discovery that an electric current affected a magnet in its vicinity. That is, a current gives rise to an inseparable magnetic field about it. Immediately following Oersted's discovery, Ampère investigated the force action between conductors carrying current. His experiments led him to the firm belief that all magnetic effects can be attributed to current flow, a viewpoint generally adopted today, and he conceived the effects of permanent magnets to be caused by atomic currents (electron motion) within the magnets. Ampère's experiments resulted in the laws governing mechanical action between currents and in the basic force equation:

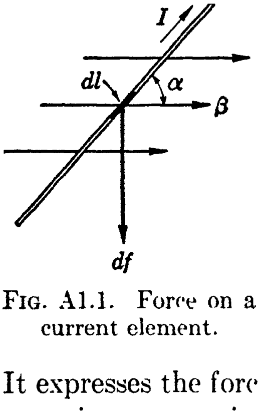


FIG. A1.1. Force on a current element.

$$df = \beta I\, dl \sin \alpha \text{ newtons.} \tag{A1.1}$$

It expresses the force on an elemental length, dl, of a conductor carrying I amperes in a magnetic field of flux density, β. α is the angle between the direction of the magnetic field and the direction of the current I. The direction of the force is perpendicular to the plane containing both β and dl, and the sense is as indicated in Figure A1.1. The rationalized mks system of units is used in all equations in the text part of this appendix. The student will recognize equations 2.3 and 2.4 as specific applications of equation A1.1.

A1.2. Magnetic Field Intensity. From Ampère's experiments we also are led to the general expression for magnetic field intensity. That is, the differential magnetic field intensity produced at a point P a distance r from an element of a conductor carrying I amperes is

$$dH = \frac{I\, dl \sin \theta}{4\pi r^2} \text{ amperes meter} \tag{A1.2}$$

where θ is the angle between the direction of the element dl and the direction of **r**. The direction of dH is perpendicular to the plane containing dl and the radius

r, and the sense is as indicated in Figure A1.2. Equation A1.2 is one form of Ampère's law. Ampère stated that experimentally one could work only with closed circuits; hence the resultant magnitude of the magnetic field intensity at any point has meaning only when integration is carried out over the complete circuit of the current. That is,

$$H = \int dH \text{ amperes/meter} \qquad \text{(A1.3)}$$

The vector nature of dH must be taken into consideration in performing the integration over the current path.

It will be noted that equation A1.1 expresses force action on a current in an existing magnetic field, whereas equation A1.2 indicates the strength of the field established by a current.

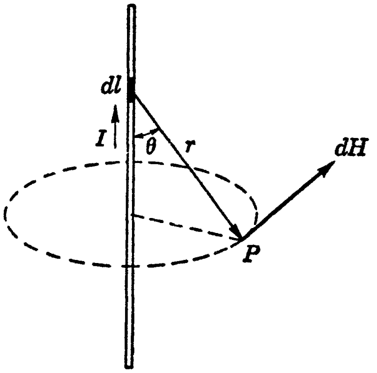


FIG. A1.2. Field produced by current element.

Ampère's law can be expressed, also, in the form

$$\oint H \cdot dl = I \text{ amperes} \qquad \text{(A1.4)}$$

where the symbolism means the line integral of H around a closed path and I is the total current enclosed by that path. The quantities under the integral form a *dot* product meaning that only the component of H in the direction of dl contributes to the value of the integral. This integral form of Ampère's law is perhaps most useful in treating magnetic circuit problems.

The relationship between the magnetic field, H, and the magnetomotive force, mmf, of a magnetic circuit follows directly from Ampère's law,

$$\text{mmf} = NI = \oint H \cdot dl \text{ amperes (or ampere turns)} \qquad \text{(A1.5)}$$

It is obvious that the integral of H around a closed path is zero unless current is encircled.

We can express the continuity of current and lines of flux by the equations

$$\begin{aligned} &\text{divergence } I = 0 \\ \text{and} \qquad & \qquad\qquad\qquad\qquad \text{(A1.6)} \\ &\text{divergence } \beta = 0 \end{aligned}$$

The first equation simply means that at any point in a conducting circuit as much current leaves that point as enters it. It is Kirchhoff's current law

$$\Sigma I = 0 \qquad \text{(A1.7)}$$

Divergence $\beta = 0$ means that for any small volume in space as much flux leaves the area enclosing the volume as enters it. An alternate statement is that flux lines are continuous—they neither emerge from nor terminate on any point source as do electric lines of flux. The above statements are true for a volume enclosing a magnet since magnetic poles always appear in pairs and experiment has not shown the existence of a *single magnetic pole analogous* to the concept of an electric charge. Furthermore, the magnetic flux is continuous within the magnet.

A1.3. Flux Density and Permeability. Magnetic flux density is the flux per unit area and is given by the equation

$$\beta = \frac{\Delta\phi}{\Delta A} \text{ webers square meter} \tag{A1.8}$$

where $\Delta\phi$ is the increment of flux passing through the incremental area, ΔA, that is normal to $\Delta\phi$. Conversely, the total flux through an area is

$$\phi = \int \beta_n \, dA \text{ webers} \tag{A1.9}$$

where β_n is the component of the flux density normal to the area dA. It is not an easy matter to compute ϕ for many magnetic systems, particularly those in which the flux is not confined to relatively simple geometric paths having a fairly uniform flux density.

For the same mmf, geometry of circuit, and flux configuration, the flux density of magnetic materials is many times that of the circuit having air only as the medium. The ratio of the flux density in the magnetic medium to the density that would be measured in free space is called the relative permeability, μ_r (a numeric), of the medium. For magnetic materials it has values ranging from a few hundred to 100,000 or more. It is unity for free space, and essentially this same magnitude for air. It is dependent upon flux density and can be evaluated only by test.

Flux density is related to magnetizing force by

$$\beta = \mu H \tag{A1.10}$$

$$= \mu_r \mu_0 H \text{ webers square meter} \tag{A1.11}$$

In the mks system of units $\mu_0 = 4\pi \times 10^{-7}$ henry per meter, the permeability of free space. The distinction between β and H lies in the fact that H depends solely upon the driving mmf of the circuit, whereas β depends upon the medium. Both quantities are vector quantities.

A1.4. Energy of the Magnetic Field. Employing equations 2.6 and 3.12, we have

$$W = \int_0^t ei \, dt = \int_0^t \left(N \frac{d\phi}{dt}\right)\left(\frac{Hl}{N}\right) dt$$

$$= \int_0^t \left(A \frac{d\beta}{dt}\right)(Hl) \, dt = Al \int_0^t H \frac{d\beta}{dt} \, dt$$

$$= Al \int_0^{\beta_1} H \, d\beta \text{ watt-seconds} \tag{A1.12}$$

which is equation 3.36, page 53. If both area and length are unity, the integral gives the energy density of the magnetic field—watt-seconds per unit volume.

It will be remembered from studies in physics that, as the magnetizing force on a sample of iron is carried progressively from zero to a positive maximum, then to a negative maximum, and so on, there is a lag of induction behind the driving mmf. This effect is called *magnetic hysteresis*, and the complete graph of the cycle is called the *hysteresis loop*. For an unmagnetized iron sample the curve starts at the origin, but once magnetized the iron generally retains some residual flux. Thereafter, the curve does not pass through the origin again unless the residual magnetism is reduced to zero. As discussed in Chapter XIII, for the self-excited d-c generator a small voltage obtained from residual magnetism is necessary if the machine is to build up its voltage.

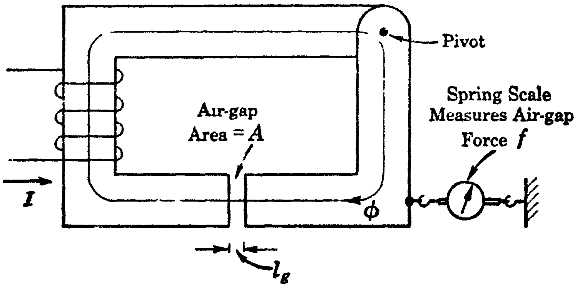


FIG. A1.3. Magnetic circuit, illustrating air-gap force.

Equation A1.12 is directly related to the hysteresis loop of the magnetic material in that, for one complete cycle, the integral gives the area of the loop which, in turn, is the hysteresis loss (heat) per unit volume per cycle. The nature of the hysteresis loop is determined solely by the kind of material, and the associated energy loss is obtained by test. This loss often is expressed as a certain number of watts per pound at a specified frequency and maximum flux density. The energy stored in the air gap of a tractive magnet (β and H are linearly related) is

$$W = \frac{\beta^2 A l_g}{2\mu_0} \text{ watt-seconds} \qquad \text{(A1.13)}$$

where A is the cross-sectional area of the core, and l_g is the length of the gap.

A1.5. Magnetic Force. Obtained directly from equations A1.1 and A1.4, the force per unit length between two parallel conductors carrying currents I_1 and I_2, separated a distance d between centers, is

$$F = \frac{\mu I_1 I_2}{2\pi d} \text{ newtons} \qquad \text{(A1.14)}$$

The derivation of this equation is left for the student. It is a very common practical application of the force equation.

As is well known from experience and experiment, a force of attraction exists between the faces of magnetic bodies carrying the same flux. The force tending to shorten the air gap (see Figure A1.3) can be determined from the energy equation (A1.13). If the length of the air gap were increased by a small increment, dl_g, the mechanical work done on the system would be equal to the increase of energy stored in the air gap provided that the energy in the rest of the magnetic circuit did not change and that no energy has been transferred to the electric circuit. These conditions would be fulfilled if the magnetizing current in the winding were increased, as the length of the air gap is increased, in such a manner that the flux in the magnetic circuit is kept constant. Under these conditions,

$$\text{mechanical work} = f\,dl_g = dW = \frac{\beta^2 A}{2\mu_0}\,dl_g$$

and

$$= \frac{\beta^2 A}{2\mu_0}\ \text{newtons} \qquad \text{(A1.15)}$$

Magnetic attraction between iron bodies exists only along the lines of flux. The rotor of a motor will have an outward radial pull over its entire lateral surface, but because of mechanical and magnetic symmetry the net outward pull is zero. Should the rotor be displaced axially within the stator, magnetic symmetry is lost and a resulting force is produced to pull the rotor back into normal running position. The force action exists at both ends of the machine and has the same direction at each end. Assuming a given displacement and constant flux, the starting pull can be calculated from the differential change in stored energy in the air gap.

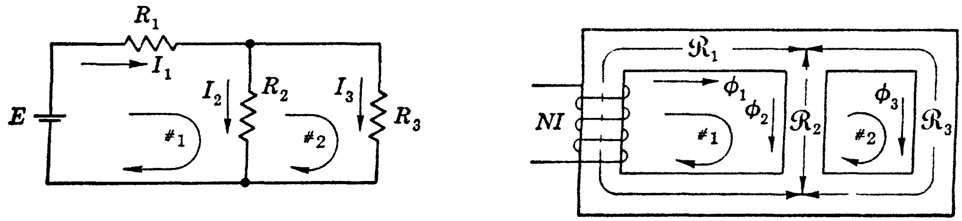
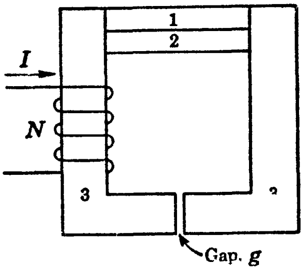


FIG. A1.4. Analogous electric and magnetic circuits.

A1.6. Electric and Magnetic Circuit Analogies. Treatment of the magnetic circuit shows the equations to be identical in form to corresponding ones for the d-c electric circuit. Further, the manner of calculating the over-all results for either system are the same as for the other. This mathematical equivalence is the basis for the analogous relations of the two systems. Figure A1.4 shows two analogous circuits, and Table A1.1 states the corresponding relations between them in both general terms and specific quantities for these two systems. The student will recognize the first two equations for the electric circuit in Table A1.1 as Kirchhoff's two basic laws for the d-c circuit. All of the analogous relations in the two systems follow from the fact that the two basic equations for the magnetic circuit are equivalent to Kirchhoff's laws.

A1.7. Example on the Magnetic Circuit. The circuit of Figure A1.5 carries the information given in the figure. For a coil of 800 turns carrying 1.5 amp, it is desired to know the flux densities in each material, the inductance of the



Material	Length	Area	μ_r
1	6 in.	0.5 sq in.	300
2	6 in.	0.5 sq in.	500
3	20 in.	1 sq in.	1000
Gap	0.05 in.	1 sq in.	1

FIG. A1.5. Magnetic circuit with air gap.

TABLE A1.1. ANALOGOUS RELATIONS BETWEEN ELECTRIC AND MAGNETIC CIRCUITS

Electric Circuit	Magnetic Circuit
I is continuous and $\Sigma I = 0$ at a junction	ϕ is continuous and $\Sigma\phi = 0$ at a junction
E = emf = ΣIR around a closed path. IR = difference in electric potential. Around loop #1, $I_1R_1 + I_2R_2 = E$ Around loop #2, $I_3R_3 - I_2R_2 = 0$	NI = mmf = $\Sigma\phi\mathcal{R} = \Sigma Hl$ around a closed path. $\phi\mathcal{R} = Hl$ = difference in magnetic potential. Around loop #1, $\phi_1\mathcal{R}_1 + \phi_2\mathcal{R}_2 = NI$ Around loop #2, $\phi_3\mathcal{R}_3 - \phi_2\mathcal{R}_2 = 0$
$I_1 = \dfrac{E}{R_t} = I_2 + I_3$	$\phi_1 = \dfrac{NI}{\mathcal{R}_t} = \phi_2 + \phi_3$
$R_t = R_1 + \dfrac{R_2R_3}{R_2 + R_3}$	$\mathcal{R}_t = \mathcal{R}_1 + \dfrac{\mathcal{R}_2\mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3}$
$R = \rho\dfrac{l}{A}$ = resistance	$\mathcal{R} = \dfrac{1}{\mu}\dfrac{l}{A}$ = reluctance
$\rho = \dfrac{1}{\sigma}$ = resistivity	$\dfrac{1}{\mu}$
σ = conductivity	μ = permeability
$G = \dfrac{1}{R}$ = conductance	$P = \dfrac{1}{\mathcal{R}}$ = permeance
J = current density	β = flux density
$J\rho l = IR$	$\beta\dfrac{1}{\mu}l = Hl = \phi\mathcal{R}$

coil, the energy storage of the circuit, and the pull between the faces of the air gap. Leakage flux and fringing at the air gap will be neglected.

The reluctances are calculated first, using conversion factors from Appendix III.

$$\mathcal{R}_1 = \frac{l}{A\mu} = \frac{6 \times 0.0254}{0.5 \times 6.45 \times 10^{-4} \times 300 \times 4\pi \times 10^{-7}} = 1.252 \times 10^6 \text{ amp/weber}$$

In like manner, $\mathcal{R}_2 = 0.752 \times 10^6$, $\mathcal{R}_3 = 0.626 \times 10^6$, and $\mathcal{R}_g = 1.565 \times 10^6$ amp/weber. From these,

$$\mathcal{R}_t = \frac{\mathcal{R}_1\mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2} + \mathcal{R}_3 + \mathcal{R}_g = 2.66 \times 10^6 \text{ amp/weber}$$

Hence

$$\phi_t = \phi_3 = \phi_g = \phi_1 + \phi_2 = \frac{NI}{\mathcal{R}_t}$$

$$\frac{800 \times 1.5}{2.66 \times 10^6} = 4.51 \times 10^{-4} \text{ webers}$$

$$= 45{,}100 \text{ lines (maxwells)}$$

From $\frac{\phi_1}{\phi_2} = \frac{\mathcal{R}_2}{\mathcal{R}_1}$, we obtain $\phi_1 = 16{,}900$ lines and $\phi_2 = 28{,}200$ lines. From these values for the flux,

$$\beta_1 = 33{,}800 \text{ lines/sq in.} = 0.523 \text{ weber/sq meter.}$$

$$\beta_2 = 56{,}400 \text{ lines/sq in., and } \beta_3 = \beta_g = 45{,}100 \text{ lines/sq in.}$$

At this point, the distribution of the magnetic potential drops follows readily: $H_1l_1 = \phi_1\mathcal{R}_1 = H_2l_2 = \phi_2\mathcal{R}_2 = 212$; $H_3l_3 = \phi_3\mathcal{R}_3 = 282$; $H_gl_g = \phi_g\mathcal{R}_g = 706$; and $(H_1l_1 \text{ or } H_2l_2) + H_3l_3 + H_gl_g = 1{,}200 = NI = 800 \times 1.5$ amp turns.

The inductance of the coil is

$$L = \frac{N\phi_3}{I} = \frac{800 \times 4.51 \times 10^{-4}}{1.5} = 0.24 \text{ henry}$$

The energy stored in the magnetic field is

$$W = \tfrac{1}{2}LI^2 = \tfrac{1}{2} \times 0.24 \times (1.5)^2 = 0.27 \text{ watt-second}$$

The pull between the faces of the gap is

$$f = \frac{{}_g^2A}{2\mu_0} = \frac{(45{,}100 \times 1.55 \times 10^{-5})^2 \times (6.45 \times 10^{-4})}{2 \times 4\pi \times 10^{-7}}$$

$$= 125 \text{ newtons}$$

$$= 28.2 \text{ lb}$$

A1.8. Magnetization Curves. The magnetization curve for a circuit is the graph, such as Figure 3.11, of the total flux, or flux density, plotted as a function of the magnetizing force. It is shown only in the first quadrant and is but the path followed in starting either from the origin for an unmagnetized circuit, or from some small residual flux if the circuit has been magnetized previously, and stopping at a chosen value of flux density. This part of the magnetization curve lies within the hysteresis loop; the return curve to zero magnetizing force is a part of the complete hysteresis loop.

The addition of very small amounts of other chemical elements to iron and steel may change the magnetic characteristics decidedly. Present-day steels produced for a variety of special applications, and having distinct magnetic properties, may contain any one, or perhaps some combination, of the following elements: chromium, cobalt, copper, manganese, nickel, phosphorus, silicon, tungsten, vanadium, etc. Further, the rolling and manufacturing processes, and heat treatment, also affect the over-all physical and magnetic characteristics of iron and steel. The magnetization curves shown in Figure A1.6 are merely representative.

Magnetization curves have several practical uses. A given flux density leads directly to the ampere turns per unit length of path for the particular kind of magnetic material and hence to the total ampere turns for that path. Again, the density can be ascertained for a given degree of saturation, and from this is obtained the associated area if the total flux is known. Further, the curves contain information on the relative permeability, μ_r, and the manner in which it varies with the degree of saturation. With a given total flux, and with materials having fixed dimensions and known magnetic characteristics, the total driving mmf for a circuit can be calculated, generally to a good degree of approximation considering the fact that there will be a certain amount of leakage flux.

Although the international mks system of units has been adopted widely, the coordinates of magnetization curves often are given in other units, for example, lines (or kilolines) per square inch and ampere turns per inch. Because these units appear in engineering practice, the scales for them are shown along with those of the mks system in Figure A1.6. For a given coil or set of coils acting on a magnetic circuit, the over-all magnetizing force is often expressed as the total ampere turns, or more simply as the exciting current in amperes.

In the case of electric machines, the magnetization characteristic usually is given by generated voltage as the ordinate and exciting current as the abscissa. For rated speed or frequency, the generated voltage is, of course, directly proportional to either the flux density or the total flux, and the exciting current is a direct measure of the magnetizing force. The volt-ampere curve so obtained is called the *saturation curve* for the machine. See Figures 3.7, 13.6, and 13.7 for example.

For most apparatus the magnetization curve is practically linear over the range from the voltage of residual magnetism to a rather high percentage of normal, or rated, voltage. The tangent, through the origin, to the face of the curve is called the *air-gap line* (see Figures 3.7 and 10.3), since the abscissa represents the excitation required primarily to force the flux across the air gaps. Some small part of this current, or corresponding number of ampere turns, is consumed, obviously, in driving the flux through the unsaturated iron of the magnetic circuit. Beyond the point at which the magnetization curve perceptibly leaves the air-gap line, *magnetic saturation* takes place. The horizontal separation between these two lines represents the increased excitation required to overcome that saturation.

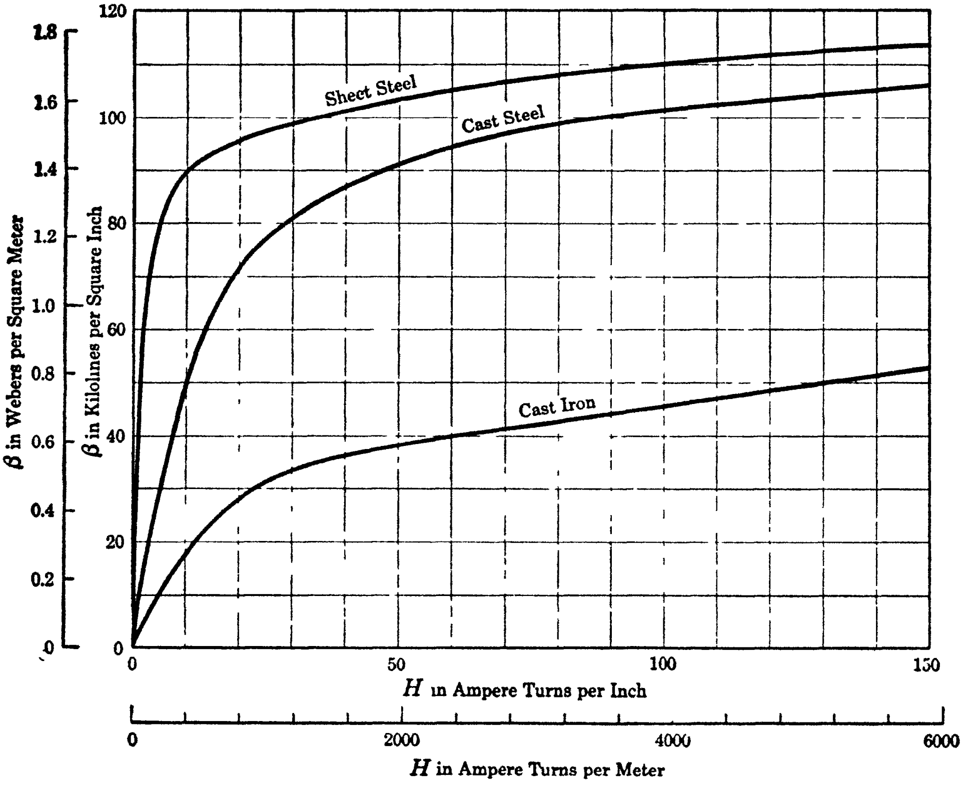


FIG. A1.6. Typical magnetization curves for selected materials

Practically all electric machines are worked with normal flux densities at, or slightly above, the bend or knee of the saturation curve. Densities just a little beyond the point at which saturation is apparent permit stability, while much above the knee of the curve far too great an excitation is required for a small increase in flux density or resulting generated voltage.

Figure 9.1 shows the cross sections of the 4-pole d-c motor, or generator, and the 4-pole synchronous machine. The field poles of either type of machine are marked as shown. For a given machine the field coils are identical and are so connected in series that the same d-c field current produces driving mmf's in opposite directions in alternate poles. The other magnetic part of each machine is the armature which is wound with coils also so connected that they give the same number of magnetic poles around the periphery as are on the field structure. Symmetry of mechanical construction is to give symmetry of the magnetic circuit, thereby giving balanced electrical performance.

A1.9. Problems on the Magnetic Circuit. The design of most electrical apparatus involves a study of the magnetic characteristics of the device together with calculations on the associated electrical circuit. The magnetic requirements may be a total flux or corresponding flux densities. The kind of magnetic material to be used, the degree of saturation desired, the lengths of paths, and the amount of air gap are all quantities that of necessity must be considered. The most usual type of problem is that in which the desired magnetic results lead to a certain number of ampere turns that must be provided. Electrical machines, which here include generators, motors, and transformers, are built to work at certain voltages and frequencies or speeds. This requires consideration of the total flux and hence densities throughout the magnetic structure. Relays, contactors, holding coils, circuit breakers, lifting magnets, etc., which are calculated mainly from the standpoint of magnetic pull, again involve the same items.

Once the driving mmf is found, the problem turns to the calculation of the best balance between turns and current. In the complete study, thought must be given to the operating voltage of the device (or perhaps it works on its own current as in the case of the circuit breaker and some holding coils), the amount of space that can be taken, the heat productivity of the coil and its allowable temperature rise, ventilation, kind of insulation, etc. In other words, the usual magnetic piece of equipment is not completed in design until everything else that goes with it also has been properly designed. For this reason, the design of magnetic apparatus is not so simple as many of the computations for the conducting circuit.

Magnetic problems are influenced in complexity by several factors. The most important, perhaps, is the nonlinear relation between flux density and magnetizing force for magnetic materials. Design must use the magnetization curves as they stand, usually through a step-by-step process, or one that requires cut-and-try methods, or that is to some measure graphical. Other complexities involve the calculation of leakage fluxes and the evaluation of fringing at gaps and pole faces. In addition, magnetic fields are three dimensional. The equations given in this appendix that can be applied to practical devices that use magnetic materials are, to some degree, approximations. How good they may be in any particular case depends primarily upon the geometry of the circuit. It must be remembered that good design merely has its start in written material. The rest comes later by way of actual practice in which experience and sound judgment enhance the art of the good designer.

PROBLEMS

1. A thin-walled cylindrical conductor is $\frac{1}{4}$ in. in diameter and carries 10 amp d-c. Sketch the field intensity from the axis of the conductor to a point 1 meter radially dis-

tant from the axis. Calculate the intensity on the axis and at the surface of the conductor.

2. Repeat problem 1 for a solid conductor $\frac{1}{4}$ in. in diameter. Calculate the total flux out to a distance of 1 meter for 1 meter length of conductor. What percentage of the total flux is within the conductor?

3. Two $\frac{1}{4}$-in. thin-walled cylindrical conductors form a parallel-wire transmission line with a separation of 1 meter between centers. For the line carrying 10 amp d-c, sketch the field intensity in the plane of the two conductors. The principle of superposition can be used in determining the net field from the two conductors. Calculate the value of the field midway between the two conductors and at a distance of 1 meter from the nearest conductor. Calculate the inductance per meter length of line.

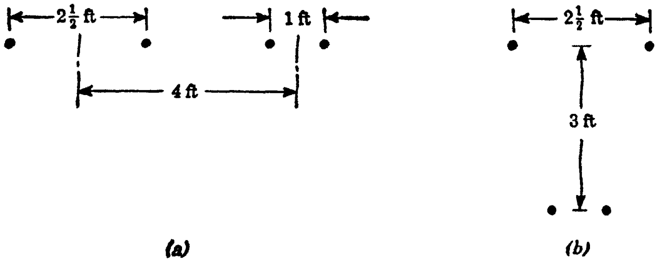
4. A concentric transmission line consists of two thin-walled conductors of $\frac{1}{4}$ in. and 1 in. diameters. Sketch the field intensity as a function of the distance from the axis of the conductors. For the line carrying 1 amp, calculate the total flux in the space between the conductors for 1 meter length of line.

5. The conductor of problem 2 is located in a uniform field of 1.2 webers per square meter and is at right angles to the field. Calculate the force per foot of length of conductor. For β in lines per square inch, l in inches, and I in amperes, show that equation A1.1 becomes

$$f = \frac{\beta l I}{11.3 \times 10^6} \text{ lb}$$

6. Two conductors, 0.375 in. in diameter, are spaced horizontally 3 ft between centers. For a 60-cycle current of 75 amp rms flowing in opposite directions in the wires, calculate the maximum flux between the conductors per mile of line. What is the rms induced voltage per mile due to this flux? What is the maximum force between the conductors per meter length of line?

7. A 60-cycle power line and an open-wire telephone line parallel each other. The power line uses No. 00 wire with $2\frac{1}{2}$-ft spacing, and the telephone line uses No. 10 wire with 1-ft spacing. For 200 amp in the power line, calculate the mutual inductance and the 60-cycle voltage induced in the telephone line per mile of line for the configurations shown in (a) and in (b).



(a) (b)

8. Develop the expression for the magnetic field intensity at a point on the axis of a circular loop of wire carrying I amperes.

9. Show that the ampere turns required for an air gap are

$$NI = 0.313\,\beta l$$

where β is in lines per sq in. and l is in inches.

10. From the magnetization curve for cast iron, calculate the relative permeability of the iron for a flux density of 40,000 lines per sq in.

11. A magnetic circuit 20 cm in length and 1 sq cm in cross-sectional area has a relative permeability of 200. Compute the flux carried by the bar for 1,500 amp turns.

12. A cast-iron ring of 1.5 sq in. cross-sectional area has a mean length of 30 in. For a total flux of 50,000 lines in the ring, how many ampere turns are required? What is the relative permeability of the iron under these conditions?

13. What is the permeance of 2 meters of sheet steel having 40 sq cm cross-sectional area for a flux density of 1.4 webers per sq meter?

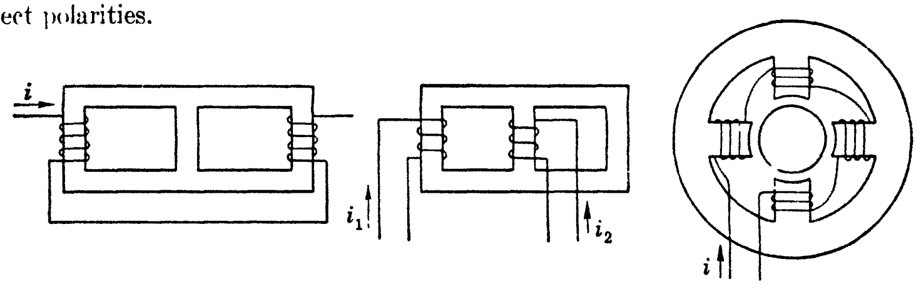
14. What is the over-all reluctance of two bars, 0.5 in. in diameter, one of cast iron and the other of cast steel, both 3 in. long, placed end to end, and carrying 6,500 lines of flux?

15. An 800-turn coil carrying I amperes drives flux through a series magnetic circuit of 15 in. iron and an air gap $\frac{1}{32}$ in. The core section is 0.75 sq in. and the relative permeability of the iron is 1,200. Neglecting fringing at the gap, what current must be carried by the coil for a flux density of 1.3 webers per sq meter?

16. An air-core solenoid of [illegible]00 closely wound turns has a mean length of 25 cm and a cross-sectional diameter of 2 cm. The reluctance of the magnetic path outside of this long solenoid can be neglected. How many amperes are required in the coil for a flux density of 10^{-2} weber per sq meter at the center of the flux path? Show that the field inside the solenoid is approximately uniform. Calculate the approximate self inductance.

17. A coil of 276 turns is wrapped uniformly around a wood toroid. The coil has a diameter of 4 cm and the core diameter is 16.5 cm. Compute the reluctance of the core and calculate the flux through the coil for a current of 3 amp. What is the mutual inductance between this coil and a second one of 276 turns wrapped closely on top of the first?

18. For the magnetic circuits shown, draw the analogous electric circuits showing correct polarities.

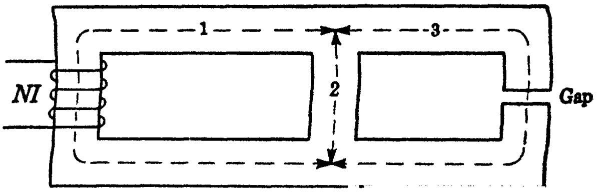


(a) (b) (c)

19. A cast-iron ring has a mean diameter of 15 in. and a section area of 1.5 sq in. Calculate the flux in the ring produced by a 1,500-turn coil carrying 1.75 amp. What is the relative permeability of the iron under this condition? Now assume the ring to be cut so as to provide an air gap of 0.20 in. What is the relative permeability of the iron for this condition?

20. A rectangular core, 8 in. by 6 in. outside dimensions, with 1 in. by 1 in. cross-section dimensions, is made of sheet steel. Stacking factor is 0.9. How many ampere *turns are required for a flux density of 100,000 lines per sq in.? If a gap of 0.25 in. in length* is cut in one side, what will be the new flux density in the circuit for the same number of ampere turns? Neglect fringing at the gap.

21. The cross-section of the magnetic circuit shown is 0.75 in. × 0.75 in. The material is cast iron. Path 1 is 14 in., path 2 is 4 in., and path 3 is 11 in. including the air gap of

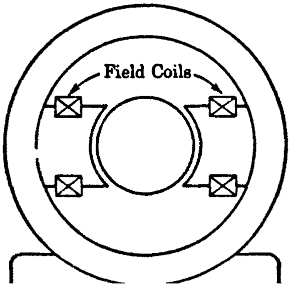


0.0125 in. Neglect leakage flux and fringing at the gap. For an air-gap flux of 6,000 lines, calculate the required ampere turns. Calculate the force between the faces of the air gap.

22. For the circuit of problem 21 and an mmf of 2,000 ampere turns, calculate the flux densities and the flux of each branch.

23. Sketch the cross-section of a 6-pole machine (d-c or a-c) and show the flux paths for normal mmf of the field coils.

24. The figure shows the cross-section of a 2-pole d-c machine having the dimensions for path length L and path area A as given below. Each pole carries a 1,200-turn coil.



An air-gap flux of 0.0165 weber gives rise to a generated armature voltage of 230 volts at rated speed. Plot the magnetization curve for the machine, that is, generated voltage as a function of field current. The field coils are connected in series. All magnetic material is of sheet steel except the yoke (frame) which is cast steel. Stacking factor of sheet steel is 0.9.

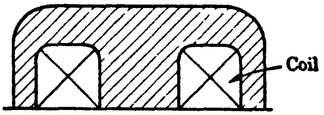
	L	A
Air gaps	0.125 in. each	25 sq in.
Teeth and core	6 in. total	28 sq in.
Poles	4 in. each	16.5 sq in.
Yoke (frame)	25 in. (half section)	10 sq in.

25. Refer to Section A1.5 and Figure A1.3. The initial dimensions of the air gap are 10 sq cm by 0.3 cm and the flux density is 0.8 weber per sq meter The air gap is increased by 0.1 cm, and during the process the current is varied to maintain the flux constant. Calculate the mechanical work done. Show that this work is equal to the increase of energy stored in the air gap and that it is also equal to the increase in $\frac{1}{2}Li^2$ of the circuit.

26. The air-gap flux density of a recording head consists of two components, a constant value of 0.5 weber per sq meter, and a periodic value of 0.02 sin $3{,}142t$ webers per sq meter. What three components of force act upon the armature and what are their relative magnitudes?

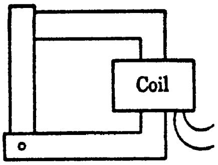
27. If the cast-iron ring of problem 19 has no gap but is cut completely through on a diameter of the ring, what is the force in pounds holding the two semicircular halves of the ring together? What total force is exerted between the halves of the ring if they are separated 0.1 in. at both cuts?

28. A lifting magnet such as shown has equal core and shell areas. The core diameter is 6 in. What steel-plate load can it lift for a flux density of 1.1 webers per sq meter?



29. A horseshoe magnet of cast steel has a path length of 20 in. through the steel. Its sectional area is 0.65 sq in. A 2,000-turn coil on the magnet carries 2.3 amp. Neglecting fringing, what is the pull of the magnet on a keeper placed 0.1 in. from the pole faces? The distance between centers of the pole faces is 5 in. Keeper and horseshoe have the same area and are of the same material.

30. The figure shows a magnetic circuit for a multi-switch motor starter. The total path length of iron is 10 in., and its sectional dimensions are 0.25 by 0.25 in. For a relative permeability of 750, what number of ampere turns are required to give a holding force of 4 lb?



APPENDIX II

APPLICATION OF THE OPERATOR j IN CIRCUIT ANALYSIS

A2.1. The Operator *j*. In general, voltages, currents, and impedances, when shown graphically, will have two components, one along the horizontal axis and the other along the vertical axis. Any such quantity then can be located by a point on a two-dimensional plane by giving its horizontal and vertical displacements measured from some chosen origin. Suppose that in the same sense that $+1$ and -1 are associated with horizontal displacements to the right and to the left, respectively, of the origin, we agree to let $+j$ and $-j$ indicate vertical displacements above and below the horizontal axis. Such a notation at this time is purely for convenience and to tell us the direction we are to take in going from the starting point of a vector to the other end of it.

Thus, for example, in Figure A2.1, there is shown the voltage $E_1 = +80 + j60$ volts in the first quadrant, and the voltage $E_2 = -50 - j85$ volts in the third quadrant. Note that in both cases the two components of each voltage are merely the sides of a right triangle, the hypotenuse of which is the actual magnitude of the voltage. That is,

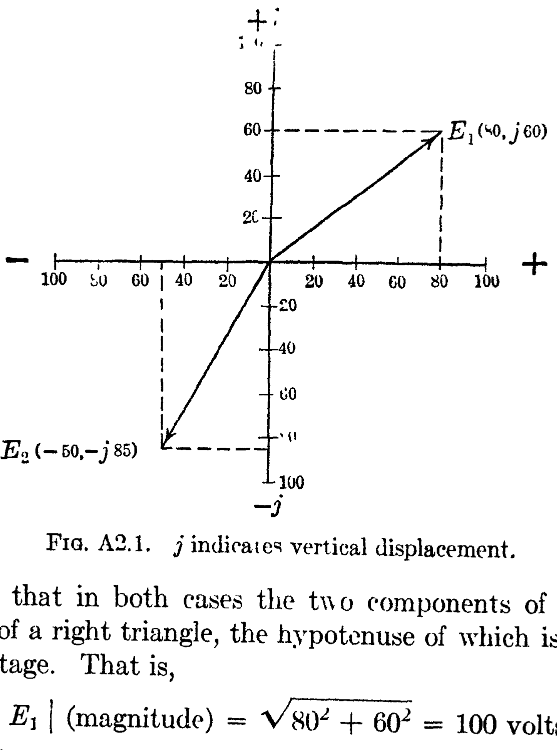


FIG. A2.1. j indicates vertical displacement.

$$|E_1| \text{ (magnitude)} = \sqrt{80^2 + 60^2} = 100 \text{ volts}$$

A current or an impedance would be shown in the same way in terms of horizontal and vertical components, and the magnitude calculated as we have for E_1.

Now observe also, as shown by Figure A2.2, that the voltage E_1 is located by laying off both +80 and +60 consecutively along the horizontal axis and then rotating the second component 90° in the counterclockwise direction. This idea immediately suggests the interpretation of $+j$ as an operator that rotates any line segment 90° in the usual positive direction, and $-j$ as the operator that rotates a line segment 90° in the negative, or clockwise, direction. Further, j^2 $(= -1)$ is a 180° operator involving two steps of 90° each.

Whereas j was introduced initially to show direction along the vertical axis, at this point it has taken on a more pronounced significance—that of an operator that rotates any radius vector forward through one right angle. In engineering it is usually so defined and applied. Since j now is identified with a rotation of 90° in the positive direction, and j^2 means movement through 180°, successive application of the operator means that j^3 $(= -j)$ is rotation through 270°, and that j^4 $(= 1)$ is a complete rotation of 360°. In this same sense, $\sqrt{j}$ is a forward movement of 45°, met in problems on communication networks. The scheme can be extended to any root or power of j, integral or fractional. Observe also that $(+j)(-j) = 1$.

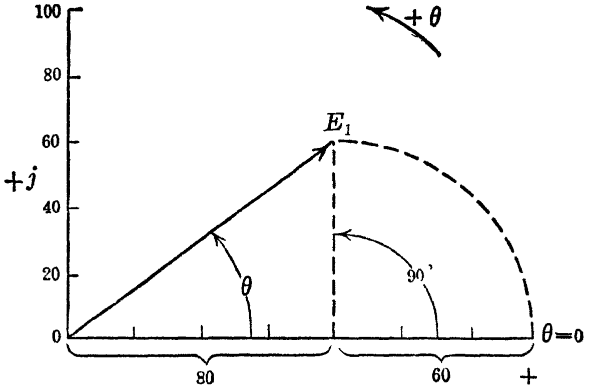


FIG. A2.2. j indicates positive rotation of 90°.

A2.2. Ways of Expressing Electrical Quantities. Comparisons of the expansions of $\epsilon^{+j\theta}$, $\epsilon^{-j\theta}$, $\cos\theta$, and $\sin\theta$ in Maclaurin's series show that

$$\epsilon^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

whence, for example, a voltage E having a horizontal component H and a vertical component V may be written in the following ways, with E_1 of Figure A2.2 used for illustration:

$$\begin{array}{llll} E & = H \pm jV & \qquad E_1 & = 80 + j60 \\ & = |E|(\cos\theta \pm j\sin\theta) & & = 100(\cos 36.9^\circ + j\sin 36.9^\circ) \\ & = |E|\epsilon^{\pm j\theta} & & = 100\epsilon^{j36.9^\circ} = 100\epsilon^{j0.64\text{ rad}} \\ & = |E|\underline{|\pm\theta} & & = 100\underline{|36.9^\circ} \end{array}$$

The first equation of the group expresses the voltage in rectangular coordinates as a complex number, so-called, in which the horizontal component is called a real number and the vertical component an imaginary number. It should be fully evident that the terminologies *real, imaginary,* and *complex* are purely technical. They simply are names given arbitrarily to these respective quantities by way of definition, and they carry no literal interpretation. The words *imaginary* and *complex* are perhaps unfortunate, but nevertheless they have been accepted in all scientific work and are used universally.

The second equation above shows the trigonometric form from which, in a great number of instances, the first form is obtained. Two electrical quantities of the same kind cannot be added to, or subtracted from, each other without first displaying them in the rectangular form, perhaps obtained from the trigonometric relation. The third manner of expression is exponential, or polar. The angle θ is to be given in radians for many computations and algebraic manipulations, although for convenience it often is shown in degrees.

The last form also is said to be polar and is read "E volts at angle θ." It is very useful, both in revealing quickly to the eye the magnitude and position of a quantity, and in calculations involving multiplication and division.

A2.3. Algebraic Operations. Electrical quantities involving the operator j follow all the fundamental operations of algebra in addition, subtraction, multiplication, and division, and in obtaining powers, roots, and logarithms. One should not forget that in any problem the algebraic solution and the geometric solution, by way of the associated vector diagram, are merely complementary. Applied to electrical circuits, algebra that includes the operator j often is called vector algebra. The basic algebraic steps that follow are described mainly by specific example. The procedures are general, should be self-evident, and are given without proof.

a. Equality of Quantities. Two quantities are equal only when their respective horizontal and vertical components are equal. For example, two impedances $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$ are equal only when $R_1 = R_2$ and $X_1 = X_2$. This simply means that a horizontal component cannot represent a vertical component. Note that the signs before the respective components also must be alike. In the polar form, two quantities are equal only when their respective magnitudes and angles are equal. Any quantity is zero only when both of its components are zero or when, in the polar form, the magnitude is zero.

b. Addition and Subtraction. In these two operations, after first expressing quantities in the rectangular form, respective horizontal and vertical components are added and subtracted independently. The final result then may be shown in the polar manner if desired. Thus, if $I_1 = 6 - j5$ amperes, $I_2 = 7 + j2$ amperes, and $I_3 = 4 - j9$ amperes,

$$I_1 - I_3 = (6 - 4) + (-j5 + j9) = 2 + j4 = 4.47\underline{|63.4^\circ}$$

and

$$I_1 - I_2 + I_3 = (6 - 7 + 4) + (-j5 - j2 - j9) = 3 - j16 = 16.28\underline{|-79.4^\circ}.$$

c. Multiplication and Division. In rectangular components, multiplication of two quantities is straightforward in that each term of the first is multiplied by each term of the second and like components then collected. If several quantities are to be multiplied together, they are taken two at a time. Division requires rationalization in order to remove the operator j from the denominator of the expression and leave it as a so-called real number only which then can be divided into each separate term of the numerator.

As an illustration, suppose that two impedances

$$Z_1 = 8 + j6 = 10\angle{+36.9^\circ}$$

and

$$Z_2 = 5 - j5 = 7.07\angle{-45^\circ}$$

are in parallel. The equivalent impedance of the combination then is

$$\begin{aligned} Z_{\text{eq}} &= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(8 + j6)(5 - j5)}{13 + j1} = \frac{70 - j10}{13 + j1} \\ &= \frac{(70 - j10)(13 - j1)}{(13 + j1)(13 - j1)} = \frac{900 - j200}{170} \\ &= 5.29 - j1.18 = 5.42\angle{-12.5^\circ} \end{aligned}$$

Observe that rationalizing simply means multiplying and dividing by the conjugate of the denominator, the conjugate of $(x + jy)$ being $(x - jy)$. The equality of the entire expression is not changed, because the ratio of the conjugate to itself is unity.

When multiplying in the polar form, the direct product is taken of the magnitudes, and the angles are added. Conversely, in division, the magnitude of the numerator is divided by the magnitude of the denominator, and the angle of the denominator is subtracted from that of the numerator. For the above example in polar form,

$$\begin{aligned} Z_{\text{eq}} &= \frac{10\angle{36.9^\circ} \times 7.07\angle{-45^\circ}}{10\angle{36.9^\circ} + 7.07\angle{-45^\circ}} = \frac{70.7\angle{-8.1^\circ}}{13 + j1} = \frac{70.7\angle{-8.1^\circ}}{13.03\angle{+4.4^\circ}} \\ &= 5.42\angle{-12.5^\circ} = 5.29 - j1.18 \end{aligned}$$

Observe that in the polar form addition and subtraction of quantities require the intervening step of expressing them in rectangular coordinates so as to obtain a single horizontal and a single vertical component, upon which one then may revert to the polar form. The final results, secured by either approach, must be exactly equal, and either type of solution can be converted readily to the other as shown by this example.

d. Powers and Roots. From DeMoivre's Theorem met in college algebra, the quantity $A\underline{|\theta}$ raised to the nth power is

$$(A\underline{|\pm\theta})^n = A^n\underline{|\pm n\theta} = A^n\epsilon^{\pm jn\theta}$$

$$= A^n(\cos n\theta \pm j\sin n\theta)$$

If the power is fractional, that is, $1/n$, then

$$(A\underline{|\pm\theta})^{1/n} = [A\underline{|\pm(\theta + 2m\pi)}]^{1/n} = A^{1/n}\epsilon^{[\pm j(\theta+2m\pi)]/n}$$

$$= \sqrt[n]{A}\left(\cos\frac{\theta + 2m\pi}{n} \pm j\sin\frac{\theta + 2m\pi}{n}\right)$$

where $m = 0, 1, 2, 3, \cdots (n-1)$. Thus for each succeeding root θ is increased by 2π, and is done so $(n - 1)$ times to obtain all the n distinct nth roots. All complex numbers have n roots equal in magnitude and evenly spaced by angle $2\pi/n$ around the circle of radius $\sqrt[n]{A}$ on the complex plane. The three cube roots of 1, for example, are $1\underline{|0^\circ} = 1 + j0$, $1\underline{|120^\circ} = -0.5 + j0.866$, and $1\underline{|240^\circ} = -0.5 - j0.866$.

e. Logarithms of Complex Numbers. As with powers and roots, this section is given only for completeness in the brief treatment on complex numbers. In obtaining logarithms of such numbers it is convenient to use the exponential form. Thus.

$$\log A\underline{|\theta} = \log A\epsilon^{j\theta} = \log A + j\theta = \log A + j\arctan\frac{y}{x}$$

where x and y are the horizontal and vertical components of A. As an example,

$$\log(4 - j3) = \log 5 + j\arctan(-0.75) = \log 5 + j1.796\pi.$$

f. Power in Symbolic Form. The application of symbolic notation to circuit problems at times requires special interpretation when results are to have particular physical significance. The illustration given here lies in the calculation of power. For example, a voltage $E = 120\underline{|60^\circ} = 60 + j103.91$ volts will force a current $I = 12\underline{|23.1^\circ} = 11.04 + j4.71$ amperes through an impedance $Z = 10\underline{|36.9^\circ} = 8 + j6$ ohms. The power taken by the circuit is 1,152 watts, with 864 reactive volt-amperes lagging, obtained from I^2R and I^2X_L, or from $EI\cos\theta$ and $EI\sin\theta$ where θ is the impedance angle.

In order to obtain these power components through a product of voltage and current in complex notation, it is necessary to multiply the current by the conjugate of the voltage. Thus,

$$(\text{conjugate of } E)(I) = (60 - j103.91)(11.04 + j4.71)$$

$$= 1{,}152 - j864 \text{ volt-amperes}$$

the so-called vector power for the circuit. This step has been known for many years and the development of it is given in Section 18 of Chapter IV.

PROBLEMS ON THE OPERATOR j

1. Evaluate the product $(+j)(-j^3)(-j)(\sqrt{+j})$ and express the result in 5 ways.

2. Evaluate a and b in the equation

$$10\underline{/-36.9^\circ} = (a + jb)(2 + j3).$$

3. Rotate the current $I = 7 - j3$ amperes forward through 35° and express the result in both rectangular and polar form. If this forward rotation yields a voltage of magnitude 115 volts as the product of the current and an impedance, calculate the impedance and show it in both rectangular and polar form.

4. Express the impedances $Z_1 = 3 - j5$ and $Z_2 = 6 + j2$ ohms in polar form. Calculate the resultant impedance of the circuit if they are in series; if they are in parallel.

5. The currents in 3 parallel branches of a network are $I_1 = 12\underline{/20^\circ}$, $I_2 = 8\underline{/-35^\circ}$, and $I_3 = 15\underline{/40^\circ}$ amperes. Calculate the resultant current. If the driving voltage is $118.2 + j20.8$ volts, calculate the equivalent impedance of the network. Calculate the vector power taken by each branch, and by the entire circuit.

6. A current $I = 3 + j2$ amperes flows through 2 impedances, $Z_1 = 20\underline{/30^\circ}$ and $Z_2 = 35\underline{/15^\circ}$ ohms, connected in series. Calculate the voltage across each impedance, and the voltage applied to the combination. Calculate the vector power taken by the circuit.

7. Calculate the current and vector power of a series-parallel circuit having a voltage $115\underline{/-30^\circ}$ volts applied. The series impedance is $20 + j15$ ohms, and the 2 in parallel have the values $35 - j20$ ohms and $25 + j30$ ohms, respectively.

8. A circuit takes a vector power of $500 + j200$ volt-amperes with a current of $5\underline{/15^\circ}$ amperes. Calculate the circuit voltage and impedance, and state the character of the impedance.

9. Express the 3 cube roots of $\sqrt{-j}$ in both polar and rectangular form. Locate the 6 sixth roots of $A = -32 - j55.42$ on the complex plane.

10. Express the voltage $e = E_m \sin(\omega t + \alpha + \theta)$ and the current $i = I_m \sin(\omega t + \alpha)$ in exponential form. Show that each component of voltage involves the conjugate of the other and is true also for the current. From the expression $p = ei$ for power, show that the average power is

$$P_{\text{avg}} = \frac{(\text{conjugate of } E)(I) + (E)(\text{conjugate of } I)}{2} = EI \cos\theta$$

where E and I are effective values. Show, also, that the average power is the in-phase component of (conjugate of E)(I) or (E)(conjugate of I). What interpretation would you place on this second product, so as to differentiate it from the first?

APPENDIX III

A. UNITS, DIMENSIONS, CONVERSION FACTORS

Newton: The unit of force in the MKS system of units. One newton gives a mass of one kilogram an acceleration of one meter per second per second. One newton = 10^5 dynes.

Joule: The unit of work or energy in the MKS system of units. One joule is the work done by one newton of force acting through a distance of one meter. One joule = one watt-second = 10^7 ergs. 1,000 joules = one kilowatt-second. 3.6×10^6 joules = one kilowatt-hour.

Watt: The unit of power in the MKS system of units. One watt is the power required to do work at the rate of one joule per second. One kilowatt = 1,000 watts.

Dielectric constant ϵ in the rationalized system of MKS units:

$$\epsilon = \epsilon_r\epsilon_0 \text{ farads per meter}$$

$$\epsilon_0 = \text{dielectric constant of free space}$$

$$= 8.854 \times 10^{-12} \simeq \frac{1}{36\pi} \times 10^{-9} \text{ farads per meter}$$

$$\epsilon_r = \text{relative dielectric constant (a numeric)}$$

Permeability μ in rationalized system of MKS units:

$$\mu = \mu_r\mu_0 \text{ henries per meter}$$

$$\mu_0 = \text{permeability of free space}$$

$$= 4\pi \times 10^{-7} \text{ henries per meter}$$

$$\mu_r = \text{relative permeability (a numeric)}$$

Velocity c of electromagnetic waves in free space:

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2\,998 \times 10^8 \text{ meters per second}$$

Charge q_e on the electron:

$$q_e = 1.602 \times 10^{-19} \text{ coulombs}$$

Mass m_e of the electron:

$$m_e = 9.107 \times 10^{-31} \text{ kilograms}$$

B. Dimensions of Electromagnetic Quantities

The quantities are expressed in terms of mass M, length L, time T, and charge Q.

Quantity	*Symbol*	*Dimensions*	*MKS Unit*
Capacitance	C	$M^{-1}L^{-2}T^2Q^2$	farad
Charge	q, Q	Q	coulomb
Current	i, I	$T^{-1}Q$	ampere
Dielectric constant	ϵ	$M^{-1}L^{-3}T^2Q^2$	farads per meter
Electric displacement	D	$L^{-2}Q$	coulombs per square meter
Electric field intensity	$\mathcal{E}$	$MLT^{-2}Q^{-1}$	volts per meter
Energy	W	ML^2T^{-2}	joule, or watt-second
Flux density	β	$MT^{-1}Q^{-1}$	weber per square meter
Force	F	MLT^{-2}	newton
Frequency	f	T^{-1}	cycles per second
Inductance	L	ML^2Q^{-2}	henry
Magnetic field intensity	H	$L^{-1}T^{-1}Q$	ampere turns per meter
Magnetic flux	Φ	$ML^2T^{-1}Q^{-1}$	weber
Magnetomotive force	mmf	$T^{-1}Q$	ampere turn
Permeability	μ	MLQ^{-2}	henries per meter
Potential difference	e, E	$ML^2T^{-2}Q^{-1}$	volt
Power	p, P	ML^2T^{-3}	watt
Reluctance	$\mathcal{R}$	$M^{-1}L^{-2}Q^2$	ampere turns per weber
Resistance	r, R	$ML^2T^{-1}Q^{-2}$	ohm

C. Selected Conversion Factors

To Convert	*To*	*Multiply by*
Length:		
meters	feet	3.281
inches	centimeters	2.54
Angstroms	centimeters	10^{-8}
Angular:		
radians	degrees	57.30
Area:		
acres	square feet	43,560
square inches	circular mils	1.273×10^6
square meters	square feet	10.76
square meters	square inches	1,550
Volume:		
cubic feet	gallons	7.481
cubic feet of water	pounds	62.43
gallons of water	pounds	8.345
Mass:		
kilograms	pounds	2.205
pounds	kilograms	0.454
Force:		
newtons	pounds	0.2248
newtons	dynes	10^5
pounds	newtons	4.45
Energy:		
watt-seconds	joules	1
watt-seconds	newton-meters	1
watt-seconds	ergs	10^7
kilowatt-hours	British thermal units	3,413
kilowatt-hours	horsepower-hours	1.341
British thermal units	foot-pounds	778

To Convert	*To*	*Multiply by*
Power:		
kilowatts	horsepower	1.341
kilowatts	British thermal units per minute	56.89
kilowatts	kilogram-calorie per minute	14.33
watt	newton-meter per second	1
watts	foot-pounds per minute	44.26
horsepower	watts	746
horsepower	foot-pounds per second	550
horsepower	foot-pounds per minute	33,000
Magnetic:		
webers	lines, or maxwells	10^8
webers per square meter	gauss, or lines per square centimeter	10^4
webers per square meter	lines per square inch	6.452×10^4

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